OPTIMAL ROYALTY RATE FOR COAL MINING IN COLOMBIA: AN EMPIRICAL ANALYSIS

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ABSTRACT

Revenue mobilization is one of the most important restrictions for economic growth in middle and low-income countries, given the high dependence on natural resources. Colombia is not an exception, and in fact, the current coal industry’s position contributes close to 1.6% of the GDP. As a consequence, the royalties systems and tax collection structure from 20 years ago are still valid. Yet, in the last few years, due to high coal prices, some arguments consider that the income produced by current mining and royalty rates is insufficient and should be increased. This paper evaluates the current Royalty Policy of Coal Mining in Colombia using a Mixed Integer Linear Programming Model. This strategy allows for the estimation of the optimal royalty rate under an approach that maximizes the present value of the revenue from private sector payments to the Colombian Government. The results show that under the current royalty scheme and assumptions applied, the royalty rate is quite close to optimal. However, the royalty system represents a huge risk to the development of exploration activities and mining projects given the lag of neutrality for investments, and approaches based on company profit offer interesting alternatives.

Keywords: Royalties, Colombia, Optimization

JEL Codes: H27, L72, C61
1 INTRODUCTION

Mining taxes and royalties have evolved from an approach in which individuals hold all rights to exploitation of renewable resources to one of extreme nationalism in which governments exercise their sovereign rights to impose taxes on an industry to promote the common welfare of its citizens. Regardless of the approach considered, natural resources are a key to economic development, and this situation calls for the need to build an adequate structure that allows governments to benefit from these resources and increase the welfare of the people. To this end, it is of utmost importance to adopt an optimal revenue structure that considers both the efficiency of the firm and government revenues.

This paper contributes to the existing literature by examining the current policy and practices of coal mining royalties in Colombia and develops a methodical application of the theoretical concepts developed by Otto, Cawood and Tilton 2006 on royalty and mining taxation. This methodology allows estimation of the optimal compensation to mining companies and to the government for exploitation of those resources as framed by maximizing the welfare of its citizens. The analysis is based on maximization of the net present value of the cash flow produced by revenues from private firms. In addition, the paper presents a review of the most common practices worldwide in compensation via coal royalties, an analysis of optimality of the current methodology for compensation for coal royalties in Colombia, a methodological proposal for the collection of these royalties, and estimation of the optimal royalty rate for coal mining in Colombia.

In recent years, it is generally agreed that the mining industry justifies special treatment for the sector due to large periods without revenues (exploration, pre-stripping\(^1\) and reclamation), large capital investments, cyclical revenues due to commodity prices, and substantial risks due to geological uncertainty. Therefore, the complementarities between the private and public sector in terms of revenue mobilization from natural resources should be exploited. Moreover, public policies should be designed to attract public and private investment to maximize welfare through the exploitation of new deposits and exploration of new prospects.

Colombia is one of the countries in which the nation has an important role in the coal mining industry worldwide. The Mining and Energy Planning Unit (UPME)\(^2\) has estimated that Colombia contains 0.8% of world coal reserves with 6593 Mton\(^3\), ranking in twelfth place worldwide, as shown in Figure 1. Colombia is the country with the largest reserves in Latin America and is located in a privileged place given its geological potential and its strategic location relative to the main consumption centers of Asia Pacific and North America.

\(^1\) Defined as the required movement required in the mine to expose the ore in the mine.
\(^2\) UPME Unidad de Planeación minero Energético.
\(^3\) Extracted from (Unit of Planning of Mining and Energy, 2012).
The purpose of this study is to evaluate the current policy of coal royalties in Colombia (methodology and rates) under an approach that maximizes the revenue from tax payments to Colombians and to propose a methodology aligned with this concept. The coal industry in Colombia has strongly contributed to the economic dynamics observed in the last decade, and its contribution to the Gross Domestic Product (GDP) has been growing. In 2011, these contributions accounted for 1.57% of the GDP, which was leveraged by a sustained increase in domestic production, an increase in global energy demand, and good prices in recent years, as shown in Figure 2.
In addition, exports of coal and Foreign Direct Investment (FDI) in the sector present similar behavior. According to the Colombian Central Bank, coal exports between 2001 and 2011 amounted to approximately 14% of total exports. Moreover, FDI in the sector is on the rise together with the development of deposits under exploitation and exploration of the territory. Between 2001 and 2011, the FDI in the sector was approximately M$US 17000, which is close to 20% of the total foreign direct investment in the country.

Over the last decade, coal mining has accounted for more than 75% of the royalties paid by the mining sector to the government. Between 2009 and 2011, coal mining paid 1.2 billion pesos per year on average to the Colombian government, whose resources represent half of the current agricultural budget of the country or the cumulative budget of the sectors of environment and development, culture, and science and technology. In addition, this amount could finance current government flagship programs such as the “100,000 free houses for the people”, as shown in Figure 3.

Figure 3. Royalties paid by Colombian mining

[Graph showing royalties paid by Colombian mining]


With these basic facts in mind, it is clear that the coal industry is of primary importance for Colombia’s development given its contributions to macroeconomic factors as well as its importance in terms of government revenue.

The structure of the paper is described as follows. Section 2 presents a summary of the development of Colombian royalty payments for coal mining since the early 20th century to the present. Section 3 describes the theoretical foundations of the main approaches to royalties worldwide and uses dynamic optimization theory as an alternative to understand and estimate the optimal rate for a given scheme. Section 4 develops a conceptual and mathematical model used in the estimation of an optimal royalty rate under different
schemes and summarizes the results. Section 5 presents the conclusions and policy recommendations based on the obtained results.

2 MINING ROYALTIES IN THE COLOMBIAN COAL INDUSTRY

2.1 Evolution of Coal Royalties

The mining of coal in Colombia began with the first steam railroads in the early years of the twentieth century. Subsequently, energy was consumed by the industrial sector, but the demand was small and was supplied by small mines in the center of the country. Electrical generation and the construction of the “Acerías Paz del Río” steel firm significantly increased the demand for coal and the production of mines located close to the projects and also introduced the first big jump in the coal industry in Colombia (Franco, 2007). In the late 1960s, almost all of the production of intermediate goods was based on coal-based energy.

In the early seventies, the global oil crisis, expansion of the electrical demand and a lack of clear policies for exploration and exploitation of oil created the need for a new energy policy. At that time, the country established development and exploitation of the potential coal deposits as the main energy strategy to supply the domestic demand, and the surpluses were marked for trade on the world market. As result of the promotion, dissemination and marketing of the coal potential of the country, at the end of this decade, the first large-scale coal mining contracts were signed for the northern region of Colombia.

