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Understanding the Physical Optics Phenomena by Using a Digital Application for Light Propagation

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Abstract. Understanding the light propagation on the basis of the Huygens-Fresnel principle stands for a fundamental factor for deeper comprehension of different physical optics related phenomena like diffraction, self-imaging, image formation, Fourier analysis and spatial filtering. This constitutes the physical approach of the Fourier optics whose principles and applications have been developed since the 1950's. Both for analytical and digital applications purposes, light propagation can be formulated in terms of the Fresnel Integral Transform. In this work, a digital optics application based on the implementation of the Discrete Fresnel Transform (DFT), and addressed to serve as a tool for applications in didactics of optics is presented. This tool allows, at a basic and intermediate learning level, exercising with the identification of basic phenomena, and observing changes associated with modifications of physical parameters. This is achieved by using a friendly graphic user interface (GUI). It also assists the user in the development of his capacity for abstracting and predicting the characteristics of more complicated phenomena. At an upper level of learning, the application could be used to favor a deeper comprehension of involved physics and models, and experimenting with new models and configurations. To achieve this, two characteristics of the didactic tool were taken into account when designing it. First, all physical operations, ranging from simple diffraction experiments to digital holography and interferometry, were developed on the basis of the more fundamental concept of light propagation. Second, the algorithm was conceived to be easily upgradable due its modular architecture based in MATLAB® software environment. Typical results are presented and briefly discussed in connection with didactics of optics.

1. Introduction

The development of a suitable computational platform allows simulating optical experiments in which technical problems referring the laboratory conditions are not of concern or can influence the results under predetermined constraints. Indeed, in some cases, virtual or digital experiments, or a combination of digital and analog experiments, go further than the analog-alone setups, due to the combined advantages of digital information processing and storage capacity. Also the capacity to achieve digital results which are able to represent adequately real situations, in short time and by using a relatively simple routine, are highly appreciated characteristics associated with digital or virtual platforms for optics applications.

Although there are several computational platforms for light propagation simulation available in the market, due to the specificity of the experimental optics applications, some of these computational

tools are intended to fit a relatively narrow application spectrum, and the multi-purpose platforms are quite expensive. Connected with this, each new application in the field of simulation imposes new conditions on the algorithms, sometimes requiring alternative developments and modifications to be consistent with a new set of experimental parameters.

Similarly, some specific requirements must be accomplished to achieve a digital tool able to assist different learning processes, in particular in the optics field, but in this case the pedagogical pillars or principals of the didactics are also of concern, mainly because they are key factors determining the effectiveness of the complete learning-understanding process.

The scope of this paper is to present the implementation of a digital application, versatile and adaptable, easy to use and update, to be used for both simulating and experimenting with light propagation related phenomena, under different conditions. Indeed, the application is intended to be used as a tool assisting the process of learning and understanding a variety of physical optics concepts and phenomena, taking in advantage concepts and tools associated with the Fourier Optics approach.

This tool allows, at a basic and intermediate learning level, exercising with the identification of basic phenomena, and observing changes associated with modifications of physical parameters. It also assists the user in the development of his capacity for abstracting and predicting the characteristics of more complicated phenomena. At an upper level of learning, the application could be used to favor a deeper comprehension of involved physics and models, and experimenting with new models and configurations.

This software was developed based on the computational techniques of digital holography and its theoretical background, and the MATLAB® environment was used to implement it. The platform allows the user setting the parameters, constraints and initial conditions by using a friendly graphic user interface (GUI) that makes the application easy to use for almost any user, ranging from novices in optics and computation, to advanced users in both fields. Moreover, several algorithms to accomplish different tasks are integrated, making the application easier to be updated when needed.

In this paper, first the principals of our approach are outlined, starting from some considerations on the characteristics of the Fourier Optics (FO) and the Digital Optics (DO) fields. Then the principals on the diffraction and its integral formulation are considered, to establish the discrete model for propagation through the Discrete Fresnel Transform (DFT). Afterwards some technical features on the computational algorithm are discussed. Finally, a variety of simulated results are presented in connection with the role of the application in the field of learning and understanding physical optics.

2. Fourier optics and digital optics for didactics in optics

There are several prominent characteristics associated with the Fourier Optics (FO) that could favor the learning processes in the field of ondulatory optics, in order to achieve a deeper understanding of light phenomena. In this sense, we can associate with the Fourier Optics approach some adjectives highlighting both the scope and explicative capacity of the concepts and the mathematical structures involved; for example, when referring to the accuracy of meanings, natural and rigorous usage of mathematics, the inherent power of incorporating the theory of linear systems, wider range in the applicability of the concepts, the capacity to go deeper in the explanations, based on the scope and versatility of the theory, among others. Indeed, by focusing on what we are mainly interested, let us consider the case of free propagation and diffraction, to show how this approach allows us to understand a variety of different phenomena, not only from a unified perspective, but also on the basis of a theoretical model relatively simple, as well as a not too complicated mathematics.

