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Optimization of V-Trough photovoltaic concentrators through genetic algorithms with heuristics based on Weibull distributions

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HIGHLIGHTS

- A study of genetic algorithms to optimize the parameters of photovoltaic V-Troughs.
- A new genetic algorithm with heuristics based on Weibull distributions.
- The new algorithm resulted significantly superior against standard metaheuristics.
- New indicators and indices were proposed as multi-objective fitness functions.
- Genetic optimization compared with an interactive software in a case study.

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ABSTRACT

Photovoltaic V-Troughs use simple and low-cost non-imaging optics, namely flat mirrors, to increase the solar harvesting area by concentrating the sunlight towards regular solar cells. The geometrical dispositions of the V-Trough's elements, and the way in which they are dynamically adjusted to track the sun, condition the optical performance. In order to improve their harvesting capacity, their geometrical set-up can be tailored to specific conditions and performance priorities. Given the large number of possible configurations and the interdependence of the multiple parameters involved, this work studies genetic algorithms as a heuristic approach for navigating the space of possible solutions. Among the algorithms studied, a new genetic algorithm named "GA-WA" (Genetic Algorithm-Weibull Arias) is proposed. GA-WA uses new heuristic processes based on Weibull distributions. Several V-Trough performance indicators are proposed as objective functions that can be optimized with genetic algorithms: (i) $\overline{C_e}$ (average effective concentration); (ii) *Cost* (cost of materials) and (iii) T_{sp} (space required). Moreover, from the integration of these indicators, three multi-objective indices are proposed: (a) I_{COE} ($\overline{C_e}$ versus *Cost*); (b) MI_{COE} ($\overline{C_e}$ versus *Cost* and $\overline{C_e}$ versus T_{sp} combined) and (c) MDI_{COE} (similar to MI_{COE} but with discretization considerations). The heuristic parameters of the studied genetic algorithms are optimized and their capacities are explored in a case study. The results are compared against reported V-Trough set-ups designed with the interactive software VTDesign for the same case study. It was found that genetic algorithms, such as the ones developed in this work, are effective in the performance indicators improvement, as well as efficient and flexible tools in the problem of defining the set-up of solar V-Troughs in personalized scenarios. The intuition and the more holistic exploration of a trained engineer with an interactive software can be complemented with the broader and less biased evolutionary optimization of a tool like GA-WA.

1. Introduction

The International Energy Agency estimated that almost one-fifth of the world's population, mostly from developing countries, lacks access to electricity [1]. This acute absence of basic living conditions contrasts with the rapid rate at which developing countries are becoming industrialized with fossil-fueled economies. Such scenario has taken the world to a point where "Developing nations are driving the ongoing increase in global CO_2 emissions"[2]. Most of these countries are located within the tropics [3], where there is higher solar insolation and hence, higher solar harvesting potential. However, the cost of residential solar Photovoltaic (PV) devices is still prohibitive for a widespread adoption [4–8]. Therefore, it is crucial to pursue solutions, from design and engineering, that might favor the transition of these nations towards sustainable solar energy.

In this regard, V-Trough concentrating devices present the

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opportunity to improve the cost-effectiveness of solar energy. They increase the energy that can be harvested with a solar absorber by means of low-cost non-imaging optics. These devices use flat mirrors, located on the borders of a PV absorber, in order to concentrate the sunlight from a greater effective solar-collecting area. As the flat mirrors and the additional supporting structure tend to be considerably less expensive than the PV material, V-Troughs have the potential to reduce the costs overall [9–11]. Moreover, the low concentration factors that they usually reach, mean that it is possible to use regular and widely available commercial solar cells [9,11–13]. V-Troughs are an effectively simple solar technology [6,14] compared to other concentration approaches, such as Fresnel lenses [15], solar power towers (heliostats) [16], high-concentration dishes [17,18] or parabolic troughs [19]. This simplicity is favorable in order to face the social-adoption barriers in developing countries because it can ease the manufacturing, installation, operation and maintenance processes [9,11,20].

The effectiveness of a V-Trough, to focus the sunlight, depends on several geometrical parameters: the lengths and angular positions of the mirrors, relative to the PV absorber, and the angular position of the device, relative to the sun. Since the sun follows an apparent path through the sky, such parameters can be dynamically adjusted in order to track the sun and enhance the harvesting effective area throughout the day. This can be achieved with automatized, yet technologically complex, solar tracking systems. On the other hand, to favor adoption among non-industrialized societies, it is worth exploring the potential of manually tracking the sun with a few adjusting steps along the day [21,22].

To get the most out of the V-Trough technology, the parameters that define the geometrical features of a V-Trough, and the way in which they are to be adjusted for manual tracking, can be tailored to the specific conditions and priorities of a given family. Personalized V-Trough set-ups can be interactively explored and evaluated with "VTDesign", a generative software developed by Arias-Rosales and Mejía-Gutiérrez [23]. VTDesign is based on a computationally-inexpensive, yet flexible, geometrical and analytical model [24]. This model can be used to simulate the interactions between the V-Trough's elements and the solar beam radiation, which is typically taken as 90% of the global radiation on average [25].

Nevertheless, a tool such as VTDesign may require the intuition of an engineer trained in V-Trough technology and may be susceptible to human bias. This may, in turn, limit a widespread exploration and implementation of V-Trough personalized solutions. Accordingly, optimization methods may be used to converge into satisfactory set-ups without requiring a deep understanding of the optical phenomena behind the simulations. However, the broad space of possible solutions renders optimization methods like exhaustive-search impractical. Despite the simplicity of these devices, there is a vast amount of possible configurations of V-Trough dynamic geometries. Moreover, a V-Trough analytical model such as the one used by VTDesign [24] is not a continuous function, so it cannot be optimized with derivatives. Other simpler analytical models [26] could be optimized in such a way, but they lack the parametric flexibility needed for a broad exploration.

When more traditional methods are not applicable, Genetic Algorithms (GAs) can be an effective way to navigate an extensive space of possible configurations looking for an optimal (or near-optimal) solution in design and engineering matters [27–31]. GAs are based on the biomimicry of Darwinian evolution principles [29]. Following this biological metaphor, V-Trough devices can be regarded as animals whose DNA (genetic information) determines the geometrical parameters of their mirrors and the PV absorber and the way in which they move in reaction to the solar apparent movement. Within a genetic algorithm, these "V-Trough beings" would be arranged in a diverse population where every individual may have a different physiology (set-up). By means of a series of processes, inspired by sexual reproduction and biological mutation, this population can evolve throughout a given amount of generations. This evolutionary progression seeks to converge into the best-explored solution in terms of a given objective function. GAs, as opposed to VTDesign and other optimization methods, offer the opportunity to overcome possible design biases, find non-intuitive solutions and to converge effectively and consistently for personalized scenarios without the need of a solar-qualified engineer behind a software.

This work studies genetic algorithms as tools for determining the V-Trough's geometrical set-up in scenarios which are personalized in terms of the context conditions, the performance priorities and the user's routine. The developed GAs can be used to optimize the system's performance regarding the effective optical concentration, the cost of materials, the space occupied and the multi-objective cost-effectiveness from several perspectives. Among the GAs studied, a new genetic algorithm is proposed based on Weibull frequency distributions; named as "GA-WA" (Genetic Algorithm-Weibull Arias). GA-WA is a general-purpose genetic algorithm and, therefore, it can be used in other design-engineering problems. However, it is believed to be of special use in the V-Trough design problem. Its performance is compared in a V-Trough case study against more traditional genetic algorithms and previously published results achieved with VTDesign [23]. To the best of the authors' knowledge, the following are the contributions of this work to the state of the art:

- Unprecedented flexibility in the multi-parameter optimization and geometrical exploration of V-Trough solar devices (Sections 3 and 4.2).
- New indicators and indices proposed as fitness functions for assessing a V-Trough's performance in terms of the space required and the cost-effectiveness from various multi-objective perspectives (Section 3).
- First evaluation of genetic algorithms as heuristic tools for determining the V-Trough parameters in personalized scenarios.
- Introduction of a new genetic algorithm, GA-WA, with the novel heuristic capabilities of initializing the first population around intuitive values, implementing various stagnation strategies and a greater control and flexibility regarding random processes in mutation and elitism (Section 4).
- First optimization of the heuristic parameters of three genetic algorithms in the V-Trough engineering problem, assessing the effect of each heuristic parameter on the optimization performance (Section 5).
- Applied case study were both GA-WA and the software VTDesign are evaluated and compared from the perspective of three different design goals (Section 6).

2. Literature review

This section seeks to address what has been done in the literature concerning the optimization of V-Trough solar devices. Heuristic optimization methods are presented as a suitable approach, in particular, genetic algorithms, which are classified as metaheuristics. As follows, there is a review on GAs implementations in solar matters and specific genetic heuristics which are of interest for this study. As defined by Pearl [32], "heuristics stand for strategies using readily accessible information to control problem-solving processes in man and machine". Regarding optimization, metaheuristics can be seen as general algorithmic frameworks that select among different heuristic strategies and can be applied to a wide set of different design/engineering problems [33].

In the majority of the V-Trough studies found, the optimization is based on performance curves obtained numerically [34,35,9,12,10,36–41]. In this approach, a given geometrical parameter is iterated within an allowable range while leaving the other parameters fixed. An analytical model or a ray-tracing simulation [14] is then successively recalculated. This way, it is possible to generate families of performance curves, or nomograms [39], where the optimum points are usually evidently distinguishable. For instance, Maiti et al. [11] assessed a V-Trough based on a numerical Ray-Tracing model, which they iterated in order to find an optimum value for the inclination of the mirrors. Nevertheless, the previous study does not specify how the iteration was performed or how it led towards an optimum. This simple iterative optimization procedure has also been reported for other types of concentrating devices, such as parabolic troughs [42]. By means of these iterative methods, parameters such as the aperture angle [14,9], the inclination of each mirror [35,36,40,41], as well as the inclination of the solar absorber [9,39], have been reportedly optimized. Satisfactory results can be achieved in highly limited case studies, where the iteration process can be constrained to only one or two parameters varying within narrow ranges. However, this approach may be unpractical or too restrictive for a more flexible set-up exploration. It is not possible to simply find the optimum value of one parameter and then to continue with the next one because the optimum value of every parameter is in function of the others. For instance, the best inclination for the left mirror depends on its length, as well as on the disposition of the other elements relative to that mirror and the solar rays.

On the other hand, Taha and Eldighidy [43] explored the angular position of a V-trough's solar absorber and one of its mirrors with Pattern Search, an optimization metaheuristic. This method allows for a progressive variation of multiple parameters in the direction of improvement. Nevertheless, the nature of Pattern Search makes it vulnerable to getting stuck in a local optimum. Other metaheuristic methods have been used for optimizing more than one parameter in other types of concentrators. For instance, Yu et al. [16] discretized the parameters of a solar heliostat and then used a TABU meta-heuristic to navigate the space of solutions. Also, seeking to improve the focal shape and its uniformity, Giannuzzi et al. [17] used a downhill simplex metaheuristic to optimize optical aberrations applied to the multiple mirrors of a Dish concentrator.

A genetic algorithm metaheuristic may offer a way to perform a broader and more unbiased exploration of the solution space. GAs have already been used for solar applications. Khlaichom and Sonthipermpoon [44] used a GA to actively fine-tune the tracking position of a solar panel. Similarly, Chen et al. [45] optimized the installation angle of a fixed solar panel with a GA. Also, studying the parameters of a Dish Stirling system, Caballero et al. [18] performed a multi-objective optimization with a genetic algorithm to obtain a Pareto Front of possible solutions. Still, no study has been found in the literature in which genetic algorithms are used to optimize solar V-Troughs. This is relevant because the efficacy and efficiency of genetic algorithms are problem-dependent [27,46]. Moreover, GAs can use different heuristics for each of the evolution steps; heuristics which are themselves more or less effective depending on the problem and the other complementing heuristics within the GA.

Several heuristics have been reported to improve the performance or the adaptability of a GA. For instance, with Elitism, a proportion of the best performing individuals passes to the next generation as part of the offspring [45]. Elitism can speed up the convergence process and avoid losing the best solutions of a given generation [47]. Mutation, which introduces diversity and reduces the premature-convergence tendency, can also be implemented with different heuristics [48]. One interesting heuristic approach is "non-uniform mutation", where the mutation values progressively become smaller in order to advance from a wide exploration to a local search [49].

Some of the key functions in the steps within a GA rely on randomness or semi-randomness. In order to enable such functions, the genetic heuristics involved are based on random variables modeled through Probability Density Functions (PDFs). For instance, the mutation values are usually determined through a Uniform PDF, where any value within a constrained range has the same probability of occurring [44,50]. As with biological mutation, this is not always reasonable because small mutation values are usually expected to be more likely than large values. To address this issue, mutations have reportedly been controlled by other PDFs. The most common for this application is the Gaussian (normal) distribution [51], which is the default distribution for modeling randomness in general. However, several studies on metaheuristic methods have explored different PDFs and found a considerable effect on the optimization performance. Yao et al. [52,53] explored the use of the Cauchy PDF to control mutation in Evolutionary programming. They found that this heuristic procedure can result in a superior optimization performance in certain problems where bigger mutation values are beneficial. Lee and Yao [54] then used the Lévy distribution, which is the generalization of the Cauchy PDF, and further analyzed the advantages of having an adjustable variation in the heuristics related to mutation. Using the Cauchy PDF, as opposed to the Gaussian PDF, was also found to produce a faster convergence in certain optimization problems with the Simulated Annealing metaheuristic [55]. The Weibull PDF was also reported as significantly beneficial when used in a heuristic to control perturbations to a population of possible solutions in Differential Evolution [56].