Over eighty years, the interest of foreign capital in the Colombian deposits increased significantly, and huge investments in thermic coal projects in the departments of Guajira and Cesar significantly increased the production of the operating companies and exploration activities. In subsequent years, these sectors have continued with similar dynamics, and new capital has been invested in this area to maximize production and take advantage of the high price cycles, as shown in Figure 4.

Figure 4. Coal Production in Colombia.

The coal production of the country has subsequently set records over the last ten years in accordance with corporate strategies designed to take advantage of high prices supported by expansions of production capacity and the resources invested in exploration over previous years, as illustrated in Figure 4 and Figure 5.

**Figure 5. Coal Resources and Foreign Direct Investment (FDI) in Colombia.**

![Coal Resources and FDI in Colombia](image)

Source: Republic Bank, adaptation by Geoperforaciones and Mining LTDA, UPME

The coal industry in Colombia has invested its surpluses in improving processing technology and knowledge of deposits. As a result, new coal reserves have been discovered, thus creating value for stakeholders in the long term. However, is it necessary to continue this dynamic, which requires aligning fiscal policy with this aim. In the royalty case, a basic relationship must be determined between the coal royalty rate and the net present value of government income to determine the optimal policy of this aspect. A disproportionate coal royalty rate will decrease the amount of investment and produce an adverse effect on the net present value of government revenue, but a notably small rate will drastically decrease this effect.

### 2.2 Current Policy of Coal Royalties

The 1991 Colombian Constitution established in article 332 that the state owns the subsoil and non-renewable resources, and the articles 360 and 361 stated that the exploitation of these resources requires “economic compensation” favor of the state, i.e., royalties. The concept of “economics” framed the royalties in a social context. This instrument was designed and included as a constitutional mandate because its implementation would positively impact the welfare of individuals. However, it appears that the current regulation of the payment methodology is not aligned with this concept, and the economic nature of this instrument requires that the payment methodology and rates should reflect an optimization approach for citizen revenues, i.e., the payment must maximize the net
The present value of the benefits for the people, which must be analyzed over a long-term framework and not only focused on short-term interests.

Coal royalties are an example of this divergence. Law 141 of 1994 created the National Royalties Fund and the National Royalties Commission but also regulated the state rights to receive royalties from the exploitation of non-renewable natural resources and established rules for settlement and distribution. Article 16 of this Law, which was subsequently amended by Law 756 of 2002, set a royalty for the exploitation of coal as a national property, i.e., 10% if the exploitation exceeds 3 million tons per year or 5% if the exploitation is less than 3 million tons per year, as applied to the value of production in the Run of Mine⁴ or pit edge, as appropriate, which is summarized by the formula

\[
Royalty = P_d \times P_m \times P_l \quad (1)
\]

where

\( P_d \) = Mine production in a time period.

\( P_m \): Base price of raw mineral fixed by the Mines Ministry of Colombia⁵.

\( P_l \): Royalty percentage fixed by Law 141 of 1994 (modified by Law 756 of 2002), 10% or 5% depending on the coal production. Table 1 shows the royalty rate fixed by the Colombian government in Law 141 of 1994 (modified by Law 756 of 2002) for the main minerals.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Measure Unit</th>
<th>Royalty percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal productions &gt; 3MTon per year</td>
<td>%</td>
<td>10%</td>
</tr>
<tr>
<td>Coal productions &lt; 3MTon per year</td>
<td>%</td>
<td>5%</td>
</tr>
<tr>
<td>Nickel</td>
<td>%</td>
<td>12%</td>
</tr>
<tr>
<td>Iron and Copper</td>
<td>%</td>
<td>5%</td>
</tr>
<tr>
<td>Gold and Silver</td>
<td>%</td>
<td>4%</td>
</tr>
<tr>
<td>Platinum</td>
<td>%</td>
<td>5%</td>
</tr>
<tr>
<td>Salt</td>
<td>%</td>
<td>12%</td>
</tr>
<tr>
<td>imestone, Plaster, Clays y Gravel</td>
<td>%</td>
<td>1%</td>
</tr>
<tr>
<td>Radioactive Ore⁶</td>
<td>%</td>
<td>10%</td>
</tr>
<tr>
<td>Metallic Ore</td>
<td>%</td>
<td>5%</td>
</tr>
<tr>
<td>No metallic Ore</td>
<td>%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Source: Colombian Congress (1994)

The elements in the liquidation formula are transparent; its structure is quite simple and has been consistent for 20 years, thus demonstrating stability over time, which are characteristics that favor the administration and management of these terms. However, the law is not clear on the criteria used for the determination of rates of payment for a set of

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⁴ Run of Mine (ROM): Raw ore mined and transported to the limits of mine does not include processing.
⁵ The Ministry of Mines and Energy fix the price to be used in royalty liquidation through periodic resolutions.
⁶ Ore: Portion a mineral deposit that can be extracted with an economical benefit.
minerals; whereas such minerals as nickel and coal have higher rates exceeding 10%, the rates for other minerals mined in the country do not exceed 5%, which reveals inequities in the design of this parameter. Royalties paid by any coal-mining firm in Colombia are proportional to the value of production (revenue or rent), which reflects a lack of efficiency and neutrality. The World Bank (Otto, et al., 2006) has studied these types of royalties based on rent and argue that the absence of neutrality lies in the lack of importance that this methodology gives to whether “the mine is making a profit or losing money”. Royalties based on rent fluctuate following commodity prices, and thus, when prices are high, the government will enjoy more revenue than when prices are low (Otto, et al., 2006), which adversely affects the interests of the state in certain cases compared with royalties based on the profit of the companies, especially in times of low prices.

The current methodology for coal royalties assumes that all projects have the same economic reality, which varies widely in practice, unlike that of other businesses. Geological variables make each deposit unique, and therefore, the cost structure associated with mining and processing of minerals contributes to the concept of shared risk between different operators and the operators and the state, which is essential to attract investment capital to develop projects in the country. From the perspective of resource utilization, the current design of royalty payments for coal can distort decisions on production, cut off, and economical stripping ratios, and therefore the amount of reserve reported because all mines must pay regardless of whether they make a profit. In the early years of a production plan, when stripping ratios are low, the cash flow is impacted, but this impact is still positive. However, in the final years, when stripping ratios are high, cash flow is marginal and thus an extra variable cost can reduce the life of the mine and therefore the ore reserves.