In the ondulatory optics field, the basic notions of light propagation and diffraction are explained by incorporating the Huygens-Fresnel principle. By following this approach, when an ondulatory field travels through free space, it is possible to know the evolution of the wavefront by superimposing a number of spherical waves to be associated with each point onto a former wavefront. Furthermore, by using the information on the complex amplitude of the field concerning all the points onto a given input plane, we can evaluate the complex amplitude of the field after it propagates in free space through an arbitrary distance; this is, we can find the complex amplitude distribution onto an exit plane, parallel to the input plane, through the interference of all the spherical waves associated with the input. Moreover, these ideas are extended in a straightforward way to the diffraction explanation in the context of the Fourier Optics, and from the diffraction, in turn the same ideas can be extended to other contexts. This is because of the fundamental role that plays diffraction on the explanation of a number of ondulatory related phenomena, like the auto-imaging to be achieved by free propagation, the reconstruction of images by holographic methods or by using lenses, the filtering and image processing, and many others.

The first of the two pillars supporting our pedagogical approach is this unique capacity of the Fourier Optics to provide a comprehensive way of understanding a number of quite different phenomena, as was described, from the unified and relatively simple perspective of the Fresnel diffraction approximation.

The second pillar refers to the instrumental capacity of depicting a variety of results which are coherent with real experiments, in short time, avoiding the sometimes tricky laboratory work and the available equipment constraints, as a consequence of the usage of a convenient computational tool; which does not mean that we underestimate experimenting in the laboratory (through experiences in real conditions and with real resources). Indeed, we are conscious about the essential role of experimentation in the construction of the natural sciences (particularly in physics) and also on its significant role in teaching and learning sciences. However, nowadays the developments in different knowledge fields are favored by the capacity of achieving reliable simulations, and the experimental physics is not the exception. On the basis of the computation in optics, the scientific discipline known as digital optics unifies the theory, the methodology and the technical means for signal processing, making possible the study of optics phenomena in a flexible, simple and natural way [1].

By combining the theoretical approach of the Fourier Optics and the instrumental advantages associated with the usage of computational methods for learning and experimenting in optics, in our proposal we develop a digital platform, able to assist teaching and self-learning, as well as experimenting in optics.

3. Diffraction and Discrete Fresnel Transform

The light diffraction phenomenon was defined by Sommerfeld as the deviation of the light rays from its rectilinear paths that cannot be defined by the phenomenon of reflection or refraction [2]. Thus, if a wave front finds in its path an obstacle, which locally modifies its amplitude or phase, the wave will be diffracted. The diffracted light that propagates through the obstacle will interfere between them beyond.

The scalar diffraction theory concerns to the physical mathematics formalism describing the diffraction phenomena whenever the effects related with the polarization can be ignored. The light diffraction, as expressed on the basis of the Huygens-Fresnel principle, can be transformed in a simple expression by means of the Fresnel approximation; this leads to the Fresnel Integral Transform [3], expressing the complex signal at the output plane as a rescaled version of the Fourier Transform of the product of the complex field $E(\xi, \eta)$ existing onto the right of the input plane and the quadratic factor $\exp[(ik/2z)(\xi^2 + \eta^2)]$:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)} \int \int_{-\infty}^{\infty} E(\xi,\eta) e^{\frac{ik}{2z}(\xi^2+\eta^2)} e^{-\frac{2\pi i}{\lambda z}(x\xi+y\eta)} d\xi d\eta \quad (1)$$

where ξ and η are the coordinates at the input plane, x and y are the coordinates at the output plane, λ is the wavelength, k is the respective wave number, and z is the propagation through free space distance.

For computing the Fresnel Integral, the input and output planes are divided in small areas (pixels), all set in a matrix arrangement of dimensions $m \times n$, on which the transform operates. Be a matrix with dimensions N_x and N_y corresponding to the input plane $\xi\eta$, where all its pixels have height and width Δx_h and Δy_h in the directions ξ and η respectively; if we express the coordinates of the input plane as $k\Delta x_h$ and $l\Delta y_h$, and the coordinates of the output plane as $m\Delta x_i$ and $n\Delta y_i$, where k, l, m and n are entire numbers, the Fresnel Transform (1) turns into the Discrete Fresnel Transform (DFT), given by:

$$E_0(m,n,z) = \frac{iE_0 e^{-ikz}}{\lambda z} e^{\left[-\frac{i\pi}{\lambda z} \left(\frac{m^2}{N_x^2 \Delta x_h^2} + \frac{n^2}{N_y^2 \Delta y_h^2} \right) \right]} \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} H(k,l) e^{\left[2\pi i \left(\frac{mk}{N_x} + \frac{nl}{N_y} \right) \right]} \quad (2)$$

being

$$H(k,l) = h(k,l) e^{\left[-\frac{i\pi}{\lambda z} (k^2 \Delta x_h^2 + l^2 \Delta y_h^2) \right]} \quad (3)$$

where $m=0,1,2,\dots,N_x-1$, $n=0,1,2,\dots,N_y-1$, Δx_i and Δy_i are the dimensions of the pixels in output plane, E_0 represents the complex field that impinges onto the input plane, and $h(k,l)$ is in general a complex amplitude transmittance representing the optical elements modifying the incident field at the input plane. Some of these optical elements that were implemented are: Apertures, obstacles, diffractive gratings represented by both horizontal and vertical vectors, lenses, random diffusers, spiral phase plates, optical wedges, and images that can be load as amplitude or phase transmittance. An example of the optical elements arrangement is depicted in Figure 1, where in the input plane is superimposed a lens, an aperture and a random diffuser to obtain a speckle field at the output plane, each of the elements are simulated by using the complex function representing them.