From the previous studies, it is well known that more frequent large mutation values can be beneficial depending on the optimization problem. If the mutations are too large, this can be detrimental in a systematic progression towards an optimum. Conversely, excessively frequent small mutations can render the search too local and frustrate the exploration of a broad solution space. Therefore, as the selected PDF controls those frequencies, it is desirable to pursue further flexibility in how such distributions adapt towards different levels of mutability. However, little has been reported in this regard when it comes to genetic algorithms. The Weibull PDF has already been used in GAs but not for controlling its heuristics. On the contrary, GAs have been used to find the parameters of Weibull PDFs as the optimization problem itself [57], which has had applications related to reliability [58] and wind speed modeling [59].

It can be therefore stated that there is a need for an assessment of genetic algorithms, exploring different heuristics, as tools for determining the geometrical parameters of solar V-Troughs. Additionally, there are certain heuristic procedures, for flexibly controlling mutation, which could benefit the GAs performance in this assessment but which are missing in the reviewed literature.

3. Description of the applied problem

This section describes the applied engineering problem of determining the geometrical parameters of V-Trough solar concentrators by means of heuristic optimization. Also, several performance indices and indicators are proposed as fitness (objective) functions that can be used depending on the optimization priorities. Lastly, the role that genetic algorithms can play in this problem is explained.

The engineering problem is contextualized in the developing world with rural families seeking to satisfy their energy needs with solar-harvesting systems. As presented in Section 1, V-Trough devices can be dynamically adjusted along the day with manual tracking in order to improve the cost-effectiveness of photovoltaic solar energy. V-Trough technology has the potential to improve the cost-effectiveness because it uses low-cost flat mirrors to increase the effective harvesting area by concentrating the sunlight towards the PV absorber. The effectiveness of this sunlight concentration depends on the geometrical set-up of the V-Trough's elements and their disposition with respect to the incident solar rays (see Fig. 1(a)). The main geometrical parameters that define

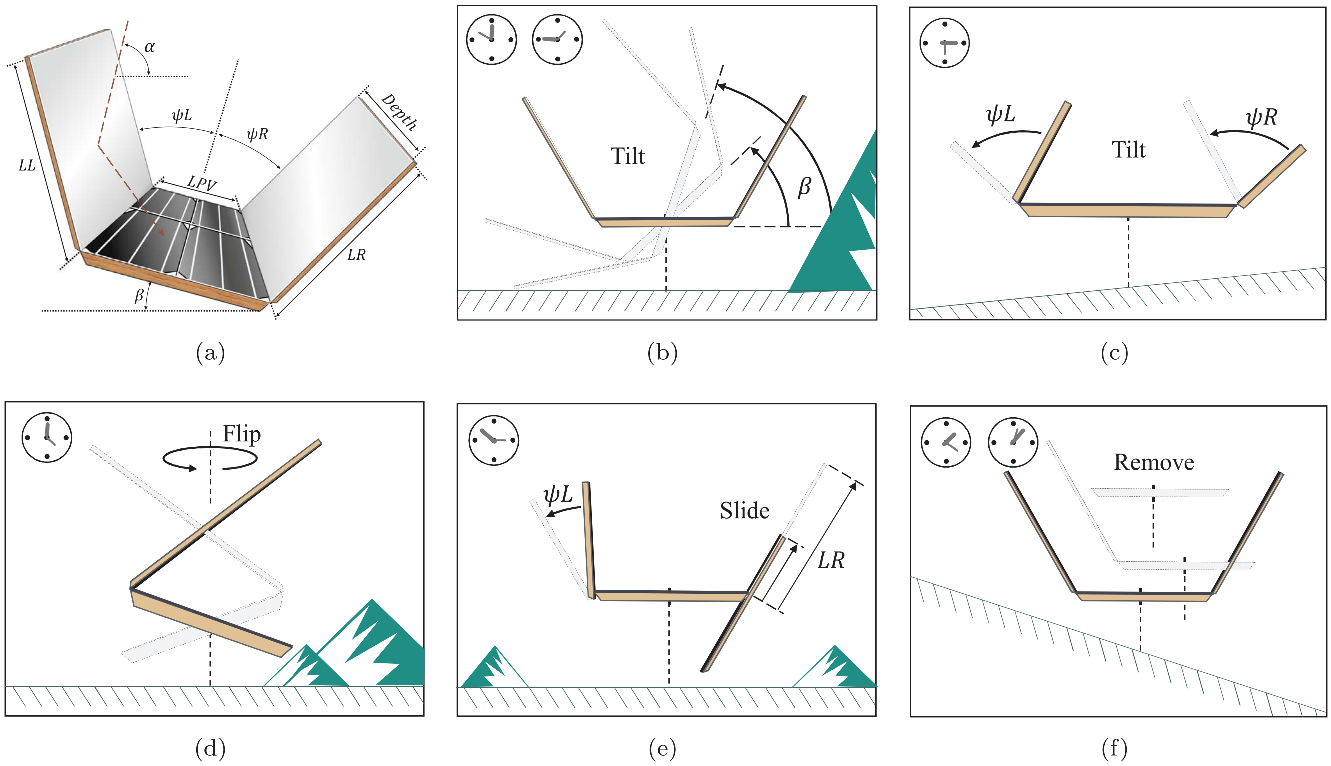

Fig. 1. (a) Diagram of a V-Trough photovoltaic device. (b–f) Possible Tracking movements and procedures.

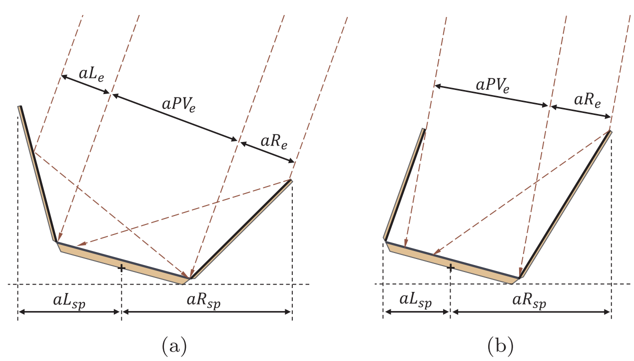

Fig. 2. Diagram of a V-Trough photovoltaic device; shown in two different geometrical set-ups in (a) and (b).

the V-Trough set-up are: the *Length of the PV absorber* (*LPV*); the *Length of the Left* (*LL*) and *Right* (*LR*) mirrors; the *Depth* of the device (*Depth*); the *Angular Position of the PV absorber* (β); and the *Angular Position of the Left* (ψL) and *Right* (ψR) mirrors. In this work, the lengths are defined as proportions of *LPV* and the angular positions are set in degrees.

These geometrical parameters can be adjusted manually, a few times along the day, in order to track the sun. The flexibility of allowing these parameters to be dynamically and independently manipulated can lead to a plurality of possible practical tracking movements and procedures. For instance, the whole device can be tilted several times a day (Fig. 1(b)); its mirrors can be independently tilted (Fig. 1(c)); its position can be flipped (Fig. 1(d)); the mirrors can be slid to different lengths (Fig. 1(e)); or the mirrors can be removed at strategic times of the day (Fig. 1(f)). Despite the geometrical simplicity of V-Troughs, as their parameters are continuous dimensions, there is an infinite number of possible V-Trough set-ups. For a more practical perspective, if each parameter is discretized into only ten possible values, and two tracking movements are allowed, then each parameter has to be defined for three positions. Then, there would be $1E15$ possible set-ups.

The parameters that determine the geometrical features of a V-Trough, and the way in which they are to be adjusted for the manual tracking, can be tailored to the specific conditions and priorities of a given family. These personalized design scenarios can differ, for instance, in the available space for locating the device, the allowable cost of materials, the energy needs, the timing and number of manual tracking adjustments that the family is willing to perform per day, and the possible shadows at the location that may limit the available solar elevation range (see Fig. 1(b)–(f)). Hence, an optimal V-Trough set-up depends on the personalized conditions and constraints but also on the performance priorities of the potential users of the system. Seeking to provide a broad optimization framework that can be adapted to different priorities, this work proposes a series of performance indices and indicators.

The following performance indicators were established: (i) $\overline{C_e}$ (the average effective concentration); (ii) *Cost* (the cost of materials); (iii) T_{sp} (the space required). Moreover, from the integration of these indicators in proportional comparisons, three multi-objective indices were proposed: (a) I_{COE} ($\overline{C_e}$ versus *Cost*); (b) MI_{COE} ($\overline{C_e}$ versus *Cost* and $\overline{C_e}$ versus T_{sp} combined); (c) MDI_{COE} (the same comparison as the previous one but also including the discretization effects related to the solar cells). Depending on the given performance priorities, any of these indicators and indices can be established as the fitness function to be optimized by a genetic algorithm. These indicators and indices are an extension of the V-Trough modeling framework proposed by Arias-Rosales and Mejía-Gutiérrez [24]. Therefore, they are dependent on the same assumptions, namely, the optical performance is solely based on the solar beam radiation interactions and no temperature losses are considered.

As depicted in Fig. 2(a) and (b), not all the solar rays that reach the device manage to make contact with the PV absorber. Several optical phenomena can affect the *Effective Concentration* (C_e), such as shadows or rays that are reflected back out after one or two bounces off the mirrors. The mathematical model presented by Arias-Rosales and Mejía-Gutiérrez [24] provides the detailed equations to assess these phenomena and estimate the optical effective contributions from each V-Trough's elements, namely aL_e for the left mirror, aR_e for the right mirror and aPV_e for the PV absorber. As a common simplification, this modeling problem is analyzed in a 2D plane, so aL_e, aR_e and aPV_e are defined as optical aperture lengths which are perpendicular to the solar incidence and are traced in a transverse plane. These optical lengths depend on the geometrical positions and lengths of each element, as well as on the *Solar Elevation Angle* (α). In the case of the mirror contributions, an *Index of Reflection* (ρ) must be considered.

Eq. (1) defines C_e as the added effect of the three contributions, in proportion of *LPV*.

$$C_e = \frac{aPV_e + aL_e + aR_e}{LPV} \tag{1}$$

In order to obtain an indicator of the *Mean Effective Concentration* of the device throughout the day ($\overline{C_{e(\alpha f,\alpha c)}}$), considering only beam radiation, Ce is averaged over an angular α range from α_f to α_c. For the same photovoltaic area, the greater $\overline{C_e}$ is, the greater the solar energy that can be harvested throughout the given α range. Hence, in order to maximize the mean optical effectiveness, $\overline{C_e}$ can be assigned as the fitness function of a GA.

Although a V-Trough can increase the optical concentration of a solar absorber, it can also increases the amount of materials required due to the mirrors and additional supporting structure. For a V-Trough to be preferable over a flat solar panel, it must at least increase $\overline{C_e}$ in a greater proportion than it increases the cost of materials. This can be addressed by the *Cost-effectiveness Index* (I_{COE}) [24] in Eq. (2), which must be calculated for a given (αf,αc) range as well.

$$I_{COE\,(\alpha f,\alpha c)} = \left(\frac{\overline{C_{e(\alpha f,\alpha c)}}}{\overline{C_{eREF\,(\alpha f,\alpha c)}}}\right) * \left(\frac{LPV}{LPV + \lambda(LL + LR)}\right) \tag{2}$$

where

$$\lambda = \frac{CO_s + CO_m}{CO_s + CO_{pv}} \tag{3}$$

with CO_{pv}= Area cost [USD/m^2] of the PV absorber, CO_m= Area cost [USD/m^2] of the mirrors, CO_s= Area cost [USD/m^2] of the supporting structures, and $\overline{C_{eREF}}$= Mean Effective Concentration of a reference horizontal and flat panel. I_{COE} can be assigned as a multi-objective fitness function, where a GA would pursue the maximization of $\overline{C_e}$, while minimizing the proportional cost of materials.