From the elements of the liquidation formula, the percentage of royalty (royalty rate, Pl) is based on public policies; therefore, its application must be addressed to maximize the welfare of the citizens. However, given the intergovernmental nature of the mining business, its application cannot be based on nationalist arguments in which the government policy is removed in the short term and the most profit possible is collected at the cost of sterilizing the coal deposits in the medium- or long-term. However, the answer is also not the capitalist argument in which private investment becomes an end in itself and therefore the compensation received by the country in exploiting its coal resources is minimal. As summarized by J. Tilton (Tilton, 2004), “the sovereign states, mineral producing nations should pursue taxation and other policies that achieve, to the maximum extent possible, the goals and objectives they have for their mineral sector. For private companies, economics normally assumes that the goal, or objective function, is to maximize profits. Over time, this means maximizing the net present value of a firm or its wealth creation”.

7 Cut off: Concentration of the element of interest in the mineral at which mining activity becomes economically attractive.
8 Stripping ratio: Relationship between the number of units of waste material that must be removed to obtain a unit of mineral interest.
9 A mineral deposit could survive more than one government.
In Colombia, although certain studies and legislation have been carried out based on this issue, efforts have been oriented more towards the most efficient distribution of resources from the royalty payments, but few efforts have analyzed whether the current payment methodologies adequately represent the spirit that the constituents of 1991 attempted to capture in the articles written on this subject, which is to provide welfare to the owners of non-renewable resources, i.e., the citizens. For the specific case of coal, the law in this sense dates back to 20 years ago and even offers signs of our fiscal stability framework for investors in the sector and obscures efficiency optimization criteria in its structure. Therefore, it is necessary to analyze this situation from an economic point of view, understand these criteria, and if necessary, adjust the policies to the main objective, which is to produce welfare for the people.

3 MINING TAXATION AND OPTIMIZATION THEORY
3.1 Mineral Resource Rents and Taxation

The literature defines economic or Ricardian rents as the return to a firm or a factor of production over and above the amount expected by its owner, and therefore, it does not affect its economic behavior, i.e., it is “neutral”. Any taxation system will seek to collect rent from mineral resources exploitation due to its practical meaning and potential source of neutrality. This situation allows the owner of the resources to extract the rent without affecting production plans, which contribute to maximization of national welfare.

In the early 19th century, Henry George argued that Ricardian rents are particularly appropriate targets for taxation, in part because taxes on rents do not alter economic behavior and thus do not introduce inefficiencies into the economy.

Figure 6. Ricardian Rent on mining taxation

In addition, for the taxes shown in Figure 6 (Rent for Mine B), the rents seem fair or equitable. Mineral deposits differ in quality and incur different production costs, i.e., Mine C. The classes in Figure 6a depict different mines, and the horizontal axis measures the mine capacity. Mine A has the lowest production costs due to its rich ore and other factors. Mine B has the next lowest, and so on. For each mine, the area under the price line and
above its costs reflects its Ricardian rent. Figure 6b portrays the nature of the Ricardian rent associated with Mine B in additional detail and assumes that the mine’s costs (OCb) are its cash or variable costs of production. These costs reflect the minimum possible variable costs of production plus the portion of the potential rent that the government collects through taxes and other measures. If the price is at or above its cash costs (OCb), Mine B has (at least over the short run) an incentive to remain in operation. Consequently, at the price P2, is the mine earns a substantial amount of Ricardian rent. This rent can be divided into three components. The first (quasi-rent) component reflects the mine’s cost of capital and other fixed inputs and exists only in the short run. If the mine does not recover its cost of capital, including a competitive rate of return, it will not replace its capital, and in the long run, it will cease to operate.

Whenever a case arises in which it appears appropriate to tax a rent, one must be careful that the apparent rent is not what economists since Marshall have referred to as ‘quasi-rents’. Quasi-rents are payments that provide a certain incentive to maintain an economically valuable allocation of resources in the long term (Garnaut, Principles and Practice of Resource Rent Taxation, 2010).

The taxation on mining companies mostly affects these quasi-rents. A current mine will not be closed because a tax does not allow generation of a satisfactory return on exploration, but new exploration will be affected. In the same way, a new tax on a mine in operation could transfer part or all of the quasi-rent of the pre-stripping\(^\text{10}\) activities from mine to the government without affecting the production plan; however, it is possible that this action will affect new investments.

3.2 Methods of Mining Taxation

As mentioned in several papers, the mining sector contains unique features and differs from other businesses, i.e., “quite risky, capital intensive, prone to wide commodity price fluctuations, and in nations where mineral ownership resides with the state, exploits a component of the national patrimony”\(^\text{11}\), which cause its own tax treatment to represent these features; thus, in certain cases, legislation taxes this activity as royalties or other granting incentives that attempt to compensate for the risk inherent in such projects.

The literature argues for several criteria and classifications of taxes on mining activities around the world. Given the relevance of the application method for this work, we review two of the most relevant.

First (Garnaut & Clunies, 1983) identified six main forms of mineral rent taxation: 1) a flat fee in which the investor makes a once-for-all payment for the rights to extract minerals from a leased area, 2) specific or ad valorem royalties that are applied to the volume or value of production, 3) a higher rate of proportional profits or income tax that applies a

\(^{10}\) Initial overburden removal prior to the commencement of mining operations in an open pit.

\(^{11}\) Extracted from (Otto, 2000).
higher rate of taxation to income received from the corporate sector, 4) a progressive profits tax that applies a higher rate of income tax in tax periods in which the amount of income (usually calculated as the rate of return on a selected measure of investment in the project) exceeds a specified level, 5) a resource rent tax that allows a mining firm a deduction for all expenditures against revenue in the year in which the expenditure was incurred focusing on net cash flows, 6) a brown tax that is structurally similar to resource rent tax except that instead of any negative cash flows carried forward with interest, the negative cash flows attract a payment equal to the product of the tax rate and the amount of the negative tax flow.

In contrast, (Otto, et al., 2006) classified the mining taxes into two groups according to their application method: 1) In rem: Charges assessed against the mineral deposit or against the inputs and actions needed to exploit it; and 2) In personam: Charges against a given definition of net revenues, i.e., revenues less qualifying costs.

Each of these views have arguments for and against, yet there is agreement that because the rem type is charged only for the fact that the ore is mined, these taxes typically create additional distortions on the decision of operators relative to production plans, cut offs, and mineable reserves that affect the survival of projects, especially those with marginal cash flows.

The global mining sector has identified a representative group of taxes charged to investors. However, although these taxes seek a steady income and help to finance government spending, which is achieved in most cases, in others, it creates an unsustainable burden due to the lack of systemic vision of fiscal policies. Table 2 identifies a subset of policies and the objective behind collection.