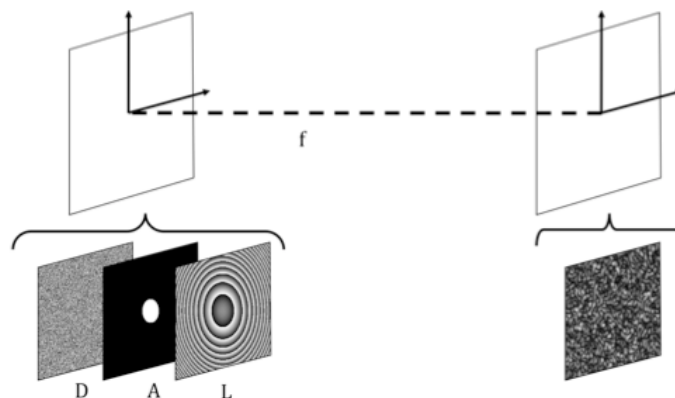


Figure 1. Schematized experimental set-up for light propagation. At the input plane (left), a set of different holographic optical elements which diffract the illuminating beam are superimposed, and at the output plane (right) the complex amplitude of the propagated field through any given distance f is observed.

The Discrete Fresnel Transform can also be expressed in terms of the Fast Fourier Transform, as follows:

$$E_0(m, n, z) = \frac{iE_0 e^{-ikz}}{\lambda z} e^{\left[-\frac{i\pi}{\lambda z} \left(\frac{m^2}{N_x^2 \Delta x_h^2} + \frac{n^2}{N_y^2 \Delta y_h^2} \right) \right]} FFT[H(k, l)] \quad (4)$$

where $FFT[H(k, l)]$ represents the Fast Fourier Transform to be evaluated in the frequency coordinates $f_x = k \Delta x_h / \lambda z$ and $f_y = l \Delta y_h / \lambda z$ [26].

4. Software proposal

The simulation of the light propagation by means of the Discrete Fresnel Transform implies the implementation of algorithms that are commonly used for digital holography. Although powerful, these algorithms are not always easy to use and setup. On the other hand, the software intended for didactics needs to be affordable for all kind of users. In our case, this is achieved by the implementation of a graphic user interface (GUI) to input the parameters and setup the conditions in the algorithm to run. Moreover, in our proposal the usage of the software resembles the hands-on laboratory work, because of the order in which the parameters must be introduced, as it is apparent from the following paragraph.

It is assumed that a monochromatic wave impinges onto the input plane, and the user must select its wavelength and the shape of the wavefront, for a plane or a spherical wave, and in the last case, the radius of curvature (positive or negative for convergent or divergent waves) also must be specified. Also the dimensions of the area that will be illuminated must be chosen given the options by default. At this point, a set of optical elements, including different apertures, wedges, lenses, diffractive gratings and different holographic elements are specified in order to synthesize the transmittance function at the input plane, which in turn diffracts the light. Afterwards, the propagation from the input to the output plane, through an arbitrary chosen propagation distance, is achieved; see Figure 2. The amplitude and phase-modulo 2π bi-dimensional distributions onto the output plane, with the corresponding line-profiles passing through the center of the distributions, are the algorithm outputs. Also the real and imaginary parts of the analytical signal at the output plane could be displayed if needed.

The developed software comes with the possibility to realize multiple propagation processes. It is useful when the simulations require several successive propagations, going through different optical elements located at arbitrary planes. For example, when forming an image by employing a single lens of focal length f , the first propagation refers to the travel of light going from the entrance plane to the plane located in front of the lens. In the second propagation, the lens (represented by a proper transmittance function) modifies the light that will be propagated up to the image plane.

In its actual state, the software allows the user saving the results of the propagation processes, both phase modulo 2π and the modulus of the amplitude, saving images corresponding to different optical elements, generating 3D plots from the optical elements and propagation results, obtaining digital holograms and unwrapping phases modulo 2π .