In a reported case study with the software VTDesign [23], it was found that not always the V-Trough set-up with the highest I_{COE} results in the lowest cost of materials. For instance, a given design scenario may aim at minimizing the cost of materials for a given daily energy need. In this case, a number of solar cells will be arranged according to the chosen V-Trough set-up. Solar cells are discrete units with discrete areas and, therefore, they cannot always be arranged in a solar panel so as to precisely fulfill a given energy demand. This discretization effect of the solar cells can be considered in a more detailed cost comparison. For this purpose, the *Minimum Number of Solar Cells* required (N_{pvc}) is first calculated with Eq. (4) for a given *Daily Energy Need* (E_{day}) [Wh].

$$N_{pvc} = \left\lceil \frac{E_{day}}{\overline{I} * \overline{C_e} * \eta_{pv} * \eta_s * A_{pvc} * H_{sun}} \right\rceil \tag{4}$$

where

$$H_{sun} = \frac{\alpha c - \alpha f}{15} \tag{5}$$

with $\overline{I}$= Average solar irradiance [W/m^2], η_{pv}= Photovoltaic efficiency, η_s= Electric efficiency of the system, A_{pvc}= Area [m^2] of a single solar cell, and H_{sun}= Hours of exposure to sunlight. It is worth noting that N_{pvc} must be rounded-up in order to account for the discretization of the solar cells. Eq. (4) uses E_{day} as an input. On the other hand, Eq. (6) allows the calculation of E_{day} as an energy output, given a known N_{pvc}. This is an indicator of the solar beam energy that could be harvested during a H_{sun} period of time.

$$E_{day} = N_{pvc} * \overline{I} * \overline{C_e} * \eta_{pv} * \eta_s * A_{pvc} * H_{sun} \tag{6}$$

The *Cost of Materials* (*Cost*) in *USD*, for a given energy need, can be calculated with Eq. (7). *Cost* can be assigned as the fitness function for it to be minimized by a GA. This can be used as a performance indicator, similar to I_{COE}, but considering also the discretization of the PV surface into individual solar cells.

$$Cost = N_{pvc} * A_{pvc} * \left[CO_{pv} + CO_s + \left(\frac{LL + LR}{LPV}\right) * (CO_s + CO_m) \right] \tag{7}$$

The space required for a V-Trough system can also be considered a priority for making design decisions. This space depends on the geometrical disposition of the device, as shown in Fig. 2(a) and (b). For this calculation, it is useful to assume that β changes by means of a pivot at the center of the *LPV* length. It is also assumed that the V-Trough's elements have no thickness. The *Linear Space* required can then be calculated to the *Left of the pivot* (aL_{sp}), with Eq. (8), and to the *Right of the pivot* (aR_{sp}), with Eq. (9).

$$aL_{sp} = \max\left[\left(\frac{aPV_{sp} + |aLL_{sp}| + aLL_{sp}}{2}\right), \left(\frac{|aLR_{sp}| - aLR_{sp} - aPV_{sp}}{2}\right)\right] \tag{8}$$

$$aR_{sp} = \max\left[\left(\frac{aPV_{sp} + |aLR_{sp}| + aLR_{sp}}{2}\right), \left(\frac{|aLL_{sp}| - aLL_{sp} - aPV_{sp}}{2}\right)\right] \tag{9}$$

where

$$aPV_{sp} = LPV * \cos\beta \tag{10}$$

$$aLL_{sp} = -LL * \sin(\beta - \psi L) \tag{11}$$

$$aLR_{sp} = LR * \sin(\beta + \psi R) \tag{12}$$

The previous equations (Eqs. (8)–(12)) were geometrically verified with Creo Parametric®, a Computer Aided Design Software. In order to calculate the *Total Linear Space* required (aT_{sp}), aL_{sp} and aR_{sp} must be calculated for every position adjustment of the V-Trough along the day. As stated in Eq. (14), the maximum linear spaces for the left and right are then independently identified and added together. It is worth noting that *max* aL_{sp} and *max* aR_{sp} do not have to happen at the same solar elevation. If the minimum number of cells (N_{pvc}) can be determined, the *Total Space* required (T_{sp}) [m^2] can then be calculated with Eq. (13). T_{sp} may be assigned as the fitness function for it to be minimized, by a GA, considering the discretization effects of the solar cells within the term N_{pvc}.

$$T_{sp} = \left(\frac{aT_{sp}}{LPV}\right) * N_{pvc} * A_{pvc} \tag{13}$$

where

$$aT_{sp} = max\ aL_{sp} + max\ aR_{sp} \tag{14}$$

While I_{COE} considers both harvesting energy gain and material costs, it might be useful to have a multi-objective index which also considers the space required for the device. In this work, two new such indices are proposed, namely MI_{COE} and MDI_{COE}. Both address and weight the following two questions: How does the energy harvested by a given V-Trough compare to the energy that could be harvested if the same cost of materials were used for a reference panel? and how does the energy harvested by a given V-Trough compare to the energy that could be harvested if the same space were used to allocate a reference panel? In essence, these indices address the problem of cost-effectiveness from the perspectives of both monetary cost and the space required.

The *Multi-Index of Cost-Effectiveness* (MI_{COE}), from Eq. (15), allows to compare the relative impact of any given V-Trough strategy over the optical performance, the cost of materials and the linear space needed. A weight $W1$ is assigned as the *Relative Importance of Cost* and $W2$ as the

Relative Importance of Space needed. $W1$ and $W2$ (Eq. (16)) are subjective and must be assigned by the user of the GA according to the optimization priorities. MI_{COE} may be assigned as a multi-objective GA fitness function seeking to simultaneously maximize the average effective concentration, minimize the cost of materials and minimize the linear space needed.

$$MI_{COE(\alpha f,\alpha c)} = \left(\frac{\overline{C_{e(\alpha f,\alpha c)}}}{\overline{C_{eREF(\alpha f,\alpha c)}}}\right) * \left[W1\left(\frac{LPV}{LPV + \lambda(LL + LR)}\right) + W2\left(\frac{LPV}{aT_{sp}}\right)\right] \tag{15}$$

where

$$W1 + W2 = 1. \tag{16}$$

Eq. (17) defines the *Multi-Discrete-Index of Cost-Effectiveness* (MDI_{COE}). MDI_{COE} differs from MI_{COE} in that it considers the discretization effects of the solar cells. $Cost_{REF}$ and T_{spREF} are, respectively, the cost of materials and the total area, required for a given energy need, in the case of a reference horizontal and flat solar panel.

$$MDI_{COE(\alpha f,\alpha c)} = W1\left(\frac{Cost_{REF}}{Cost}\right) + W2\left(\frac{T_{spREF}}{T_{sp}}\right) \tag{17}$$

In this section, a series of indices and indicators have been established that can be used for evaluating the performance of V-Trough systems from different perspectives. The selection of any of these as the objective function for an optimization process depends on the specific performance priorities of the application, the level of specification of the scenario and the commercial elements to be used. If the priority is energy maximization, $\overline{C_e}$ may be used as the objective function. If the priority is to reach a cost-effective set-up, I_{COE} can serve as the most appropriate objective function. If the space occupied is also a priority, MI_{COE} can weight the cost-effectiveness in terms of monetary cost of materials and space implications. Moreover, if the space occupied is the only or the main priority, aT_{sp} may be preferred. It is worth highlighting that $\overline{C_e}$, I_{COE}, MI_{COE} and aT_{sp} are all indicators proposed for an early and generic design stage and the estimates are based only on two-dimensional reasoning. Also, they do not consider the discretization implications that may arise when the solar cells to be used are selected. If the energy need and the solar cells are already established, then more realistic indicators can be obtained with *Cost*, for the cost of materials; T_{sp}, for the area space occupied by the device; and MDI_{COE}, for a proportional and weighted consideration of both *Cost* and T_{sp}.

Having established a series of possible objective functions, the main purpose of this work is to explore the use of GAs as metaheuristic tools for the optimization of the V-Trough parameters in personalized scenarios. The intention is not to find an all-encompassing Pareto Front, but to evaluate if GAs can converge to feasible set-up solutions that are highly adapted to the conditions entered and satisfactory with regards to a given selected fitness function. In further design stages, any of these set-up solutions can be translated into a dynamic product architecture with a practical tracking strategy (such as in Fig. 1). Then, the design can be detailed with simple structures, as it is explored in Section 6.

4. Development of the genetic algorithms: introduction of GA-WA

In order to assess the performance and feasibility of GAs as heuristic tools for optimizing the V-Trough's geometrical parameters, for personalized scenarios, three genetic algorithms were developed and are described in this section. From these, two GAs are reference algorithms with standard heuristic processes; named as "GAUniform" and "GAGauss". This section is focused on the development of the third one, which is a new type of genetic algorithm proposed in this work and was named as "GA-WA" (Genetic Algorithm-Weibull Arias).

The lack of flexibility for mutation control, described in Section 2, motivated the proposal for new heuristic processes. These were found useful for controlling other random or semi-random processes in other stages of the evolution scheme besides mutation. In this work, a semi-random process, variable or parameter, is one which is bounded by randomness with a given probabilistic bias which may differ from a uniform or Gaussian distribution. The integration of the developed heuristics resulted in the new GA-WA scheme. GA-WA is believed to be of special use for the V-Trough design problem, which is explored in the following sections. In particular, GA-WA was designed not only to be used in parallel to an interactive exploration with a tool such as VTDesign but also to further explore and optimize the results achieved with such a tool. However, this new genetic metaheuristic can be used to address all kinds of design-engineering problems whenever a flexible control of the random processes is desired. This section describes the concepts and procedures behind the development of GA-WA.

4.1. Heuristics based on the Weibull distribution

Genetic algorithms are inspired by biological evolution. Hence, many of their internal heuristics result from a direct biomimicry of natural processes translated into code. Among these, the heuristics related to the random or semi-random processes are of special interest. Randomness introduces variability and diversity, which ultimately allows a GA to navigate the wide solution space. As explained in Section 2, different PDFs have been used for a more reasonable control of randomness, as opposed to just uniform randomness. For the first time, this work proposes the Weibull PDF as a more flexible and adaptable approach. To the best of the author's knowledge (see Section 2), the Weibull PDF has not been used before in heuristics for controlling the random or semi-random processes within a GA. The reasons for choosing this PDF are explained as follows.

The random variables related to the heuristic procedures within a GA are usually controlled by a Gaussian distribution. As the Gaussian distribution is symmetrical, the mean is expected to fall in the middle of the variation range. The following example is presented to illustrate why this behavior is too restrictive as a control for the heuristic processes of a GA. If the magnitude of mutation for the length of a given mirror can vary from 0 to 1 m, the mean mutation magnitude would then be 0.5 m with a Gaussian distribution adjusted to the limits of the given range (see Fig. 3). This degree of mutation is excessive and unreasonable given that mutations are expected to introduce progressive perturbations that are small relative to the original value of the variable. On the other hand, the variation range can be established from −1 m to 1 m by including both the magnitude and direction of the possible mutation values. Then, the mean mutation value would be 0 with a Gaussian distribution adjusted to the limits of the full range. Even though this behavior is more reasonable, with regards to evolution, it does not allow for a mean value that is greater than 0 but not necessarily 0.5 m. As it was discussed in Section 2, PDFs that allow a greater degree of mutation more frequently can favor the evolutionary process in certain engineering problems. Given that some applications are more effectively optimized with higher degrees of mutation, while others are favored by less disruptive mutations, it is convenient to seek a more flexible approach for controlling the mean magnitude of

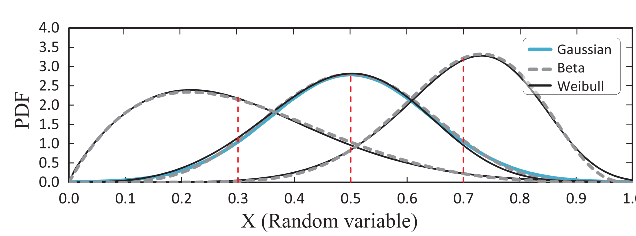

Fig. 3. Random value controlled by different PDFs.

mutation. Therefore, the intention is to control random or semi-random processes with a distribution that can be skewed so as to modify the mean at will within a given variation range.

The previously described desired behavior is illustrated in Fig. 3. It is shown how both Weibull [60] and Beta [61] PDFs can closely resemble a Gaussian distribution. Moreover, the parameters of such distributions allow their shape to be skewed towards other mean values (such as 0.3 and 0.7 in Fig. 3). The Beta and Weibull PDFs, as well as other similar distributions, offer the flexibility sought in this work with a comparable complexity in terms of their parametric control. Motivated by the successful implementation of Cárdenas-Montes [56] in a similar metaheuristic, the Weibull distribution is selected in this work as a means to control random variables within the heuristic procedures of a GA.

The Weibull PDF can adopt a broad range of different shapes by manipulating its two parameters, i.e., *Shape* (k) and *Scale* (c). Modifying k has a similar effect to skewing the shape of a Gaussian PDF. This is conceptually coherent for the current issue because it relates to a probabilistic bias that can be systematically introduced, to an originally random behavior, so as to adjust the mean towards the desired value. In fact, the Weibull distribution is the general form of other widely used distributions, such as the exponential and Rayleigh PDFs. At a given k, the Weibull PDF can also resemble a Gaussian distribution, as seen in Fig. 3. Therefore, by manipulating k and c, a GA user can have a high degree of control over several kinds of genetic steps involving randomness. Nevertheless, k and c are not intuitive parameters. If the distribution is to be adjusted to a given range and shaped towards a desired bias, there are no obvious k and c corresponding values. Consequently, the following method was proposed in order to control a Weibull distribution from a more intuitive parameter, namely the *Mean* (μ).