**Table 2. Taxes Levied in the World Mining industry: Optimal mining taxation**

<table>
<thead>
<tr>
<th>Tax</th>
<th>Basis</th>
<th>In rem taxes</th>
<th>In personam taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit – based Royalty</td>
<td>Set charge per unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ad valorem–based royalty</td>
<td>% of mineral’s value (definition of value may vary)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% of value of sales</td>
<td>% of value of sales</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property or capital tax</td>
<td>% of value of property or capital</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Import duty</td>
<td>% of value of imports (usually)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Export duty</td>
<td>% of value of exports</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Withholding on remitted loan interest</td>
<td>% of loan interest value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Withholding on imported services</td>
<td>Inc % of value of services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value-added tax</td>
<td>% of the value of the good or service</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Registration fees</td>
<td>Set charge per registration event</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent or usage fees</td>
<td>Set charge per unit area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stamp tax</td>
<td>Set charge / transaction or % of value of transaction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income tax</td>
<td>% of income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital gains tax</td>
<td>% of profit on disposal of capital assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additional profits tax</td>
<td>% of additional profits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess profits tax</td>
<td>% of excess profits</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Net profits royalty or net value royalty
% of mineral’s value less allowable costs
Withholding on remitted profits
% of remitted value
Source: Author J. Otto

As mentioned by (Otto, et al., 2006) and discussed previously, the mining sector contains special characteristics that make it different from other sectors of economic activity:

- A lengthy period of exploration takes place during which there is no revenue.
- The amount of capital required during the development and construction phases is relatively greater than that of most other businesses.
- Once the mine is built, the capital is captive and not transportable.
- Equipment tends to be specialized and is available only from a few manufacturers worldwide, and thus, it must be imported.
- Mines can have long lifecycles and will be subject to regime changes and policy instability.
- Revenues are cyclical because commodity prices move up and down more so than is experienced by most other businesses.
- The scale of operations can be quite small or notably large.
- Large costs will be incurred at the time that the project closes (reclamation).
- Substantial costs unrelated to production may be incurred, i.e., investment in community infrastructure or programs.

The mining business is associated not only with geological risk (meaning that its ore reserves might be overstated) but also the risk involved in capturing returns in the long term. A favorable environment in investor decision-making will be one in which at least the risk associated with the time horizon of the investment is as controlled as possible, and the government and its fiscal policies (among others) play a decisive role in ensuring that situation.

What is the optimal allocation between the government and the firm for the revenue created by mining? The nationalist arguments say the government should receive the maximum possible percentage because citizens must be compensated for the exploitation of non-renewable natural resources. However, given the lucrative nature of private enterprise, its policy will also seek to maintain the highest possible percentage of this income. However, both private enterprise and government know that without the other, neither will derive benefits, and under this approach, the goals of private companies and the government are not necessarily opposed.

The government of mineral-producing nations should pursue taxation and other policies that achieve the goals and objectives of welfare of their citizens to the maximum extent possible; however, these policies should allow a level of profit for private enterprise that stimulates new investment not only in the current projects in operation but also in exploration of the potential of the country, which will allow for future revenues. High tax rates will mean that companies halt exploration activities, expansions, and establishment of new mines or close mines in operation, but low taxes minimize the benefits to the
government and citizens of the host country. Somewhere between those points exists a tax rate $T^*$ measured as a percentage of profits where the NPV of the benefit of the country's citizens is maximized, as shown in Figure 7. As the tax rate rises from zero to an optimal level ($T^*$), the net present value of the stream of current and future government revenues increases. However, once the optimal level is reached, any further increase in the tax rate will cause companies to cut explorations for new deposits, halt development of new mines, and eventually, if the tax rate is pushed high enough, close their existing mines (NPV of government is zero) (Tilton, 2004).

**Figure 7. Government Tax Revenues as a Function of the Tax Rate**

![Graph showing the relationship between tax rate and government tax revenues.]

Source: (Tilton, 2004)

3.3 Mining Royalties

The dynamic that influences all royalty agreement negotiations is governed by what David E. Pierce has stated as the "royalty value theorem" (Pierce, 2002), which is to say: If compensation under a contract is based upon a set percentage of the value of an item, there will be a tendency by each party to either minimize or maximize the value. Put another way, every royalty can be expressed as $x\% (a - b)$, where "$a$" is the aggregate of what will be included in calculating the royalty and "$b$" is the aggregate of all deductions in calculating the royalty, and the theorem compels the operator to minimize "$a$" and maximize "$b$"; of course, the royalty holder will maximize "$a$" and minimize "$b$".

Nevertheless, most of the royalties paid worldwide are based on the fact that in the exploitation of mineral resources (in rem), the method of payment is applied in two different ways: per unit of raw production (unit-based) or based on the value of the unit of run of mine ore (value-based or ad valorem), thus seeking to discriminate between the scales of operations. However, as mentioned by several authors (Otto, et al., 2006), such royalties may affect the flows of marginal operations in remote locations where they are the only source of employment. In contrast, many countries levy royalties based on profit or income, and those who agree will argue that royalties should be paid under the concept of "to pay". The concepts of profit and income seek to maximize the value of these items in
the long term; however, under this scheme, it is common to find pressures by governments because the profit margin is zero or negative in the early years of a project.

Table 3 provides a summary of coal royalties (rates and method) in several countries in the world and includes the legal origin of the royalty (national law, provincial law, or negotiated agreement), which is the predominant type of royalty.
### Table 3. Summary of Coal Royalty Practices in World regions

<table>
<thead>
<tr>
<th>Tax</th>
<th>Country</th>
<th>Africa</th>
<th>Asian and Pacific</th>
<th>Australia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Format</td>
<td>Botswana</td>
<td>National law</td>
<td>National law and negotiated agreement acts</td>
<td>National law</td>
<td>National law</td>
</tr>
<tr>
<td>Coal</td>
<td>Ghana</td>
<td>Mozambique</td>
<td>National law</td>
<td>Namibia</td>
<td>South Africa</td>
</tr>
<tr>
<td></td>
<td>Botswana</td>
<td>Namibia</td>
<td>South Africa</td>
<td>Tanzania</td>
<td>Zambia</td>
</tr>
<tr>
<td></td>
<td>Mozambique</td>
<td>National law</td>
<td>National law</td>
<td>Guidelines</td>
<td>National law</td>
</tr>
<tr>
<td></td>
<td>Namibia</td>
<td>National law</td>
<td>National law</td>
<td>Guidelines</td>
<td>National law</td>
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<tr>
<td></td>
<td>South Africa</td>
<td>Guidelines</td>
<td>National law</td>
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<td></td>
<td>Tanzania</td>
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<tr>
<td></td>
<td>Zambia</td>
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<td>Provincial law</td>
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<td>Chile</td>
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<td>Peru</td>
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<td></td>
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<tr>
<td></td>
<td>Venezuela</td>
<td>None</td>
<td>Provincial law</td>
<td>Provincial law</td>
<td>None</td>
</tr>
</tbody>
</table>