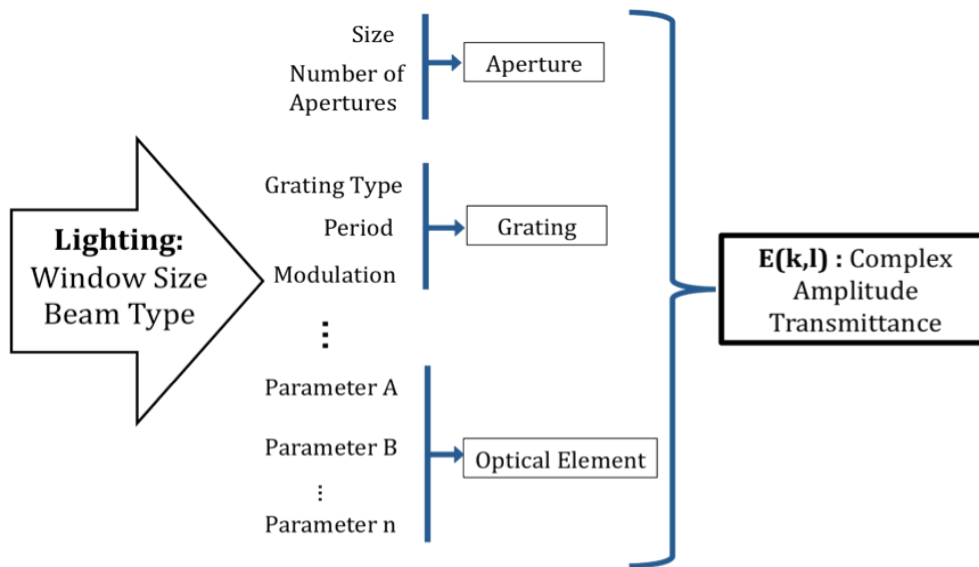


Figure 2 Input plane set up

5. Simulated results

The Going through learning levels by the simulation of different laboratory experiences, it implies to obtain results that resemble the reality, both from the qualitative and quantitative perspectives, passing from classical diffraction experiments to concrete applications of the digital and virtual optics. An insight of our typical results is given by modeling the Fraunhofer diffraction, Young experiment, diffraction through multiple apertures, image formation by lenses, digital holography and some optical vortex examples.

At a basic learning level, two classical experiences where modeled: Fraunhofer diffraction and Young experiment. Figure 3 presents the results of the light diffracted in Fraunhofer regime by a circular aperture whose diameter is 0.25 mm; the distance the light is propagated is 50 cm; it is shown in a) the amplitude module; in b) the phase modulo 2π ; in c) and d) a center line profile for a) and b), respectively. In e) it is depicted a line profile for the real amplitude, which is obtained by associating a sign change in the amplitude whenever a phase shifts of π radians occur passing from one to another consecutive ring onto de diffraction Airy pattern.

The Figure 4 presents the results of the Young interference experiment for light diffracted by 2 identical apertures with 0.25 mm of diameter, separated 0.50 mm; in this case the field is propagated a distance of 50 cm; it is shown in a) the amplitude module, in b) the phase modulo 2π , and in c) and d) the centered line profiles corresponding to a) and b). In e) it is depicted a line profile for the real amplitude, which is obtained by shifting the sign in the amplitude whenever the phase changes in π radians occur, both when going from one to another diffraction ring in the Airy pattern, and also when going from one to another interference consecutive fringe onto the diffraction pattern.

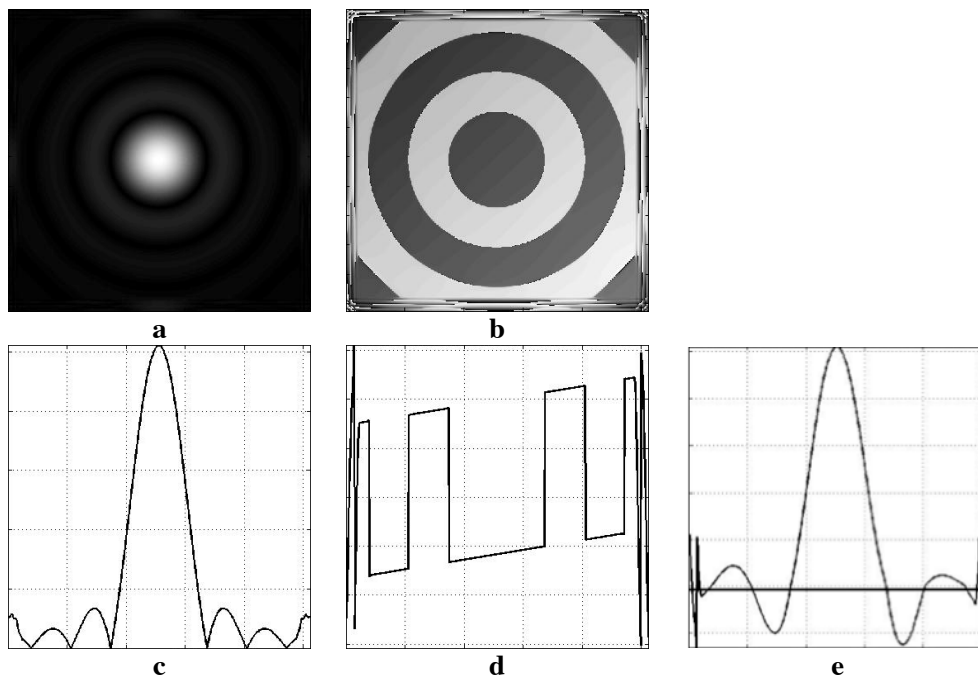


Figure 3. Light diffracted in the Fraunhofer regime from an aperture with 0.25 mm of diameter; light is propagated a distance of 50 cm. In: a) amplitude module; b) phase modulo 2π ; c) and d) centered line profiles for a) and b); e) real amplitude profile obtained from c) and d).