Eq. (18) presents the Weibull probability density in function of x, k and μ.

$$f_{(x)} = \frac{k\Gamma(1+1/k)}{\mu}\left(\frac{x\Gamma(1+1/k)}{\mu}\right)^{k-1} * e^{-\left(\frac{x\Gamma(1+1/k)}{\mu}\right)^{k}} \tag{18}$$

The intention is to constrain the distribution to a range from 0 to a *Maximum Allowed Value* (*Top*). k can be defined in order to make it highly unlikely for a value greater than *Top* to occur, as stated in expression 19.

$$\frac{k\Gamma(1+1/k)}{\mu}\left(\frac{Top*\Gamma(1+1/k)}{\mu}\right)^{k-1} * e^{-\left(\frac{Top*\Gamma(1+1/k)}{\mu}\right)^{k}} \leqslant 0.0002 \tag{19}$$

It was found that a satisfactory k value remains the same for equal proportions of $p\mu = \mu/Top$, regardless of the *Top* magnitude. Hence, seeking a more general analysis, expression (19) was normalized to *Top*, making $Top = 1$ and $\mu = p\mu$. The resulting inequality was numerically solved for k in the range $0.15 \leq p\mu \leq 0.5$ in steps of 0.01, as shown in Fig. 4(a). When $p\mu < 0.15$, a $k < 1$ would result. However, a Weibull PDF with a $k < 1$ approaches the vertical axis asymptotically, which is problematic for the intention of clearly delimiting the PDF to a given allowable range. Therefore, $k = 1$ is assigned whenever $p\mu < 0.15$. Additionally, the k search was limited to $p\mu \leq 0.5$ for a symmetry to be further created in the range $p\mu > 0.5$. From the points shown in Fig. 4(a), a polynomial regression (see Eq. (20)) was obtained with a sample standard error of 0.0038.

$$k = 15.8749972991*p\mu^3 - 6.46861529214*p\mu^2 + 5.93130385328*p\mu + 0.185331693521 \tag{20}$$

With this equation, it is possible to directly calculate the k value for a given $p\mu$ in the stated range. The Weibull PDF can then be traced with Eq. (18). With this procedure, Fig. 4(b) shows the PDF curves corresponding to various desired μ values (shown in vertical dashed lines)

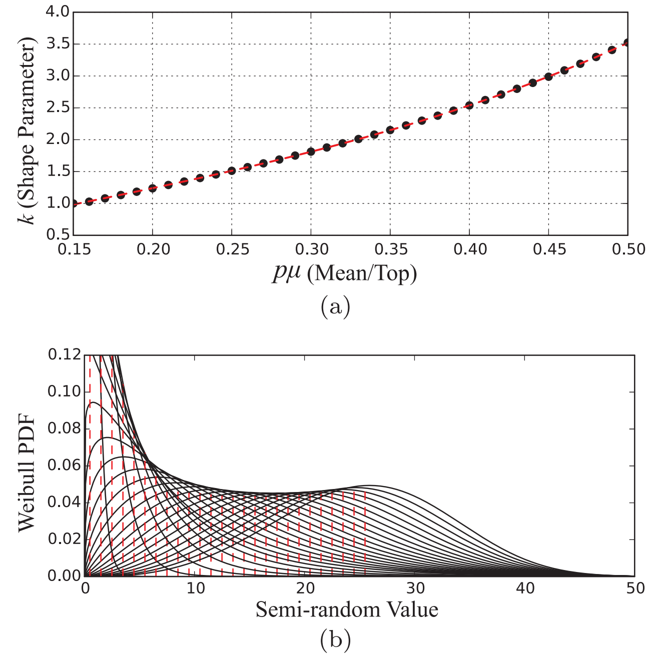

Fig. 4. Weibull Distribution controlled by the mean (μ). (a) k values calculated for a range of $p\mu$. (b) The corresponding Weibull PDF curves adjusted for a range from 0 to $Top = 50$.

and adjusted to a range $0 \leq X \leq 50$. These curves illustrate how the probability distribution of a semi-random variable can be flexibly shaped, within a given range, with the proposed procedure.

When $p\mu = 0.5$, and $3 < k < 4$, the Weibull PDF tends to be symmetrical and resembles the shape of a Gaussian distribution. This is illustrated in the far-right PDF curve in Fig. 4(b), which was calculated for a $\mu = 25$ and $p\mu = 0.5$. Weibull PDFs with $k > 4$ do not present a mirrored behavior of PDFs with $k < 3$. However, such a symmetrical behavior would be beneficial for a more intuitive control of the semi-random variables. For instance, a $p\mu = 0.8$ should present a PDF curve with a shape that is inverse and proportionally equivalent to a PDF for $p\mu = 0.2$. Accordingly, the procedure in expression (21) presents the necessary steps required to generate the desired symmetry: first the mean is flipped to its proportional inverse (μ'). Then the mean-to-Top proportion is calculated based on the flipped mean ($p\mu'$), which is used to find k' with Eq. (20). A number of values V' must be generated by satisfying a PDF calculated in function of the flipped shape (k') and mean (μ') parameters. Finally, the values V are obtained by flipping back each V' value.

$$\begin{aligned} \text{if } p\mu > 0.5:\quad & \mu' = Top - \mu; \\ & p\mu' = \mu'/Top; \\ & k'_{(p\mu')}; \\ & \text{values } V' \text{ following } f'_{(k',\mu')}; \\ & V = Top - V'. \end{aligned} \tag{21}$$

The previous procedures allow a Weibull distribution which can be adjusted within a given range and shaped with a bias towards the desired mean value. This capacity was used in the heuristic processes that differentiate GA-WA from the rest of the genetic algorithms; heuristics related to mutation control, elitism, stagnation strategies and the initialization of the first population.

4.1.1. Mutation control and elitism

Promoting mutations in a population of possible solutions is in itself a heuristic, inspired by biological evolution, for introducing diversity.

This greater diversity is meant for reducing the tendency for the algorithm to get stuck in a local optimum and keep looking for different approaches as opposed to just an incremental progression in the direction of improvement. Mutation, as in nature, is a process linked to randomness. As explained in Section 2, a procedure to control a probabilistic bias for this randomness in mutation may be beneficial. The heuristics within the mutation evolutionary step must provide an answer to the following questions: How many mutations per generation? Which individuals will be mutated? Which genes will mutate? By how much (magnitude) and in which direction will every mutation change a gene? The following heuristic process addresses these decisions based on the Weibull procedures described at the beginning of Section 4.1.

1. The user of the GA must define the *Mean Proportion of Individuals who will suffer a Mutation*; a parameter that will be known as $\mu pMut$. From the desired mean, and defining $Top = 1$, a Weibull distribution can be determined. A number of values are generated by following this Weibull distribution. Each generation, one of those values is randomly selected as the proportion of individuals who will suffer a mutation.
2. Following the selected proportion resulting from Step 1, the individuals to be mutated are randomly chosen from the offspring of every generation. Every offspring has the same probability of being selected. For every chosen child, only one of its genes is randomly selected to be mutated. Every gene of such individuals has the same probability of being selected.
3. The user of the GA must define the *Mean Mutation Magnitude* (μMut) for every gene, as well as the *Maximum* (*Top*) and *Minimum* (*Min*) values that will be allowed for every gene. A Weibull distribution is determined for every gene with the corresponding μMut values and adjusted from 0 to $0.5 * (Top{-}Min)$. A number of values are generated and stored in a list, for every gene, by following the defined Weibull distributions. Every time a gene is selected for mutation, a value is randomly chosen as the mutation magnitude from the corresponding Weibull-distributed list.
4. The direction of mutation ("+" or "−") is randomly selected. The mutation is then implemented in every chosen gene. If a mutated gene surpasses one boundary (*Min* or *Top*) of the allowable range, the value of the gene must be replaced by the value of the concerning boundary.

Fig. 5 illustrates various possible ways in which the mutation values could be distributed for a range $-1 \le V \le 1$ from Weibull, Gaussian and Uniform PDFs. The curves correspond to different Weibull distributions calculated for the shown μ values. As the mutation values might be added or subtracted, the curves are shown symmetrical to a vertical axis at 0. In the background, a random-uniform distribution shows how every sub-range of values has approximately the same frequency and probability of occurrence. In order to address the mutation distribution with a Gaussian distribution, as also seen in the figure, the mean is set to 0 and the *Standard Deviation* (*SD*) is adjusted for the allowable range. Following the "68-95-99.7 rule" of the Gaussian distribution [62], it can be calculated that approximately 99.95% of the values will be within the $3.5 * SD$ range. Therefore, the standard deviation can be calculated as $SD = 0.5{*}(\text{Top-Min})/3.5$.

On the other hand, elitism is also controlled by a heuristic process. These heuristics must address the questions: What proportion of the population will be selected as elites? can elites mutate? how many will mutate and by how much? Regarding the first decision, there is no obvious proportion of elites which would benefit the GA performance in any case study. In some situations, it may be beneficial to have an elite portion as small as possible in order to maximize the diversity that the non-elites may introduce. Conversely, some situations may get a better performance from a wider elite portion by increasing the reproduction among elites. Therefore, the proportion of elites could be better defined by a probabilistic Weibull distribution biased towards a reasonable mean proportion. In this way, while the mean introduced by the user is fulfilled overall, some generations will explore a wider range of possibilities regarding the elite proportion. Hence, the GA user must initially define a *Mean Proportion of Elites* (μn_e). Then, by following the same procedure of step I above, the *Proportion of Elites* (n_e) is to be determined for every generation from Weibull-distributed values. Once n_e is established for a given generation, the n_e fittest individuals are selected as elites.

The mutations should not necessarily be the same for the elites. For instance, a shark's DNA presents a much slower rate of accumulated mutations compared to other animals such as mammals [63]. Given the high degree of adaptation that sharks have already achieved in their environment, this can be interpreted as a biological way of preserving a highly effective solution, namely the shark's phenotype. The Biomimicry from this can be translated as: the elite portion of the population mutating less often or in smaller magnitudes. Therefore, the mutation of elites follows the same heuristic process (steps 1-4) but with different GA parameters that are unique for the elites: The *Mean Proportion of Elites to suffer Mutations* () and a *Mean Elite Mutation Magnitude* (μMut_e).

4.1.2. Stagnation

For the kinds of problems where GAs are implemented, there are usually different suitable solution approaches. Each approach can be fine-tuned to the best of its potential, but one of them is usually superior. Each of those different approaches, after fine-tuning, could result in a local optimum. Additionally, one (or more) of those local optimums could also be a global optimum in the constrained solution space. When a GA finds the hill towards a local optimum, it may focus the evolution resources towards reaching the best of its local potential instead of exploring whether there is another hill leading towards a global optimum. In other words, the GA is stuck or stagnated seeking a local optimum. It is worth noting that genetic algorithms do not necessarily achieve a global optimum ever. Moreover, a global optimum is not even guaranteed to exist in these kinds of problems. The aim of a GA is to effectively navigate a broad solution space seeking to converge towards the best solutions and heuristic strategies are implemented for trying to find a global optimum if it exists. Accordingly, mutation is used to reduce the tendency towards premature-convergence. However, sometimes regular mutation is not capable of introducing the variability needed to avoid stagnation. Therefore, this work proposes a heuristic process intended to increase the chances of reaching a global optimum:

1. The user defines the *Number of Generations without fitness improvement before considering the GA as Stuck* (*Gstuck*). A counter must be programmed in order to keep track of the generations without historic fitness improvement.
2. When the counter reaches a multiple of *Gstuck*, the GA first assumes that the evolution might be approaching a global maximum. Given this assumption, the elites are prioritized and the mutations are intensified in occurrence but decreased in magnitude. For this generation, the elites are cloned, doubling their influence and their chances of having offspring and reproducing with other elites. Mutation is temporarily set to occur twice as often and with half the mean magnitude: $2 * \mu pMut$, $2 *$, $0.5 * \mu Mut$ and $0.5 * \mu Mut_e$.
3. If the previous strategy did not improve the historic fittest value, then the GA assumes that it is stuck in a local maximum. The strategy is then to keep the mutation occurrence doubled, but to double as well the mean mutation magnitude as compared to the normal parameters: $2 * \mu Mut$ and $2 * \mu Mut_e$. Additionally, the population is temporarily doubled by means of new individuals whose genes have been randomly initialized.
4. If one of the previous strategies manages to improve the historic fittest value, then the stuck counter is set back to 0 and the GA parameters are set back to normal. Otherwise, if both strategies have been implemented without improvement, the stuck counter is left counting and the parameters are set back to normal until the counter

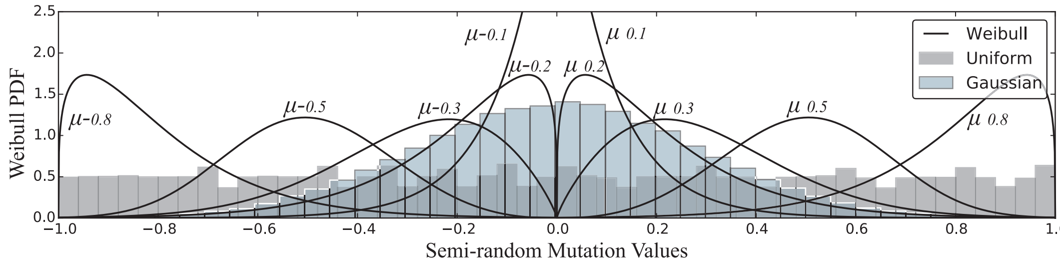

Fig. 5. Weibull-distributed mutation values compared to a Uniform and Gaussian distribution.

reaches another multiple of *Gstuck*.