| Africa     | Botswana              | National law | National law and negotiated agreement acts | National law | National law | None |
| Asia and Pacific | China               | National law | National law | National law | National law | National law | None |
| Australia  | New South Wales       | Provincial law | Provincial law | Provincial law | Provincial law | Provincial law | None |
| Latin America | Argentina            | National law | National law | None | Provincial law | Provincial law | None |
|           | Bolivia               | Brazil | Chile | Dominican Republic | Mexico | Peru | Venezuela |
|           | Argentina             | National law | National law | None | Provincial law | Provincial law | None |
|           | Bolivia               | Brazil | Chile | Dominican Republic | Mexico | Peru | Venezuela |
|           | Brazil                | Chile | Dominican Republic | Mexico | Peru | Venezuela |
|           | Chile                 | Dominican Republic | Mexico | Peru | Venezuela |
|           | Dominican Republic    | Mexico | Peru | Venezuela |
|           | Mexico                | Peru | Venezuela |
|           | Peru                  | Venezuela |
|           | Venezuela             | None | Provincial law | Provincial law | None |

**Notes:**
- Ad valorem: At a rate per unit of production.
- Sliding scale: Rates vary based on the ratio of a production measure to a reference or market value.
- National law: Legislation passed by the national government.
- Provincial law: Legislation passed by the provincial government.
- Negotiated agreement: Agreement reached through negotiation between parties.
- Model agreement: Agreement based on a model framework.
- FOB: Free on board.
- Net back value: Value after deduction of costs such as production, transportation, and taxes.
- Tax rate: Percentage of value or profits subject to tax.
- Ad valorem, creditable against income tax: Rate that can be credited against income tax.
- Exported mineral: Royalty applicable to exported minerals.
- Small scale: Royalty applicable to small-scale operations.
- Ad valorem, sliding scale: Rates vary based on a sliding scale.
- Ad valorem, sliding scale based on annual cumulative sales: Rates vary based on cumulative sales.
- Ad valorem, sliding scale based on annual cumulative sales, 0–3% (exported mineral 1–3%): Rates vary based on cumulative sales with a lower rate for exported minerals.
- Ad valorem, sliding scale based on annual cumulative sales, 0–3% (exported mineral 1–3%; if no international price 1%): Rates vary based on cumulative sales with a lower rate for exported minerals.
- Ad valorem, sliding scale based on annual cumulative sales, 0–3% (exported mineral 1–3%; if no international price 1%; small scale 0%): Rates vary based on cumulative sales with a lower rate for exported minerals.
- Ad valorem, sliding scale based on annual cumulative sales, 0–3% (exported mineral 1–3%; if no international price 1%; small scale 0%; Venezuela): Rates vary based on cumulative sales with a lower rate for exported minerals.
<table>
<thead>
<tr>
<th>North America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Format</strong></td>
</tr>
<tr>
<td>Arizona (U.S.)</td>
</tr>
<tr>
<td>Provincial law</td>
</tr>
<tr>
<td><strong>Coal</strong></td>
</tr>
<tr>
<td>At least 2% of market price</td>
</tr>
</tbody>
</table>

Source: (Otto, et al., 2006)
3.4 Optimal Royalty Rate

As mentioned previously, every royalty can be expressed as \( x\% \) \((a - b)\), where \( x\% \) is the royalty rate, the definition of which is a component of the fiscal policy of the government, which must respond to the clear criterion of maximizing the present value of the welfare of its citizens.

*Inter-temporal choice and the NPV approach*

Assume a mining firm has estimated \( R \) units of mineral as its ore reserves and wishes to produce \( q_1, q_2, q_3, \ldots, q_t \) units of ore in each time period \( t \). Furthermore, suppose that the prices of production in each period are constant at 1, such that value of production each time period is \( q_1, q_2, q_3, \ldots, q_t \) and,

\[
R = q_1 + q_2 + q_3, \ldots + q_t \quad (4)
\]

Moreover, the amount of money that the firm will take in is \( m_1, m_2, m_3, \ldots, m_t \) units of money for each time period \( t \).

If we assume that the firm must borrow money at interest rate \( r \), the budget constraint of the firm is:

\[
q_1 + \frac{q_2}{(1+r)} + \frac{q_3}{(1+r)^2} \ldots + \frac{q_t}{(1+r)^{t-1}} = m_1 + \frac{m_2}{(1+r)} + \frac{m_3}{(1+r)^2} \ldots + \frac{m_t}{(1+r)^{t-1}} \quad (5)
\]

The production plan is affordable if the present value of production is equal to the present value of income (Varian, 2006). Rearranging equation 5:

\[
(q_1 - m_1) + \frac{(q_1-m_1)}{(1+r)} + \frac{(q_2-m_2)}{(1+r)^2} \ldots + \frac{(q_t-m_t)}{(1+r)^{t-1}} = \pi(q, t) \quad (6)
\]

The firm seeks the production pattern that maximizes the present value of its profit.

\[
V = \int_0^T \pi(q, t)e^{-rt}dt \quad (7)
\]

The taxation policy affects not only the choice of extraction rates \( q(t) \) but also the resources employed in exploration for new ore deposits. Assume that a certain exploration process at a cost of \( C(R) \) could augment the initial ore reserves\(^{12} \) (Thomas, 2010), and under this assumption, the economic behavior of the firm will be described by the function:

\[
V = \int_0^T \pi(q, t)e^{-rt}dt - C(R) \quad (8)
\]

Under this approach, the path chosen is not altered, although the present value of this production plan is now smaller.

\(^{12}\) Represents the present value of past exploration and development expenses at the moment of operation.
Optimization model

The mining operator’s problem involves the choice of a length of time T for the mine to operate and determination of an extraction profile (q(t)) to solve the problem (Heaps, 1985):

\[ \text{MAX } V = \int_0^T \pi(q, t)e^{-rt}dt - C(R) \quad (9) \]

Subject to the following constraints:
\[ q(0) = R, \text{ where } R: \text{Ore reserves} \]
\[ q(T) = 0 \]
\[ R(t) = R - q(t), \text{ where } q(t): \text{Extraction rate} \]

The Hamiltonian equation for the optimization is given by

\[ \text{Max } H = \pi(q, t)e^{-rt} - C(R) + \lambda(R - q) \quad (10) \]

The first-order conditions that characterize the operator’s optimal strategy are

\[ \left( \frac{\partial \pi}{\partial q} \right)e^{-rt} = \lambda \quad (11) \]

\[ \lambda - C(R) \quad (12) \]

Condition (11) states that discounted marginal profits should be the same at all points in time. Otherwise, ore extraction could be shifted from a time of low discounted profits to a time of high discounted profits, resulting in an increase in V, whereas condition (12) states that the shadow price of extra reserves should equal to the marginal cost of finding them (Thomas, 2010).