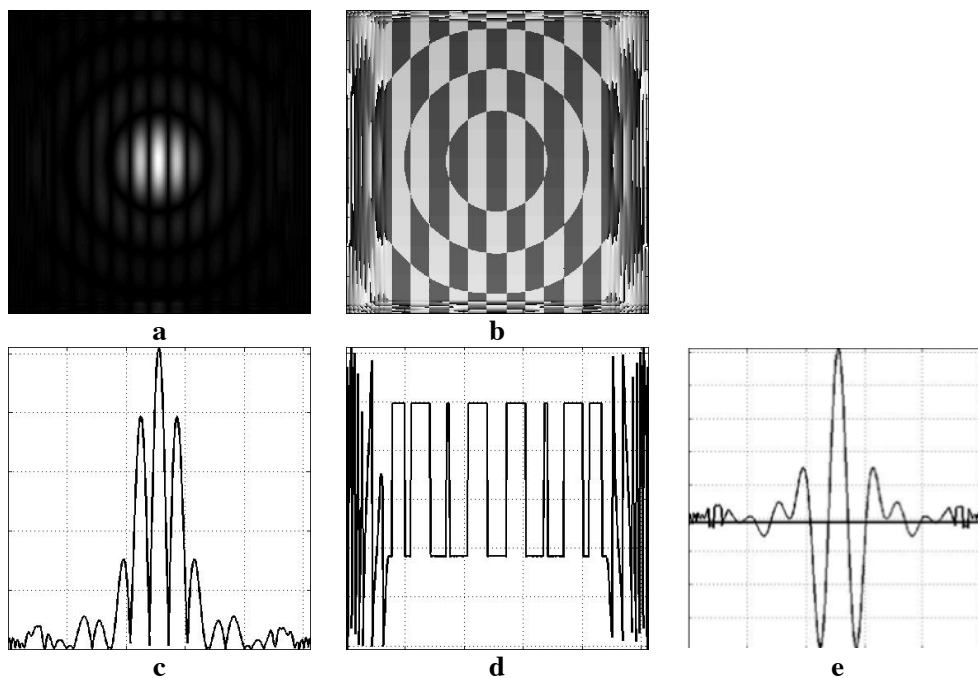


Figure 4. Young interference experiment, obtained from the light diffracted from 2 apertures with 0.25 mm of diameter and separated 0.50 mm, when light is propagated a distance of 50 cm. In: a) amplitude module; b) Phase modulo 2π ; c) and d) the centered profiles for a) and b); e) real amplitude profile obtained from c) and d).

The parameter variations allow the user obtaining a deeper understanding of the phenomena by means of the observation of the changes implied in these variations. To reach an intermediate learning level, the simulated experiences grow in complexity, which enables the user to do some predictions about the phenomena involved. An example of this consists of the propagation from the light diffracted by using multi-aperture pupils, leading to an interference experiment. In this case, the interference pattern comes from the superposition of all fringe systems associated with all the possible pairs of apertures in the pupil, whose characteristic are related by the Young experiment, but are not fully contained in it [4]. In Figure 5 are depicted the simulated results for light diffracted by pupils containing 8 and 12 apertures (left and center) respectively, in both cases for apertures of radius of 0.1 mm, whose centres are located onto a circumference of radius 1mm; and the diffraction produced by an annular aperture pupil (right) with a thickness of 0.1 mm, when light going through the ring is propagated a distance of 1.30 m. The radius of the internal circle is 0.9 mm.