The rationale for the procedures in step 2, where it is assumed that the evolutionary progress might be stuck close to a global maximum, is the following. The individuals that are more likely to be closer to that assumed global maximum are the elites, who have scored as the fittest. It is then reasonable to temporarily focus the GA's resources on them and force a more dynamic but fine-tuned exploration around their area in the solution space. The intention is to more effectively see if there is a global maximum somewhere around them. This search might be more effective with more frequent mutations but with a smaller mean mutation magnitude that does not lead the offspring too far from the local search area. On the other hand, a stagnation around a local maximum is assumed in step 3. The rationale behind the procedures in this step is to temporarily maximize variability with new random individuals and a more aggressive mutation strategy: more frequent and with more magnitude.

4.1.3. Initialization from intuition

Initialization is the first stage of a GA. During this phase, the individuals of the first generation are created with randomly defined genes. These genes are usually generated with uniform randomness within an allowable range [*Min*,*Top*], defined by the GA user, for every gene. This way, the evolution starts unbiased and broadly spread throughout the solution space. However, in some cases, it is of interest to initialize the population with a distribution which is biased towards an intuitive value of the genes (see Fig. 6). These initial gene values could be set not necessarily from intuition but from a previous parameters definition process. This capability is of great use for this work, as there was a previous exploration of the V-Trough's parameters with the interactive software VTDesign. Based on the Weibull procedures described at the beginning of Section 4.1, the following heuristic process is proposed to address a population initialization around intuitive or influential values:

1. The *Intuitive Value* (μi) is established for the desired genes. The range for the Weibull distribution must be calculated for every gene going from 0 to Top'; with $Top' = Top{-}Min$. This makes the Weibull variation range to have the same absolute size as the allowable range of every gene, but starting at 0. μi must also be displaced in order to calculate the desired Weibull mean; with $\mu i' = \mu i{-}Min$. A Weibull distribution, which satisfies Top' and $\mu i'$, can then be determined. A number of values (V') are generated by following the defined Weibull distribution. The values are then adjusted with $V = V' + Min$. The result is a Weibull distribution adjusted for the range [*Min*,*Top*] of a given gene and with a mean μi.
2. The number of non-elite individuals is calculated and their genes are randomly selected from the Weibull distributed and adjusted values.
3. The elite individuals are directly initialized with their genes being μi, namely the intuitive value which the user defined for every gene.

Fig. 6 shows a series of different ways in which a population could be distributed, from Weibull, Gaussian and Uniform PDFs, in a range $[-50,50]$ concerning one gene. The population is shown in the background with a random uniform distribution and a Gaussian distribution with mean 0 and an *SD* adjusted to the range. These two options are highly restrictive since the uniform randomness cannot be biased or reshaped and the Gaussian PDF is symmetrical, which is not always reasonable or desired. On the other hand, the PDF curves of the figure show some of the various ways in which the population could be flexibly Weibull-distributed with the procedures described in this section. Each curve is related to a desired intuitive value (μi) as the mean and they comprise only the non-elite individuals of a first generation. The curve of the middle (which is thicker) corresponds to a Weibull distribution calculated for $p\mu = 0.5$, namely for a mean which is half the distribution range. It is evident how this curve resembles the Gaussian distribution in the background.

Pursuing a further flexibility in the initialization stage of GA-WA, it was made possible for some genes to be distributed around an intuitive value, while the others can be determined with uniform randomness as usual. This level of flexibility is important when there is not the same initial amount of information about every gene.

4.2. Proposed scheme for GA-WA

A new genetic algorithm, GA-WA, is proposed by integrating the heuristic processes described in Section 4.1. The main scheme of this algorithm is presented in Fig. 7(a). It is worth noting that this scheme is designed for maximizing a given fitness function. If on the contrary, the objective is to minimize a given function, the same scheme may be used with its reciprocal (1/Fitness). Also, it is shown how three genetic processes are controlled by Weibull probability density functions: The initialization of the first population regarding a gene for which an "intuitive" starting value (μi) has been provided; the establishment of the proportion of top individuals, for every generation, who will be considered elites; and the mutation procedures in both non-elites and elites.

Every time the *Highest Fitness of a Generation* (Fittest-*G*) is not an improvement over the *Highest Historic Fitness of the Run* (Fittest-Run), a counter (Stuck) is increased by one. When Stuck becomes a multiple of *Gstuck*, the first stagnation strategy is activated. If this "Possible Global Maximum" strategy does not result in an improvement for the next generation, a second strategy increases variability. This "Possible Local Maximum" strategy is implemented as a subsequent resource because it is riskier than the first stagnation strategy. The increased magnitudes of mutation of the second stagnation strategy may cause the best solutions to be lost for the next generation, but this increased variability may also allow the population to escape from a local stagnation. The parent selection is performed with the "Roulette wheel" method and the sexual reproduction process swaps the genes at a randomly defined single point [50].

The parameters needed to control the heuristic processes of GA-WA are described in Table 1. The ones that are shown with (∗) are mutually exclusive, so the user of the algorithm must choose whether to control the mean mutation magnitude, for elites, at the general or the gene level. If $fMut_e$ is chosen, the mean mutation magnitude of the elites

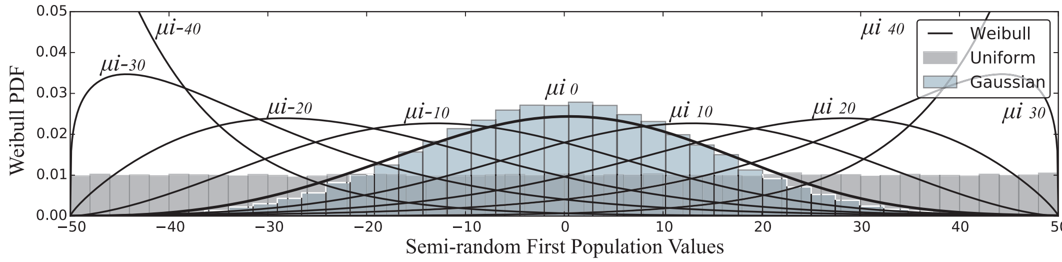

Fig. 6. Weibull-distributed values for the initial population of a gene; compared to Uniform and Gaussian distributions.

would be defined relative to the mean mutation magnitude of the non-elites. This approach may be more straightforward and intuitive. For instance, the user may want the elites to mutate, on average, half the magnitude of the non-elites ($fMut_e = 0.5$). Additionally, the parameter μi is optional and can be defined only for the genes that have an intuitive or pre-defined value.

For comparison purposes, two other reference genetic algorithms with standard heuristic processes are described, i.e., "GAGaussian" and "GAUniform". Fig. 7(b) illustrates the scheme which they both follow. The initial population is always defined with uniform randomness, the elites are selected according to a *Fixed Elite Proportion* (n_e), and there is no stagnation strategy. These reference genetic algorithms differ from each other only in the mutation phase. Their procedure for distributing the mutation magnitude is illustrated in Fig. 5 and described in Section 4.1.1. A parameter *pMut* is entered as the *Probability that any given Gene Mutates*.

Any of the performance indicators and indices presented in Section 3 can be defined as the fitness function for GA-WA or for the two GAs used as references. These potential fitness functions all depend on a set of parameters/variables which can be assigned to one of two categories, i.e., the needs and constraints of a given case study, or the geometrical set-up of a V-Trough device. The latter is the category of interest to be iterated, by the genetic algorithms, in the optimization process. These variables, which define the geometrical set-up, are the lengths LL, LR and LPV; and the angular positions ψL, ψR and β. Seeking to simplify the optimization problem, LPV can be set as 1 and the other lengths are then defined as a proportion of LPV. Therefore, a solution obtained with the GAs should fully determine the behavior of the five geometrical variables: LL, LR, ψL, ψR and β. Additionally, each of those variables can be defined as a fixed value or as a value which dynamically varies in function of the solar elevation (α). Then, depending on how each variable is defined, they must be translated into genes which can feed the evolutionary process. The following modalities are proposed as ways to define the behavior of such geometrical variables:

- **DF**: The variable is defined as a fixed value entered by the user. No genes are assigned to this variable.
- **DL**: The variable is defined as a list entered by the user. The values of the list determine the value of the variable for every α value to be considered in the calculations. No genes are assigned to this variable.
- **RF**: The variable is defined as a random fixed value to be specified by the GA. Since it is fixed, it does not depend on the solar elevation α. One gene is assigned to this variable.
- **RD**: The variable is defined as a random dynamic value, to be specified by the GA, according to a user-defined list of α tracking points. The user must specify a list with an arbitrary number n of solar elevations α, for which the variable will change to a new position. $(n + 1)$ genes are assigned to this variable, corresponding to the positions which the variable will adopt in function of α.
- **RRD**: The variable is defined as a random dynamic value without a user-defined list of α tracking points. The user must specify the number n of α tracking points where the variable should change its position. $(2 * n + 1)$ genes are assigned to this variable, corresponding to the positions which the variable will adopt in function of α, as well as the α tracking points themselves.
- **RRDsame**: The variable is defined in the same way as in **RRD**. However, if there are several **RRDsame** variables, they will share the same α tracking points as soon as they are defined by the GA genes.

These different modalities allow an unprecedented flexibility, in terms of geometrical exploration, that was not found in other reported V-Trough studies.

5. Optimization of the genetic heuristic parameters

The effectiveness and efficiency of a GA can be greatly affected by the tuning of the parameters which control its heuristic processes [29]. Therefore, this section presents the results of a partial optimization of the genetic parameters for GA-WA and both reference algorithms, i.e., GAUniform and GAGauss.

The most relevant GA parameters were selected and their variation was constrained to three discrete levels [0.1, 0.5, 0.9], corresponding to a subjective interpretation of [low, intermediate, high]. Although those are originally continuous parameters, they were discretized in order to reduce the computational expense and for a more practical initial optimization. Consequently, it was possible to explore all possible combinations of the discretized GA parameters and run each combination 10 times. The performance of each combination was measured according to the fitness function I_{COE} in a simple and common scenario for V-Trough devices.

Table 2 defines the optimization scenario. The GA parameters control the heuristic processes, while the set-up parameters control how the genetic algorithms explore the V-Trough geometry. The parameters marked with (∗) were the ones which were partially optimized in discrete levels. There were hence nine possible combinations (two parameters with three levels each) for GAUniform and GAGauss, and 243 combinations (five parameters with three levels each) for GA-WA. Within the GAs, there were seven genes iterated in the evolution process: four genes defined the RF length and angular position of each mirror, and three genes defined the RD β angular positions which changed according to the provided α tracking-points [60°,120°]. In order to calculate I_{COE} as the fitness function, other needed fixed parameters were defined as $LPV = 1, \rho = 0.85, \alpha f = 0°, \alpha c = 180°, CO_{pv} = 600 USD/\mathrm{m}^2, CO_m = 13.33 USD/\mathrm{m}^2$, and $CO_s = 62.23 USD/\mathrm{m}^2$ [24].

Fig. 8 shows the performance of all the GA parameter combinations for the three genetic algorithms. Three different curves appear, corresponding to different perspectives for measuring the performance of a GA:

- *Fittest/Combination* considers only the highest fitness achieved during the 10 runs of each combination.
- *Mean Fittest/Run* averages among the highest fitness achieved for every run.