Further differentiating the previous equations, we obtain the slope of the extraction path, which is given by

\[ (\partial^2 \pi / \partial q^2)(dq/dt) = r \left( \frac{\partial \pi}{\partial q} \right) - (\partial^2 \pi / \partial q \partial t) \quad (13) \]

Rearranging, we obtain

\[ \frac{dq}{dt} = \left[ r \left( \frac{\partial \pi}{\partial q} \right) - (\partial^2 \pi / \partial q \partial t) \right] / (\partial^2 \pi / \partial q^2) \quad (14) \]

which is less than 0 unless the price of the refined product rises too rapidly. Extraction rates will decline over time.

If the royalty is σ(t) at time t, then the present value of a firm’s extraction plan becomes
\[ V = \int_0^T (\pi - \sigma q) e^{-rt} \, dt - C(R) \quad (13) \]

and \( V \) is maximized when
\[ \left( \frac{\partial \pi}{\partial q} - \sigma \right) e^{-rt} = \lambda \quad (13) \]

The slope of the extraction path is now given by
\[ \frac{dq}{dt} = \left[ \left\{ \left( \frac{\partial \pi}{\partial q} \right) - \left( \frac{\partial^2 \pi}{\partial q \partial t} \right) \right\} / \left( \frac{\partial^2 \pi}{\partial q^2} \right) \right] + \left[ \frac{\partial \sigma}{\partial q} \frac{\partial \pi}{\partial q} \right] \quad (14) \]

The slope of extraction path is now increased by the last term \( \left[ \frac{\partial \sigma}{\partial q} \frac{\partial \pi}{\partial q} \right] \). The extraction path with a royalty is therefore flatter than the no-tax extraction path because a firm will tend to extend the duration of extraction. The royalty reduces the exploration effort and causes the extraction rates to fall, at least initially. If marginal exploration costs increase rapidly, the royalty will not cause significant changes in exploration effort, and thus reduced extraction rates will lead to a longer mine life. If marginal exploration costs increase slowly, larger adjustments to the exploration effort will occur, and the mine life will be shortened (Thomas, 2010).

Based on the extraction rates obtained from solving the optimization problem proposed by Equation 13, it is possible to generate a number of simulations by varying the rate of royalties and obtain a functional relationship between this value and the net present value of government revenues.

### 4 OPTIMAL COAL ROYALTY IN COLOMBIA

#### 4.1 Optimal Royalty Rate

Assuming that the proven reserves of thermic coal in Colombia can be exploited by a “representative firm”, the setup of the problem is designed to find the production plan that maximizes the present value of the cash flow received from mining of mineral deposits subject to the restrictions of mineral depletion (non-renewable condition), the amount of the reserves and the operational and financial profile of the “representative mine” over the life of mine.

A block model\(^{13}\) is presented in which each block represents a proven reserve reported by the main producer regions (see Table 4). The model assumes an average overburden of 10.2:1 (see Figure 8).

\(^{13}\) Developed using the geo-statistical software “SGEM”.

---

[13]: Developed using the geo-statistical software “SGEM”.
Table 4. Proven Reserves of Colombian Thermic Coal

<table>
<thead>
<tr>
<th>Zone</th>
<th>Department</th>
<th>Proved Reserves (Mton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic Zone</td>
<td>Guajira</td>
<td>3695</td>
</tr>
<tr>
<td></td>
<td>Cesar</td>
<td>1771</td>
</tr>
<tr>
<td></td>
<td>Cordoba</td>
<td>378</td>
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</tbody>
</table>

Figure 8. Block model of Representative Coal Deposit

The financial and operational profile was extracted from the "Comparative Analyses of State participation in the Gold mines and Coal in Colombia", the information of which is based on mines located in South America, interviews with mine operators, projects in Colombia and the international guidelines for coal mines (ERNST & YOUNG, 2012), Table 5 summarizes the data used in the analysis.
Table 5. Operating profile of a Representative Coal mine

<table>
<thead>
<tr>
<th>Profile</th>
<th>Units</th>
<th>Open Pit Coal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Ore Production</td>
<td>Mton</td>
<td>250</td>
</tr>
<tr>
<td>Average Stripping ratio(^\text{14})</td>
<td></td>
<td>10.2:1</td>
</tr>
<tr>
<td>Processing Recovery(^\text{15})</td>
<td>%</td>
<td>92</td>
</tr>
<tr>
<td>Life of Mine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exploration Years</td>
<td></td>
<td>3</td>
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<tr>
<td>Execution Years</td>
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<td>3</td>
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<td>Operation Years</td>
<td></td>
<td>24</td>
</tr>
<tr>
<td>Capital Investment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial investment</td>
<td>MUS$</td>
<td>657</td>
</tr>
<tr>
<td>Exploration Investment % Initial investment</td>
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<td></td>
</tr>
<tr>
<td>Sustaining Capital % Initial investment</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Project financing</td>
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<tr>
<td>Interest rate on debt %</td>
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<td>Coal Price</td>
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</tr>
<tr>
<td>Long Term Price</td>
<td>US$/ton</td>
<td>90</td>
</tr>
<tr>
<td>Operating Costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mining &amp; Processing Cost US$/ Ore ton</td>
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<td>34.4</td>
</tr>
<tr>
<td>Selling Cost US$/ Ore ton</td>
<td></td>
<td>23.2</td>
</tr>
<tr>
<td>Reclamation Cost US$/ Ore ton</td>
<td></td>
<td>2.4</td>
</tr>
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</table>

An annual production rate of 250 Mtons was chosen to maintain the life of mine at less than 25 years. This rate assumes an operating cost adjustment factor over the life of mine of form \(t^\alpha\), where \(t\) is the time of the life of mine and \(\alpha\) is assumed as 0.1, which simulates the increasing maintenance and operating costs due to depletion of the useful life of the equipment and higher future energy and diesel costs.

To solve the optimization problem and estimate the coal production profile, we use Mixer Integer Linear Programming. This algorithm assumes that the deposits can be modeled as a set of blocks characterized by tons and qualities that can be sequenced to solve the previously discussed optimization problem, i.e., maximization of the net present value of the mine reserves subject to restrictions and mineral depletion. The formulation of the algorithm can be described as \(^\text{16}\):

Objective function: In this case the objective function seeks to maximize the net present value of the mine in the specific period

\(^\text{14}\) Stripping Ratio: Number of Waste Tons mined by each ton of Coal extracted.