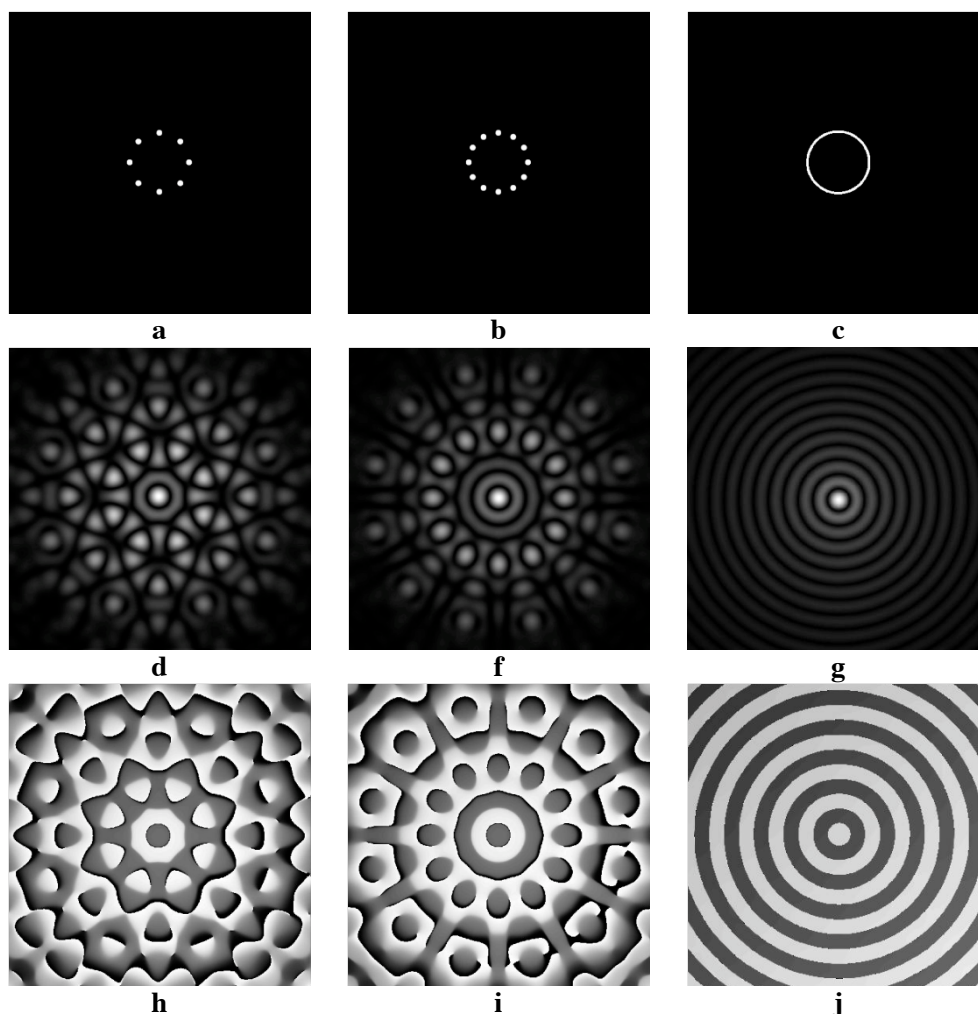


Figure 5. Simulated results for light diffracted by an: a) 8 apertures-pupil, b) 12 apertures-pupil, c) annular aperture pupil; the amplitude module for each experience are depicted in d), e), f), and the phase modulo 2π in g), h), i), respectively.

Another example we might use for the intermediate learning level simulation is the image formation by using a single lens. In this experience, three successive steps should be taken into account: first, the light diffracted from an image (which stands for the amplitude transmittance at the input plane) is propagated to reach the plane located just in front of the lens; second, light goes through the lens,

which is associated with a quadratic phase function; and third, light emerging from the lens is propagated again up to the output plane. Figure 6 presents a single lens imaging simulation; in the first row, a high quality image (left) is imaged with magnification -1; in the second row, the experience is repeated, but a speckled noise signal is added to the original image (left) as an insight of virtual optics applications.

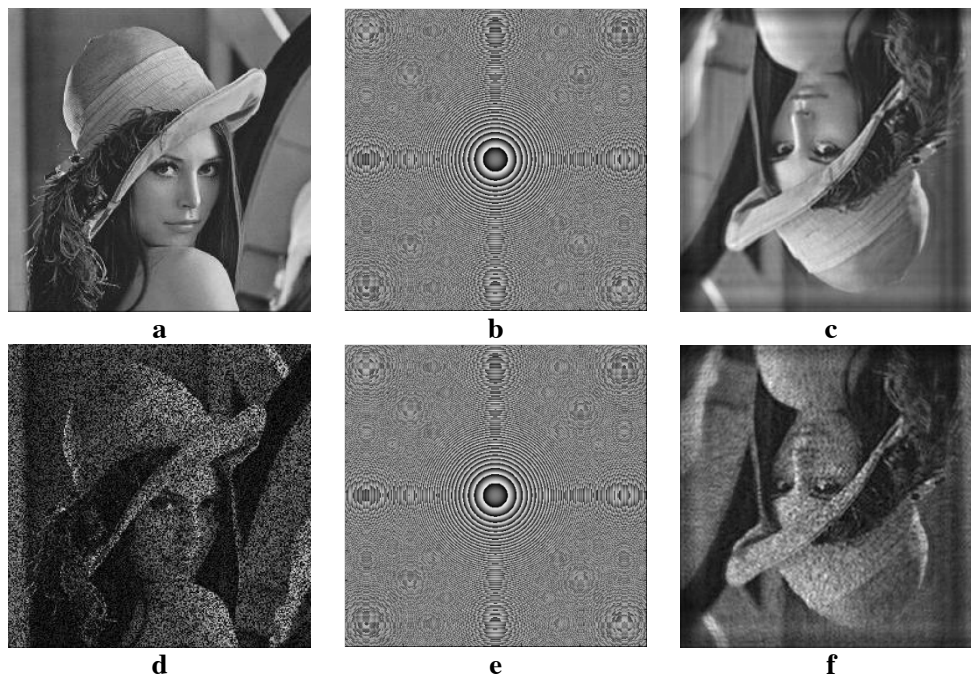


Figure 6. Single lens image formation, with both object and image distances equal 55 cm, and by simulating a lens (central row) with 275mm of focal length. In: a) the original high quality image, c) the reconstructed image. In d) a speckled image, f) the reconstructed image from d). In b) and e) the lens is represented in terms of a quadratic phase function.

Other way to resemble the real world in the simulations, in the perspective of approaching to virtual optics, is the introduction of optical aberrations in the lenses by means of Zernike polynomials; Figure 7 presents the experiment of focusing a coherent plane wave by using lenses with spherical (left), coma (centre) and astigmatism (right) aberrations. In the three cases the focal length is 150 mm. Also these lenses and lenses with other aberrations can be employed to form images like in Figure 5.