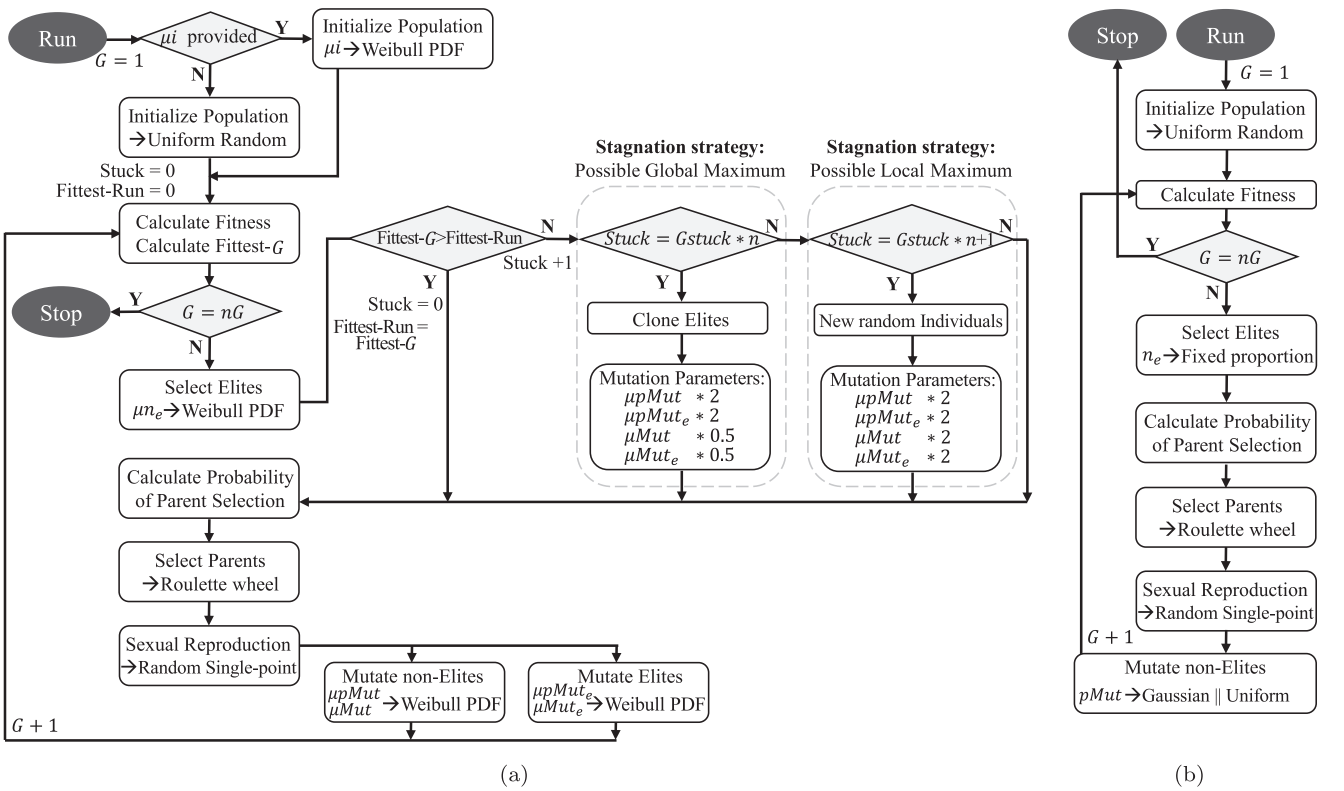

Fig. 7. Main scheme of (a) the proposed genetic algorithm GA-WA and (b) a standard genetic algorithm. n is any positive integer greater than 0.

- *Mean Fittest/G* averages among the highest fitness achieved for every generation of the evolutionary process.

Unlike the others, *Mean Fittest/G* is affected not only by the end result of each run, but also by how fast the GA converges throughout the generations. For GAUniform and GAGauss, combination 2 ($n_e = 0.5$ and $pMut = 0.1$) achieved the highest performance in *Fittest/Combination*, while combination 1 ($n_e = 0.1$ and $pMut = 0.1$) performed best in terms of *Mean Fittest/Run* and *Mean Fittest/G*. In the case of GA-WA, combination 208 ($\mu n_e = 0.5, \mu pMut = 0.9, \mu pMut_e = 0.9, fMut = 0.1$ and $fMut_e = 0.1$) performed best in terms of *Fittest/Combination*, combination 175 ($\mu n_e = 0.1, \mu pMut = 0.9, \mu pMut_e = 0.5, fMut = 0.1$ and $fMut_e = 0.5$) performed best in terms of *Mean Fittest/Run*, and combination 183 ($\mu n_e = 0.1, \mu pMut = 0.9, \mu pMut_e = 0.9, fMut = 0.9$ and $fMut_e = 0.1$) performed best in terms of *Mean Fittest/G*.

Fig. 9 illustrates the importance of tuning the GA parameters. The graphs here show the highest I_{COE} fitness, achieved for every generation, for the worst and best (optimized) combinations of the three algorithms. Every curve corresponds to the progression of a GA for one run. The so-called "worst" (combination 9 for GAUniform and GAGauss, and combination 162 for GA-WA) present evolutionary paths which are more dispersed, converge to different points and the highest fitness tends to be smaller in comparison.

As there are random processes involved in the performance of any GA, it is worth analyzing the results from a statistical point of view: are the differences in performance between the three GAs and the effects of the combinations of heuristic parameters significant? An appropriate statistical method was chosen for addressing this inquiry. Parametric

Table 1
Parameters for controlling GA-WA, GAGaussian and GAUniform genetic algorithms.

Parameter	Level	Genetic Algorithm	Description
nG	General	all	No. of generations
Gstuck	General	GA-WA	No. of generations, without improvement, for stagnation
nP	General	all	No. of individuals in the population
μn_e	General	GA-WA	Mean proportion of elites
n_e	General	GAUniform, GAGauss	Fixed proportion of elites
$\mu pMut$	General	GA-WA	Mean proportion of non-elites who will mutate
$\mu pMut_e$	General	GA-WA	Mean proportion of elites who will mutate
$fMut_e$	General*	GA-WA	Elite mutation factor. $fMut_e = \mu Mut_e/\mu Mut$
μMut	Genes	GA-WA	Mean mutation magnitude for a gene in non-elites
μMut_e	Genes*	GA-WA	Mean mutation magnitude for a gene in elites
μi	Genes	GA-WA	Desired intuitive value for a gene; Optional
Top	Genes	all	Maximum value allowed for a gene
Min	Genes	all	Minimum value allowed for a gene
pMut	Genes	GAUniform, GAGauss	Probability for any given gene to mutate

Table 2
GA optimization parameters.

GA Parameters	Genetic Algorithm	Definition
nG, nP	all	100
$Gstuck$	GA-WA	5
μn_e, $\mu pMut$, $\mu pMut_e$, $fMut_e$	GA-WA	*Levels [0.1, 0.5, 0.9]
$fMut$	GA-WA	*Levels [0.1, 0.5, 0.9]; $\mu Mut{=}(Top/4){*}fMut$
n_e, $pMut$	GAUniform, GAGauss	*Levels [0.1, 0.5, 0.9]
Set-up Parameters	**Genetic Algorithm**	**Definition**
LL, LR	all	RF. Top = 2.5, Min = 0. 2 Genes
ψL, ψR	all	RF. Top = 90°, Min = −90°. 2 Genes
β	all	RD. Top = 180°, Min = −180°, α [60°,120°]. 3 Genes

tests could not be used because the residuals of the obtained data presented clear violations of normality and homogeneity of variance. Several transformations of the data were implemented, i.e., $(1/y)$, $ln(y+1)$, $log_{10}(y+1)$, $\sqrt{y}$ and arcsin; but none could satisfy the parametric assumptions for the residuals. Therefore, the Kruskal-Wallis rank sum test was used as a non-parametric approach for discrimination of stochastic dominance, namely "The probability that a randomly drawn observation from one group will be greater than a randomly drawn observation from another" [64].

With Kruskal-Wallis tests, it was possible to verify that the combination of the GA heuristic parameters had a highly significant effect on the *Fittest/Run* performance of GAUniform, with *P*-value = 1.39E−12; of GAGauss, with *P*-value = 4.15E−13; and of GA-WA, with *P*-value <2.2E−16. It is therefore highly unlikely that the variations in GA performance, shown in Fig. 8, were caused by randomness alone and it is hence assumed that the combinations had a real effect on the measured GA performance.

The combinations involve a selection of values for various parameters. It is also of interest to explore the isolated overall effect of each of those parameters. Fig. 10 presents how the *Fittest/Run* results are distributed, for every GA, and for every level of the GA heuristic parameters. In GAUniform, the parameters n_e and $pMut$ both had a significant effect; with *P*-values 1E−4 and 5.75E−12, respectively. Likewise, in GAGauss, the parameters n_e and $pMut$ both had a significant effect; with *P*-values 2.31E−8 and 4.13E−9, respectively. In GA-WA, the parameters μn_e, $fMut$ and $fMut_e$ had a significant effect; with *P*-values <2.2E−16, 1.49E−4 and 1.56E−4, respectively. On the other hand, the parameters $\mu pMut$ and did not have a significant effect, with *P*-values 6.1E−2 and 7.7E−2. Going further, Dunn's test, with the Benjamini-Hochberg adjustment [64], was used as a non-parametric pairwise comparison of the levels of each parameter. The pairs of level-groups that presented a significant difference are highlighted in the

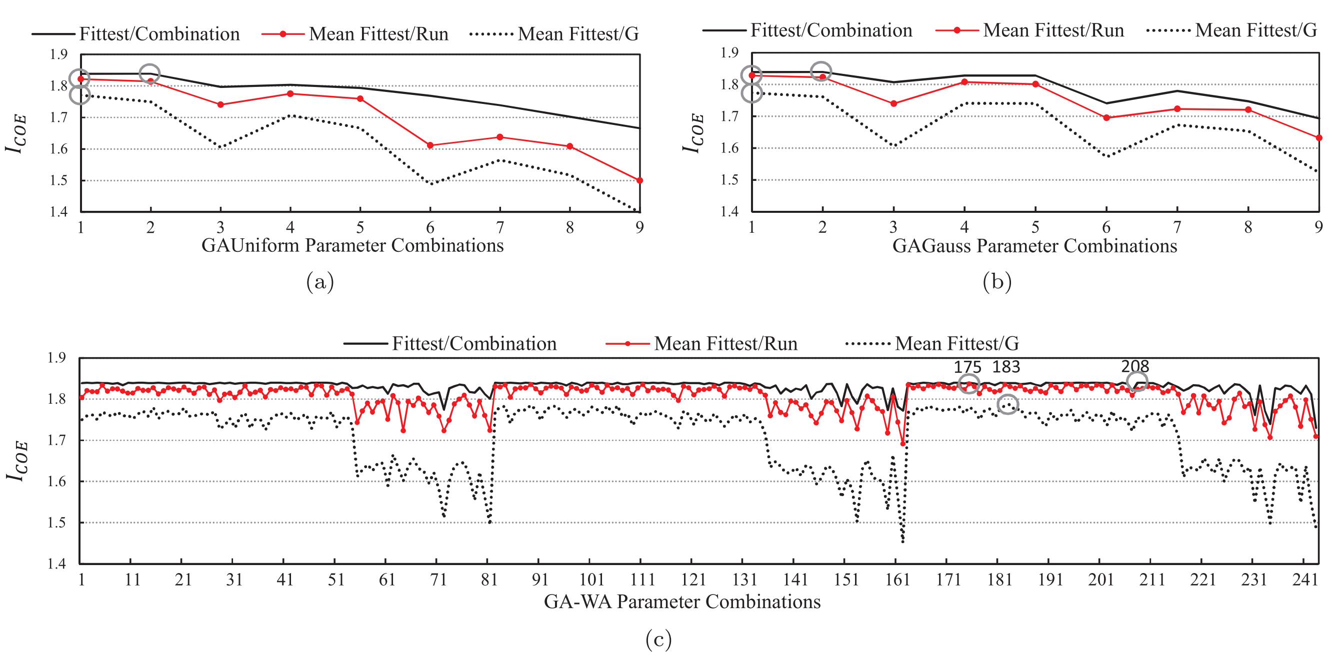

Fig. 8. Performance of the GA parameter combinations for (a) GAUniform, (b) GAGauss and (c) GA-WA. The combinations with the highest performance are highlighted with circles.

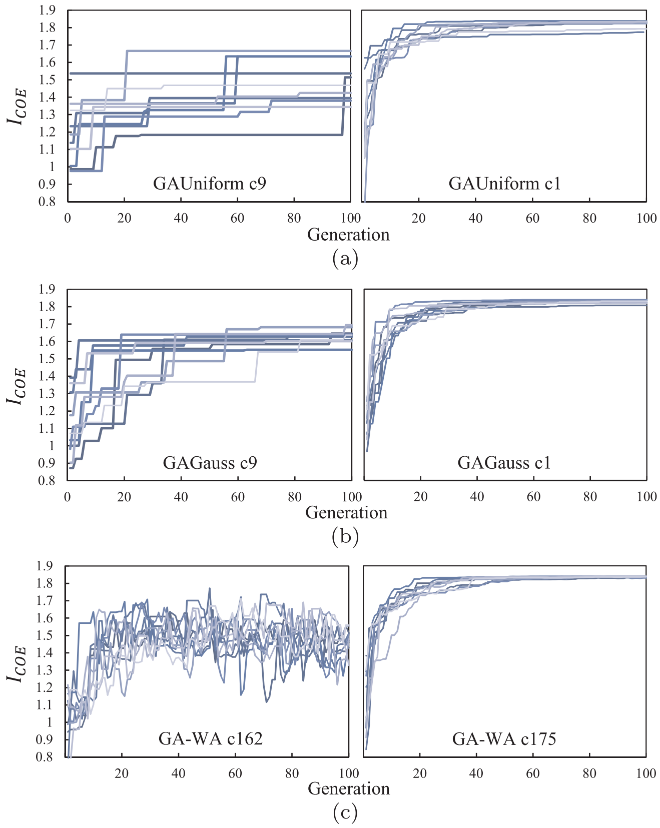

Fig. 9. Worst and best parameter combinations for (a) GAUniform, (b) GAGauss and (c) GA-WA.

figure with (∗).