\(^\text{15}\) it has assumed Coal needs to be washed.

\(^\text{16}\) The algorithm formulation has been extracted from (Rafiee & Asghari, 2008).
Total NPV = Max $\sum_{t=1}^{T} \sum_{i=1}^{N} V_i^t x X_i^t$ \hspace{1cm} (15)

$T$: Maximum number of scheduling periods,
$N$: Total number of blocks of mineral to be scheduled,
$V_i^t$: The NPV to be generated by mining block $n$ in the period $t$ and $V_i^t = R_i^t - C_i^t$
$R_i^t$: Revenue generated by mining block $n$ in the period $t$
$C_i^t$: Revenue generated by mining block $n$ in the period $t$
$X_i^t$: A binary variable equal to 1 if the block $n$ is to be mined in the period $t$; otherwise, 0.

If we assume $\sigma$: Royalty rate = 10% (thermic coal, production > 3 Mton per year) applied on a value basis (revenue), then the objective function for the “representative coal mine” will be:

Max $\sum_{t=1}^{T} \sum_{i=1}^{N} (R_i^t (1 - \sigma) - C_i^t) X_i^t$ \hspace{1cm} (16)

Subject to:

Reserves constraint: The constraints are constructed for each of the blocks such that all of the ore blocks considered must be mined once.

$\sum_{t=1}^{T} X_n^t = 1$ \hspace{1cm} (16)

Processing capacity: The total tons of ore processed cannot be greater than the processing capacity $PC_{max}$ in a period $t$.

$\sum_{n=1}^{N} O_n x X_n^t \leq PC_{max} = 250Mtons$ \hspace{1cm} (17)

$O_n$: Tonnage in the ore block

Mining capacity: The total of material (ore and waste) to be mined cannot exceed the total available equipment capacity ($MC_{max}$) for each period $t$.

$\sum_{n=1}^{N} (O_n + Wn) x X_n^t \leq MC_{max} = 2800Mtons$ \hspace{1cm} (18)

$W_n$: Tonnage in the waste block

Value-based royalty approach: Max $\sum_{t=1}^{T} \sum_{i=1}^{N} ((1 - \sigma) \times Revenue X_i^t - C_i^t) X_i^t$

Using the mining-planning package, which uses CPLEX\textsuperscript{17} algorithms to solve mixed MILP problems, we set up the objective function and constraints. Figure 9 shows the production plan for coal and the cash flow profile of the “representative mine” using the current royalty rate that maximizes the net present value.

\textsuperscript{17} CPLEX is an optimization software package designed to solve mathematical programming problems.
The theoretical optimal royalty rate for a mining firm is zero because this choice would maximize its cash flow, but the government would not receive revenues in this scenario. Indeed, the government expects the largest possible royalty rate to maximize its revenue. Once the optimal sequence is found, a set of simulations is carried out by varying the royalty rates applied to the optimization algorithm, each of is used to calculate the net
present value of the “representative mine” and taxes\textsuperscript{18} paid to the government. Figure 10 and Figure 11 show the cash flow profile of the “representative mine” and the discounted revenue received by the government for tax payments.

**Figure 11. Discounted revenue of the government**

![Discounted revenue plot](image)

**Figure 12. Net Present Value Revenues of the Government**

![NPV revenues plot](image)

Using the discounted revenue of the government, it is possible to derive a relationship between the royalty rate and the NPV of government revenues. Figure 12 shows this relationship.

\textsuperscript{18} To simplify the problem, we considered only the income tax (25\% of income tax and 9\% of CREE).
correlation and a polynomial regression, which has been estimated in R using ordinary least squares.

The regression model is:

$$NPV_{rg} = 11404 + 389829 \times RR - 2016223 \times RR^2$$

where $NPV_{rg}$ is the net present value of government revenues and RR is the royalty rate.

|         | Estimate | Std. Error | t value | Pr(>|t|) |
|---------|----------|------------|---------|----------|
| (Intercept) | 11404    | 2039       | 5.592   | 0.000338 *** |
| RR      | 389829   | 48819      | 7.985   | 2.25e-05 *** |
| RR^2    | -2016223 | 256312     | -7.866  | 2.53e-05 *** |

Table 6. Coefficients of Regression

Assuming $\alpha = 0.05$ and given p-value < 0.05, the variables (royalty rate) and (royalty rate)^2 are statistically significant in explaining the behavior of the NPV of government revenues. The F values and p-value of the regression show statistical significance.

Based on the function obtained and use of differentiation techniques, the calculated maximum occurs when the royalty rate is 9.67%, i.e., the government revenue reaches a maximum of MUS$ 30347 when the royalty rate is 9.67%.

4.2 Mining Royalty approach for Colombian Coal

As mentioned previously, most approaches to royalty payment can be grouped into three categories: unit-based, value-based and profit- or income-based (Otto, et al., 2006). To determine the best methodology for the NPV of government revenues and private cash flows, we select a subset of the most commonly applied approaches in the industry for royalty determination to calculate the coal royalty in Colombia; these choices were analyzed using the previous methodology. The alternative approaches chosen were:

- Profit-based payment
- Adjusted-profit-based payment
4.2.1 Profit-based payment

As stated previously, this methodology is based on the operational profit of a "representative mine". The objective function continues to maximize the net present value of the mine in a specific period

\[
\text{Total NPV} = \text{Max} \sum_{t=1}^{T} \sum_{i=1}^{N} V_i^t \times X_i^t
\]

T: Maximum number of scheduling periods,
N: Total number of blocks of mineral to be scheduled,
\(V_i^t\): The NPV to be generated by mining block \(n\) in the period \(t\) and \(V_i^t = R_i^t - C_i^t\)
\(R_i^t\): Revenue generated by mining block \(n\) in the period \(t\)
\(C_i^t\): Revenue generated by mining block \(n\) in the period \(t\)
\(X_i^t\): Binary variable equal to 1 if the block \(n\) is to be mined in the period \(t\); otherwise, 0.

However, if we assume \(\sigma\): royalty rate = 10% (thermic coal, production > 3 Mton per year) applied to a profit-based approach, then the objective function for the "representative coal mine" will be:

\[
\text{Total NPV} = \text{Max} \sum_{t=1}^{T} \sum_{i=1}^{N} (1 - \sigma)V_i^t \times X_i^t
\]

The constraint of this optimization problem is the same, i.e.:

\[
\sum_{t=1}^{T} X_n^t = 1
\]

\[
\sum_{n=1}^{N} O_n \times X_n^t \leq PC_{\text{max}} = 250Mtons
\]

\[
\sum_{n=1}^{N} (O_n + Wn) \times X_n^t \leq MC_{\text{max}} = 2800Mtons
\]

Figure 13 and Figure 14 show the cash flow profiles for a "representative mine" and the discounted revenue received by the government for tax payments.
Figure 13. Discounted cash flow of a Representative Mine

Figure 14. Discounted revenue of the government
Figure 15 shows the relationship between royalty rate and the NPV of government revenues as well as a polynomial regression, which was estimated in R using ordinary least squares.