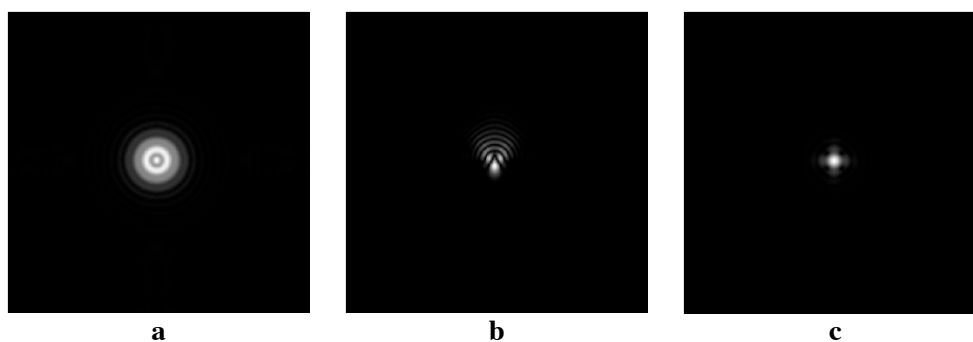


Figure 7. Focusing of a plane wave by using lenses with aberrations. In a) spherical, b) coma, and c) astigmatism. The focal length is 150 mm.

A concrete application concerning the usage of the software for an intermediate learning level is the generation of computer generated holograms (CGH) and the reconstruction of digital holograms. Figure 8 presents an example of a digital hologram recording and reconstruction. For hologram recording, a transmittance as depicted in a) is simulated in the input plane, by employing an image to generate the object beam, and a square aperture overlapped with an optical wedge for reference beam generation. The input plane is illuminated by a plane wave and then the diffracted light is propagated through free space; the dihedral angle of the optical wedge and its refractive index are automatically determined taking into account the propagation distance; at the output plane the digital hologram is obtained, as presented in Figure 8 b). This is the first step, being the second step the original image reconstruction. The hologram reconstruction is obtained by using the interference pattern that was achieved in the previous process. This is a new diffractive process in which the hologram has to be located at the input plane and illuminated with the conjugated of the reference beam that was employed for recording. Then, at the corresponding output plane an image corresponding to the holographic reconstruction of the original can be observed (Figure 8 c).

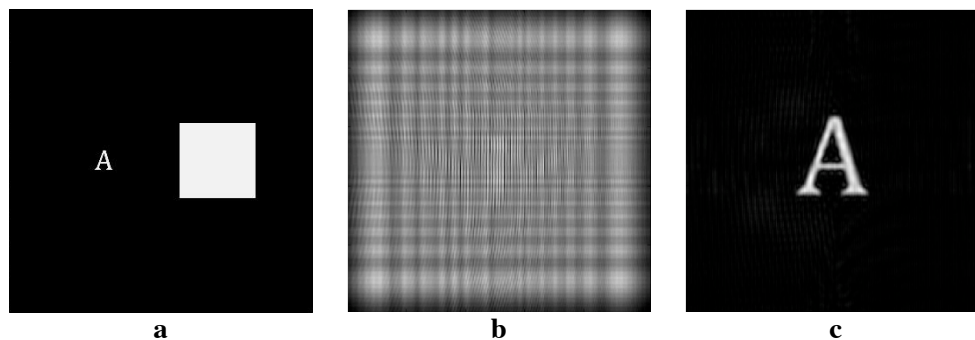


Figure 8. Synthesis of a computer generated hologram and holographic image reconstruction. In a) the transmittance at the entrance plane with an image and a square aperture, b) generated hologram, c) reconstructed image.

Finally at an advanced learning level, the user is able to simulate more complicated experiments, both employing the optical elements by default and for the generation of new algorithms for the simulation of different set-ups and components; an example of this is the experience of generating optical waves exhibiting a phase vortex or networks of vortices, by the combination of a diffracting opaque mask and a spiral phase plate. Let us consider that an annular aperture and a spiral phase plate are superimposed to form the input diffracting transmittance. If a plane wave impinges onto this transmittance, which in turn is diffracted by it, after propagating through free space, a phase singularity can be observed at the center of the distribution onto the output plane. Figure 9 depicts an annular aperture pupil with a radius of 1 mm and a thickness of 0.1mm, which is overlapped with a single level spiral phase plate, and the corresponding amplitude module and the phase modulo 2π , when light is propagated 3 m from the entrance to the exit. In the second row of Figure 9, similar results are presented, but in this case the phase of the spiral plate was affected by a random phase noise ranging in the interval $[-0.9\pi, 0.9\pi]$.