The *Mean Fittest/Run* performance is the most decisive perspective for comparing different GAs because it indicates the fitness at which a GA will tend to converge after every time that it is used. From this perspective, the best performing configurations found were combination 1, for both GAUniform and GAGauss, and combination 175 for GA-WA. Fig. 11(a) shows a close-up of the evolutionary progression, throughout 100 generations, of the three optimized GAs. These curves illustrate how GA-WA converges most of its evolutionary progressions towards a narrower range at the top, as compared to the other two GAs. From this performance perspective and considering only the optimized combinations of parameters, was GA-WA significantly superior to the reference GAs? This inquiry is statistically addressed as follows.

The comparison of how the results of these three GAs are distributed is shown in Fig. 11(b). The shown Box Plots correspond to the fittest results achieved in each of the 10 times that the GAs were Run. By performing the Dunn's test, with the Benjamini–Hochberg adjustment, it was found that GA-WA (combination 175) presented a significant superiority when compared to both GAUniform (combination 1) and GAGauss (combination 1). Moreover, there was no significant difference in the case of GAUniform when compared to GAGauss.

6. Implementation of GA-WA in a case study

This work is contextualized in the general applied problem of optimizing the V-Trough geometrical parameters, from multiple performance perspectives, for a personalized design scenario. However, it is also of interest to explore the capabilities of the proposed optimization framework in a specific case study. In this section, GA-WA is implemented to approach a personalized V-Trough scenario; the same design scenario that was reportedly used to explore the capabilities of the interactive software VTDesign [23]. For this analysis, GA-WA was set with the combination of heuristic parameters found (in Section 5) to have the best performance: $\mu n_e = 0.1, \mu pMut = 0.9, \mu pMut_e = 0.5, fMut = 0.1$ and $fMut_e = 0.5$. The results obtained with GA-WA are compared against the solutions reportedly achieved with VTDesign [23].

The case study was established around a rural Colombian family seeking to fulfill their daily energy demand ($E_{day} = 1495.4$ Wh) with a V-Trough system. The average irradiance at their location is $\bar{I} = 354.17\ \mathrm{W/m^2}$, which was assumed to behave as beam radiation. Since their exposure to the sun is not particularly privileged, the family wants to maximize the performance of their harvesting system as

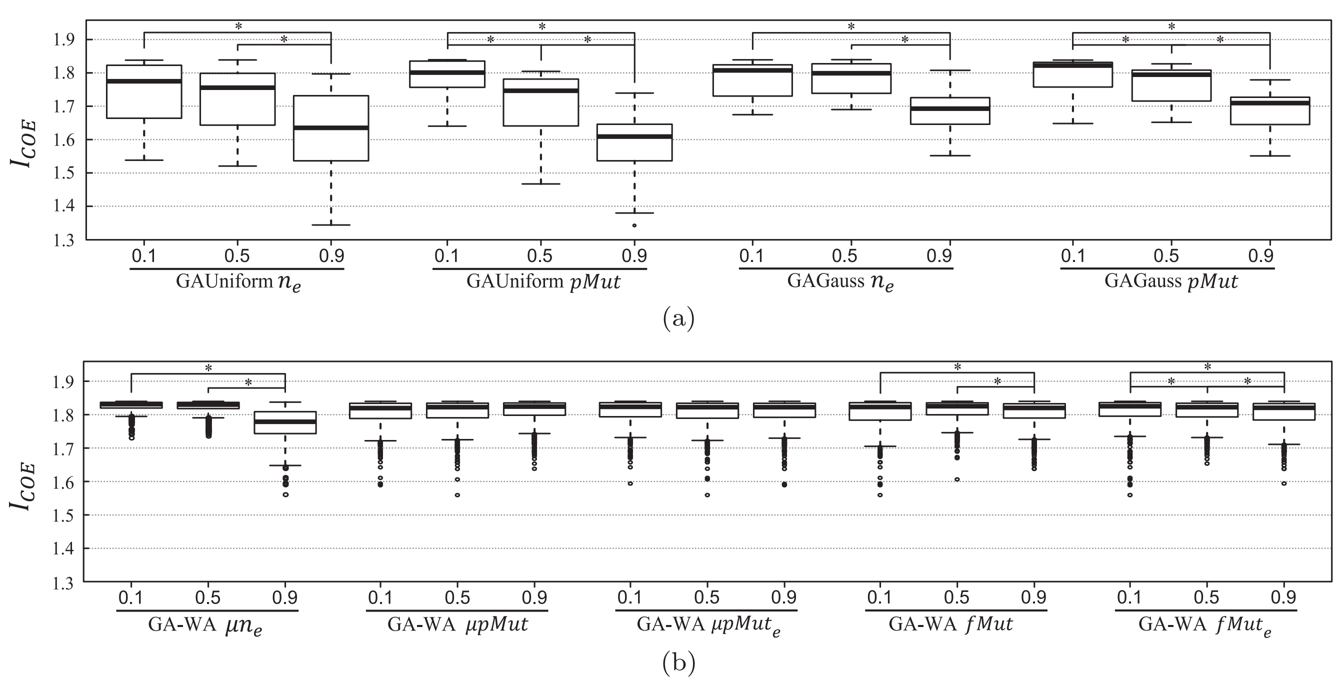

Fig. 10. Performance Box Plots in function of the GA heuristic parameters for (a) GAUniform and GAGauss, and for (b) GA-WA.

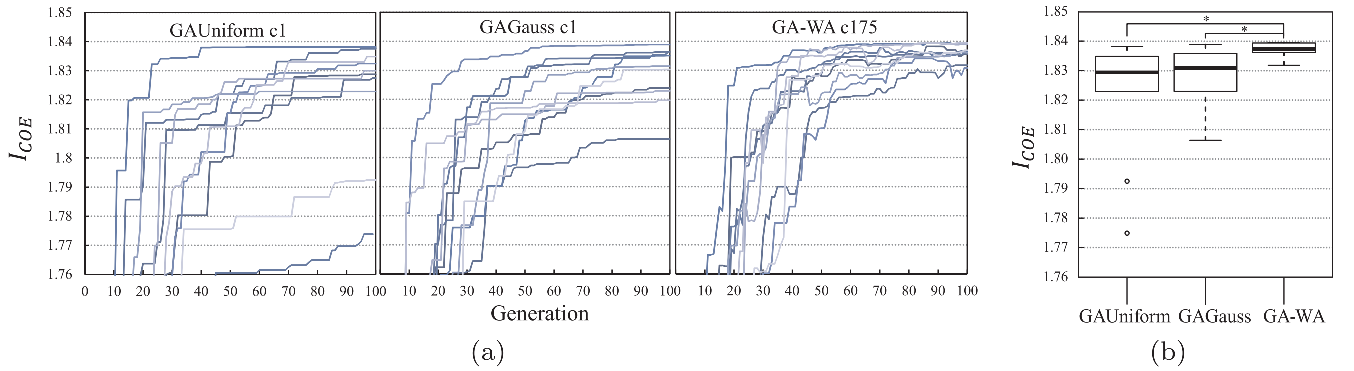

Fig. 11. I_{COE} Performance of the optimized parameter combinations for GAUniform, GAGauss and GA-WA. (a) Close-up of the evolution through the generations. (b) Box Plot.

compared to a regular fixed and flat solar panel. They do not desire the complexity and additional capital cost of an automatic tracking system. However, they are willing to manually adjust the position of the system twice a day, i.e., at 10 a.m. ($\alpha = 60°$) and at 2 p.m. ($\alpha = 120°$). In the afternoon, the sun gets blocked in the horizon by a mountain, so they have an available solar elevation range of $0° \le \alpha \le 160°$. They have access to regular silicon photovoltaic cells with an efficiency of $\eta_{pv} = 15\%$ and a cell area of $A_{pvc} = 0.024$ m^2. Based on the specifications of their power inverter, they assume a system efficiency of $\eta_s = 84\%$. The V-Trough's geometrical parameters, the mirror's index of reflection ρ and the area costs were defined in the same way as described in the optimization procedure in Section 5 (see Table 2). These parameters correspond to a case where a V-Trough has mirrors with fixed lengths and inclinations and the solar tracking is performed by adjusting β. Therefore, the optimization problem resides in determining: how β should be dynamically adjusted along the day; the lengths of the V-Trough's elements; and the angular positions of the mirrors. Considering that the lengths are established in proportion of LPV, and that three positions must be established for β to be adjusted twice, the following parameters can fully constrain the V-Trough set-up within the case study: LL, LR, ψL, ψR, and $[\beta_1, \beta_2, \beta_3]$.

In this case study, the people, their context and their needs are based on real facts taken from a series of exploratory field trips performed during 2014. Therefore, the family is defined as to represent a typical scenario for a rural and low-income household in a developing country. The design/engineering problem of satisfying the energy needs of this theoretical family was approached from three different *Design Goals* and with both VTDesign and GA-WA. The optimizations with GA-WA were performed with several Indicators and Indices (see Section 3) used as fitness functions. For each fitness function used, GA-WA converged into a particular V-Trough set-up. As follows, the performance of these set-ups, resulting from both GA-WA optimizations and interactive explorations with VTDesign, are compared according to the corresponding *Design Goal*:

Design Goal I (Cost): To minimize the cost of the system, in terms of materials, while fulfilling the given energy need (1495.4 Wh/day). Five set-ups were developed for this matter: $VT1_i$ (intuitive starting point); $VT1_{VTD}$ (from VTDesign); $VT1_{icoe}$ (from GA-WA with I_{COE} as fitness function); $VT1_{cost}$ (from GA-WA with *Cost* as fitness function); and $VT1_{icost}$ (from GA-WA with *Cost* as fitness function and with the population initialized around the intuitive set-up). As shown in Fig. 12(a) and detailed in Table 3, the three different runs with GA-WA converged into similar set-ups and with almost equivalent costs in terms of materials; differing in less than $2USD$. The set-up $VT1_{icost}$ achieved the lowest cost in terms of materials, with a cost reduction of 4.3% as compared to the intuitive $VT1_i$ and 0.6% as compared to the fittest from VTDesign ($VT1_{VTD}$). Moreover, the *Cost* with $VT1_{icost}$ was 42.2% less than the reference flat and fixed horizontal solar panel; a considerable gain in cost-effectiveness. In Table 3, MI_{COE} and MDI_{COE} were calculated with $W1 = 0.9$ and $W2 = 0.1$ for these five set-ups.

Design Goal II (Energy): To maximize the daily energy within an available 10 m^2 and with only 80 cells to allocate in the system. Five set-ups were developed for this matter: $VT2_i$ (intuitive starting point); $VT2_{VTD}$ (from VTDesign); $VT2_{ce}$ (from GA-WA with $\overline{C_e}$ as fitness function); $VT2_{micoe}$ (from GA-WA with MI_{COE} as fitness function); and $VT2_{ice}$ (from GA-WA with $\overline{C_e}$ as fitness function and with the population initialized around the intuitive set-up). As shown in Fig. 12(b), and detailed in Table 3, $VT2_{VTD}$ achieved the greatest daily energy harvesting. $VT2_{VTD}$ increased E_{day} by 19.1%, as compared to the intuitive $VT2_i$, by 4%, as compared to the fittest GA-WA set-up ($VT2_{ce}$), and by 142.7%, as compared to a reference fixed and flat solar panel with 80 cells. The set-ups $VT2_{ce}$ and $VT2_{ice}$ converged into similar geometries and energy results; differing in less than $2Wh$. Even though they achieved the highest $\overline{C_e}$ values, they were surpassed in terms of energy by $VT2_{VTD}$ because they could only use 76 solar cells without exceeding the available space for the device. On the other hand, $VT2_{micoe}$, by considering also the space minimization, achieved the highest I_{COE}, MI_{COE} and MDI_{COE} indices. Being more compact, $VT2_{micoe}$ could have allocated up to 201 cells in the available 10 m^2 and surpass the E_{day} of all the other set-ups; but the 80 cells limit greatly restricted its energy performance. Overall, the different results showed more divergence as compared to the *Design Goal I*. In Table 3, MI_{COE} and MDI_{COE} were calculated with $W1 = 0.9$ and $W2 = 0.1$ for these five set-ups.