![Figure 15. Net Present Value of the Revenues of the Government](image)

The regression model is:

\[
NPV_{rg} = 13063 + 78108 \times RR - 132039 \times RR^2
\]

where \( NPV_{rg} \) is the net present value of government revenues, and \( RR \) is the royalty rate.

### Table 7. Coefficients of Regression

|             | Estimate | Std. Error | t value | Pr(>|t|) |
|-------------|----------|------------|---------|----------|
| (Intercept) | 13063    | 1026       | 12.733  | 1.02e-08 *** |
| RR          | 78108    | 9785       | 7.982   | 2.29e-06 *** |
| RR^2        | -132039  | 19749      | -6.686  | 1.50e-05 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1559 on 13 degrees of freedom

Multiple R-squared: 0.8572, Adjusted R-squared: 0.8352

F-statistic: 39.01 on 2 and 13 DF, p-value: 3.209e-06

Assuming \( \alpha = 0.05 \) and given p-value < 0.05, the variables (royalty rate) and (royalty rate)^2 are statistically significant in explaining the behavior of the NPV of government revenues. The F values and p-value of the regression show statistical significance.
Based on the function, we calculate a maximum when the royalty rate is $29.58\%$, i.e., the revenue of government reaches a maximum of MUS$ 24614$ when the royalty rate is $29.58\%$.

### 4.2.2 Adjusted-Profit-based payment

Although this approach is based on profit firms, the payment base is adjusted by assuming a percentage of the operating cost. The current study assumes a $50\%$ operating cost, and thus the objective function can be defined as:

\[
\text{Total NPV} = \max \sum_{t=1}^{T} \sum_{i=1}^{n} (1 - \sigma) (R_i^t - 0.5C_i^t) X_i^t
\]

The constraint of this optimization problem is the same, i.e.:

\[
\sum_{t=1}^{T} X_n^t = 1
\]

\[
\sum_{n=1}^{N} O_n \times X_n^t \leq PC_{\text{max}} = 250Mtons
\]

\[
\sum_{n=1}^{N} (O_n + Wn) \times X_n^t \leq MC_{\text{max}} = 2800Mton
\]

Figure 16 and Figure 17 show the cash flow profiles of a “representative mine” and the discounted revenue received by the government for tax payments.

**Figure 16. Discounted cash flow of a Representative Mine**
Figure 17. Discounted revenue of the government

Figure 18 shows the relationship between royalty rate and the NPV of government revenues as well as a polynomial regression, which was estimated in R using ordinary least squares.

Figure 18. Net Present Value of the Revenues of the Government

The regression model is:

\[ NPV_{rg} = 11956 + 233405 \times RR - 794905 \times RR^2 \]

where, \( NPV_{rg} \) is the net present value of government revenues and \( RR \) is the royalty rate.
Table 8. Coefficients of Regression

|                | Estimate | Std. Error | t value | Pr(>|t|) |
|----------------|----------|------------|---------|----------|
| (Intercept)    | 11956    | 2000       | 5.978   | 0.000331 *** |
| RR             | 233405   | 36324      | 6.426   | 0.000203 *** |
| RR²            | -794905  | 139063     | -5.716  | 0.000446 *** |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 1

Residual standard error: 2466 on 8 degrees of freedom
Multiple R-squared: 0.849, Adjusted R-squared: 0.8112
F-statistic: 22.49 on 2 and 8 DF, p-value: 0.0005199

Assuming \( \alpha = 0.05 \) and given \( p \)-value < 0.05, the variables (royalty rate) and (royalty rate)^2 are statistically significant in explaining the behavior of the NPV of government revenues. The F values and \( p \)-value of the regression show statistical significance.

Based on the function, we calculate a maximum when the royalty rate is 14.68%, i.e., the government revenue reaches a maximum of MUS$ 29089 when the royalty rate is 14.68%.

Figure 19 illustrates that royalties based on production value require royalty rates that are less than those of the royalty approaches based on profit. The NPV of the government revenues are greater in an approach based on production value; there is no significant difference with the profit-adjusted methodology.

**Figure 19. Net Present Value of the Revenues of the Government**
5 CONCLUSIONS AND POLICY RECOMMENDATIONS

Coal mining is one of the most important areas of the Colombian economy, and the value chain contains many linkages with a significant number of other activities, which makes this activity one that generates wealth directly. Additionally, the transmission mechanisms support the generation of wealth from other lines of the economy. The Colombian government receives income from royalties, taxes and compensations given the externalities that this activity generates, which, for thermic coal, accounts for approximately a 50% share of the operational profits. Colombia applies a coal royalty based on production value; although it has demonstrated stability, this approach ignores the fact that whether the mine makes a profit is related to the joint venture concepts introduced by risk associated with the geological conditions of the ore deposit.

Using MILP, we estimated the optimal royalty rate under the current scheme of the coal royalty. The results are 9.67%, a value that is rather close to the value currently used by Colombian government, which is 10% for thermic coal with production greater than 3 Mton. This scenario suggests that the actual rate is an optimal payment rate, which is true under the assumptions used for the analysis but not necessarily under other conditions. In a possible scenario of low prices over the long term, government revenues would decrease but the income of companies would not necessarily increase because the company strategies to reduce operating costs seek to maintain the same profit performance, which reveals a clear inefficiency in the current approach.

The results indicate that the current approach to coal royalty payments maximizes the net present value of the government revenues; however, this approach represents a high risk that the income is not reinvested in the exploration of new areas and in the development of already discovered deposits. The approach based on the cash flows of private companies would return less revenue but nevertheless offer neutral conditions for investments by encouraging reinvestment in exploration and adding value in the long term due to a significant increase in the compensation taxes that companies should pay during different stages of its expansion projects (exploration, design, execution and operation). An interesting alternative to royalty payments for Colombian coal would be an approach based on profit by assuming a percentage of the operating costs, as shown in the model analyzed in this paper. This scenario represents a small difference in terms of the present value of government revenues with respect to the current scheme, but given the conditions of neutral incentives and efficient reinvestment in new areas of exploration, the alternative approach could create long-term value for the Colombian state.
6 REFERENCES


ERNST & YOUNG. (2012). *Comparative analysis of the state participation in Gold and Coal mining in Colombia*. Bogota DC: ERNST & YOUNG.