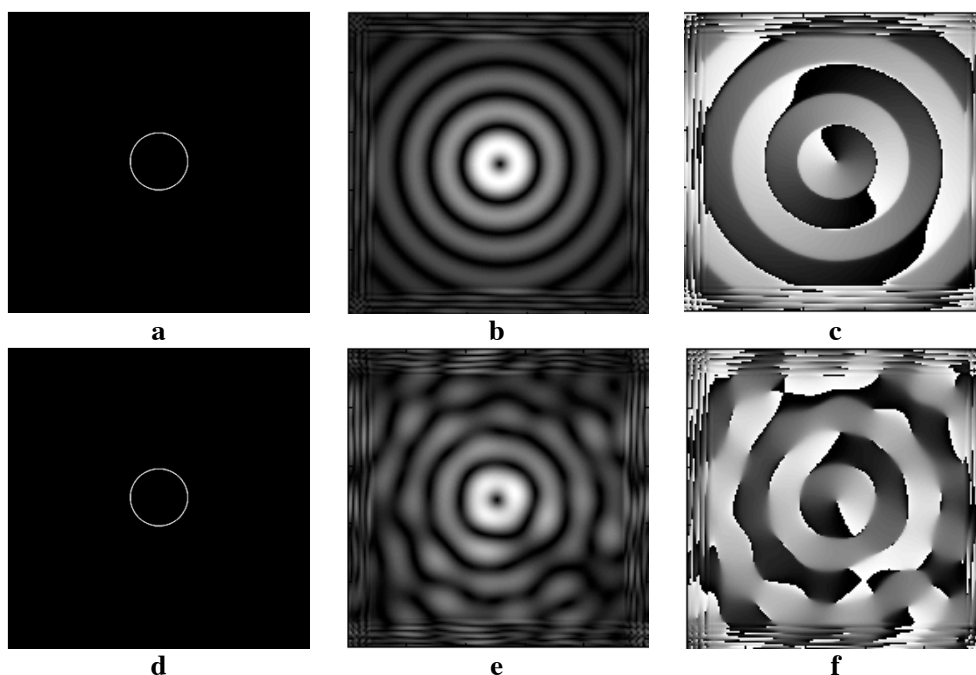


Figure 9 . Vortex generation from light diffracted by an annular aperture and a spirial phase plate. In a) and d) it is schematized the annular aperture, in b) the amplitude module and in c) the phase modulo 2π distributions, without a noisy signal; in e) the amplitude module and f) the phase modulo 2π distributions with a noise signal is presented.

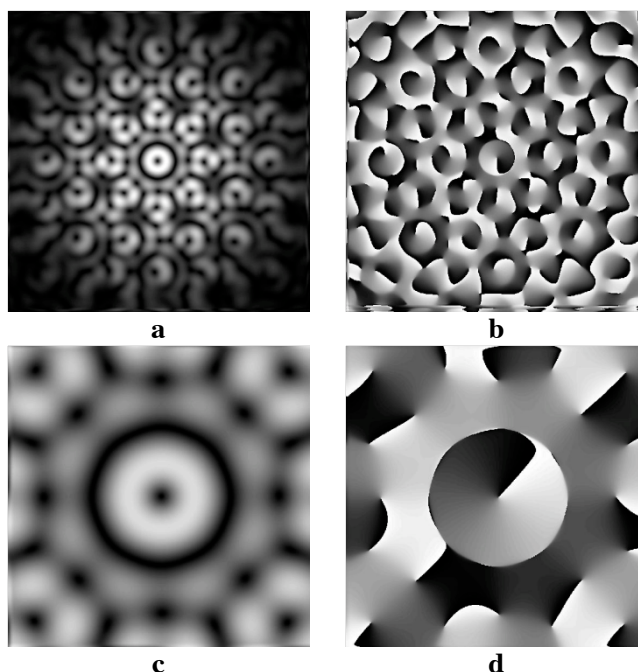


Figure 10. Results obtained by using an 8 apertures pupil which is overlapped with a single level phase plate. In a) the amplitude module, in b) the phase modulo 2π when light is propagated 1.3 m from the input plane to the observation plane. In c) the amplitude module and in d) the phase modulo 2π when light is propagated 5 m.

When the annular aperture in the previous example is replaced with a multi-aperture pupil, the interference pattern resulting from the interference of each pair of apertures (Figure 5) is modified by the presence of the phase spiral plate. Figure 10 presents the results both in amplitude module and phase modulo 2π of the light diffracted by 8 apertures pupil with a radius of 0.1mm located over a circumference with radius 1mm, along with single level phase plate.

6. Conclusions

By taking in advantage the theoretical approach of the Fourier Optics, and the practical benefits associated with the use of computational tools, we implemented a functional and versatile digital platform by employing the MATLAB® environment, to simulate the light propagation, a tool which is intended to assist different learning processes of physical optics, at the basic, intermediate and advanced levels.

Understanding the light propagation on the basis of the Huygens-Fresnel Principle stands for a fundamental factor for a deeper comprehension of different physical optical related phenomena, like diffraction, self-imaging, image formation, and spatial filtering.

The software we developed allows, at a basic and intermediate learning level, practicing with the identification of basic phenomena, and observing changes associated with modifications of physical parameters. This is achieved by using a friendly graphic user interface (GUI). At an upper level of learning, the application could be used to favor a deeper comprehension of involved physics and models, and experimenting with new models and configurations. To do this, the modular structure of this tool is easily upgradable.

This application is based on the Discrete Fresnel Transform, which simulates the physical propagation of an optical field from an input to an output plane. To achieve this in necessary to know the propagation distance and the structure of the input plane, whose characteristics are determined by the local modifications in the amplitude and the phase associated with the optical elements located at this plane. The software is addressed to assist the learning processes, by means of the simulation of experiences such as simple diffraction experiments, digital holography, interferometry experiments, among others.

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