Design Goal III (Space): To minimize the space needed to allocate the device while fulfilling the energy need (1495.4 Wh/day). Five set-ups were developed for this matter: $VT3_i$ (intuitive starting point); $VT3_{VTD}$ (from VTDesign); $VT3_{mdicoe}$ (from GA-WA with MDI_{COE} as fitness function); $VT3_{tsp}$ (from GA-WA with T_{sp} as fitness function); and $VT3_{itsp}$ (from GA-WA with T_{sp} as fitness function and with the population initialized around the intuitive set-up). As compared to the intuitive $VT3_i$, the tracking strategy of $VT3_{VTD}$ (presented in Table 3) directly decreased the area footprint by tilting the PV surface, which reduced the space required in 28%. This also increased $\overline{C_e}$, which in turn allowed the use of only 135 solar cells as opposed to the 187 cells with $VT3_i$. All five set-ups converged in that they eliminated the mirrors to reduce their occupied space. Moreover, the three GA-WA set-ups resulted in virtually the same set-up, where the space required was minimized to virtually 0 with the panel left almost vertical and then flipped in order to track the sun. As illustrated in Fig. 12(c), the genetic exploration greatly surpassed the intuitive and VTDesign set-ups. These results presented a major divergence from the intuitive strategies explored and revealed the capacity of GA-WA to "escape" from a particularly detrimental human bias in the initialization. This is the case with $VT3_{itsp}$, which was initially biased towards the parameters of $VT3_i$ but evolved into a superior strategy in terms of T_{sp}. $VT3_{itsp}$ achieved the lowest space needed to allocate the device, with a required space which was only 0.02% of the space with $VT3_i$ and 0.028% with $VT3_{VTD}$. In Table 3, MI_{COE} and MDI_{COE} were calculated with $W1 = 0.2$ and $W2 = 0.8$ for

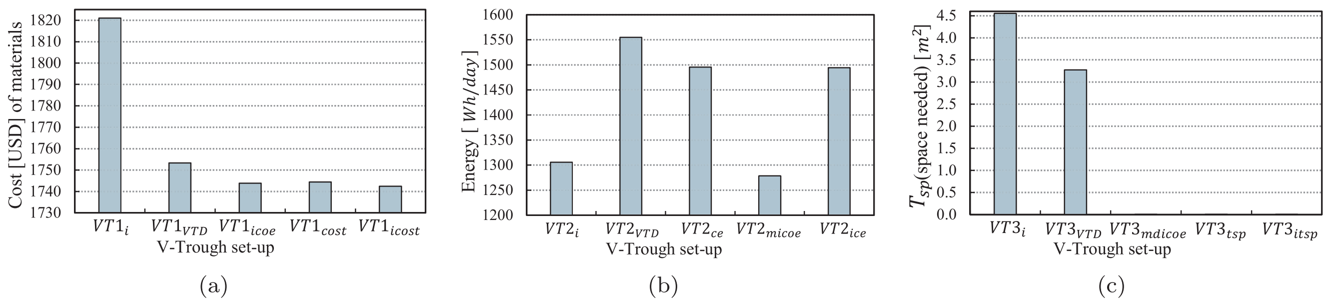

Fig. 12. Results of the case study. (a) Design Goal I. (b) Design Goal II. (c) Design Goal III.

these five set-ups.

Fig. 13 presents the optical performance of the selected set-ups for every Design Goal, i.e., $VT1_{icost}$ (*Design Goal I*), $VT2_{VTD}$ (*Design Goal II*), and $VT3_{itsp}$ (*Design Goal III*). These graphs show how the *Effective Concentration* (C_e) [24] varies along the day as the solar elevation (α) increases. A shadow in the graphs delimits the stated solar elevation range of $0° \le \alpha \le 160°$. The *Incident Optical Concentration* (C) [24] curves are also shown, which indicate the proportional suns that reach the device as a whole, regardless of whether the rays involved actually manage to reach the PV surface. The differences between the C_e and C curves are due to the solar beam radiation that is lost because of internal shadows, the rays that are reflected back out without reaching the PV surface and because of the radiation that is not perfectly reflected from the mirrors (due to their *Index of Reflection* (ρ)). The C_e and C curves are equal in Fig. 13(c) because this set-up has no mirrors. In Fig. 13(a) and (b) there are three performance peaks clearly identifiable, which correspond to the three tracking positions that the V-Trough adopts in terms of β. On the other hand, $VT3_{itsp}$ in Fig. 13(c) has only two discernible performance peaks because its set-up converged into virtually only two β positions.

The graphs in Fig. 13 also show the C_e performance of a reference device with the same PV area as the V-Troughs but without mirrors or tracking adjustments. Comparing the C_e curves against the C_eRef curves illustrates the optical advantage of using the V-Trough technology to increase the solar harvesting area. It is also evidenced how the performance curves correspond well to the *Design Goals*. $VT2_{VTD}$, designed to maximize the daily energy, converged into a set-up with large mirrors that achieves three high C_e peaks of up to 2.3 proportional suns. $VT1_{icost}$, designed to minimize the cost of materials, converged into a more balanced set-up. It is worth noting how the performance of this set-up never drops below the C_eRef curve despite having mirrors of moderate size and only two tracking adjustments. $VT3_{itsp}$, on the other hand, was designed to minimize the horizontal space required, so its performance resembles the one of the reference curve but split into two sections.

Fig. 14 illustrates some of the practical implications when applying the selected set-ups to more detailed product designs. Fig. 14(b), (e) and (h) respectively show the element's proportions and the changing β positions of V-Troughs with the set-ups $VT1_{icost}$, $VT2_{VTD}$ and $VT3_{itsp}$. Fig. 14(a), (d) and (g) respectively show the horizontal space required for devices with the set-ups $VT1_{icost}$, $VT2_{VTD}$ and $VT3_{itsp}$. This linear space (aT_{sp}), in the transverse plane, was calculated with Eq. (14) and it does not consider the elements' thickness or the footprint of the structure holding the device. However, aT_{sp} serves here as a comparative indicator that relates well to the corresponding *Design Goals*. For instance, the aT_{sp} of $VT3_{itsp}$ tends to zero while $VT2_{VTD}$ presents the largest aT_{sp}. These figures also illustrate the potential PV area reduction as compared to a reference fixed and horizontal panel. This relates to the $\overline{C_e}$ achieved by each set-up because, the larger the optical effectiveness, the smaller the PV area can be to satisfy the same energy demands. Consequently, as $VT2_{VTD}$ has the highest $\overline{C_e}$ between the three selected set-ups, it also has the greatest reduction in PV material. On the contrary, $VT3_{itsp}$ requires 25.7% more PV area as compared to the reference device because its $\overline{C_e}$ is less than 1 (see Table 3).

Table 3
Results of the case study. *LL* and *LR* as proportions of *LPV*; ψL, ψR and β in [deg]; T_{sp} in [m²]; *Cost* in [USD]; and E_{day} in [Wh]. The most relevant indices are highlighted according to the corresponding Design Goal.

	Geometrical set-up					Performance Indicators and Indices						
Set-up	*LL*	*LR*	ψL	ψR	β	$\overline{C_e}$	I_{COE}	MI_{COE}	T_{sp}	*Cost*	MDI_{COE}	E_{day}
$VT1_i$	1	1	30	30	[60, 0, −60]	1.409	1.659	1.575	5.597	**1821**	1.571	1501.4
$VT1_{VTD}$	0.8	0.8	25	25	[60, 4, −52]	1.403	1.716	1.64	4.778	**1753.4**	1.642	1495.4
$VT1_{icoe}$	1.01	1	23.55	25.31	[59.13, −0.42, −51.87]	1.471	1.73	1.641	5.441	**1743.9**	1.639	1499.5
$VT1_{cost}$	0.90	1	25.43	25.74	[60.61, 1.13, −47.66]	1.451	1.726	1.639	5.277	**1744.4**	1.641	1496.1
$VT1_{icost}$	1.04	0.96	21.73	27.33	[57.55, −3.97, −51.62]	1.468	1.728	1.639	5.429	**1742.5**	1.64	1496.1
$VT2_i$	1	1	30	30	[60, 0, −60]	1.409	1.659	1.575	4.867	1583.5	1.806	**1305.6**
$VT2_{VTD}$	2.35	2.35	20	20	[60, 4, −52]	1.678	1.58	1.469	9.943	1980.7	1.415	**1554.8**
$VT2_{ce}$	2.5	2.5	8.19	31.85	[46.44, −13.47, −63.23]	1.699	1.564	1.454	9.964	1923.6	1.456	**1495.4**
$VT2_{micoe}$	0.94	0.56	17.86	33.09	[52.15, −10.39, −59.19]	1.38	1.704	1.632	3.962	1509.7	1.911	**1278.6**
$VT2_{ice}$	2.5	2.5	19.45	21.06	[60.67, 0.71, −52.41]	1.697	1.563	1.452	9.987	1923.6	1.456	**1494.3**
$VT3_i$	0	0	–	–	[0, 0, 0]	0.691	1	1	**4.551**	3013.7	1	1497.6
$VT3_{VTD}$	0	0	–	–	[60, 4, −52]	0.96	1.389	1.392	**3.277**	2175.7	1.388	1501.6
$VT3_{mdicoe}$	0	0	–	–	[89.98, −90.01, −90.03]	0.547	0.791	1382.8	**0.0026**	3819.5	1379.5	1501.2
$VT3_{tsp}$	0	0	–	–	[89.99, 90.01, −90]	0.55	0.795	3476.3	**0.001**	3787.3	3477.6	1497
$VT3_{itsp}$	0	0	–	–	[90.01, 89.99, −90]	0.55	0.795	3947.4	**0.0009**	3787.3	3948.8	1497

Fig. 14(c), (f) and (i) show how the geometrical set-ups obtained through GA-WA and VTDesign can be used for a more detailed design and with further product-related considerations; respectively for $VT1_{icost}$, $VT2_{VTD}$ and $VT3_{itsp}$. These figures show realistic 3D renderings contextualized in the real scenario of the case study. Their design shows how these set-ups can be implemented with simple-to-build wooden structures that can be adjusted twice a day in a practical manner. With Eq. (4), the number of solar cells needed was calculated for the three set-ups and they were arranged in rows (longitudinal direction) and columns (transverse direction): for $VT1_{icost}$, $N_{pvc} = 88$ arranged in 4 rows and 22 columns of cells; for $VT2_{VTD}$, $N_{pvc} = 80$ arranged in 4 rows and 20 columns of cells; and for $VT3_{itsp}$, $N_{pvc} = 235$ arranged in 5 rows and 47 columns of cells. The applied design of $VT2_{VTD}$ was developed differently from the one of $VT1_{icost}$ in order to demonstrate how different detailed configurations can be used to implement a given V-Trough set-up. Also, the $VT3_{itsp}$ detailed system evidently contrasts with the one of $VT1_{icost}$ even though they generate almost the same daily energy (see Table 3). These differences illustrate how V-Trough devices can be tailored to specific scenarios and performance priorities. For further development of these set-ups as products, other considerations are needed, such as mechanisms to lock the devices at a given β angle and markings in their structure that indicate the user how and when to modify the β inclination. It is worth noting that these shown simple structures are suitable when only β is established as a dynamic parameter. Other axes and mechanisms should be added if, for instance, ψL and ψR were also dynamic parameters that could be adjusted for tracking purposes.

This case study serves as a demonstration of how a specific design scenario, with specific performance priorities, can be addressed with both GA-WA and VTDesign in order to obtain candidate V-Trough geometrical set-ups. These set-ups then serve as a guideline for a more detailed product design. As compared to the results achieved by a trained engineer, interactively exploring V-Trough set-ups with VTDesign, GA-WA converged into similar solutions or even superior in the case of the *Design Goals* I and III. The evolutionary capacities and the portfolio of fitness functions of GA-WA allowed a broader exploration that can be more directly tailored to specific design goals. This high competitiveness against a dedicated interactive software has positive practical implications because it entails that V-Trough systems could be designed for personalized scenarios without the need of an engineered trained in the subject. Remarkably, the average run-time for each of the GA-WA explorations in the case study was only 2 min and 36 s. These were run with the algorithms programmed in Python and in a conventional personal computer with 12 GB RAM, 2.5 GHz CPU and with Intel CORE i7. This low computational expense also has positive practical implications because it entails that GA-WA, or other similar genetic algorithms, could very well be implemented in an online or smart-phone application, which supports the sought democratization of the V-Trough technology. However, when a goal or constraint is not directly considered by the available GA-WA fitness functions, VTDesign offers a way to approach them with conscious intuition. Accordingly, these tools were found to be complementary and can be used in parallel or in series to further explore the results from one another.

7. Conclusions

The procedures presented in this work allow to adjust Weibull distributions to a specific range and to shape their function with a bias towards the desired given mean. This capability is useful for controlling, with a high detail and flexibility, semi-random processes. Randomness is critical in genetic algorithms to introduce variability and to favor a wider exploration of the solution space. The new genetic algorithm proposed, GA-WA, was shown to effectively make use of these Weibull procedures to control various heuristic processes that define: the number of individuals who mutate and by how much; the proportion of elites for every generation; the implementation of strategies to overcome stagnation; and the initialization of the first population with a bias around a pre-existing or intuitive value for any gene.

The parameters that control the heuristic processes of the assessed genetic algorithms were discretized into only three levels (0.1, 0.5 and 0.9), for a partial optimization, based on the ability to improve the index I_{COE}. Despite this discretization, the optimization procedure managed to effectively improve the behavior towards convergence in I_{COE} of the three genetic algorithms, namely, the proposed GA-WA and the reference algorithms GAUniform and GAGauss. The optimized versions were more organized in their progression throughout the generations, converged to a narrower range and achieved, on average, a higher I_{COE} stagnation ceiling. For instance, the best combination of parameters in GA-WA, as compared to the combination with the worst performance, improved the mean highest I_{COE} fitness achieved per Run in an 8.6% and the mean highest I_{COE} fitness achieved per generation in a 23.2%. The performed optimization allowed the three genetic algorithms (GAUniform, GAGauss and GA-WA) to reach, on average, solutions with a higher fitness and with a higher reliability and consistency in their results. Moreover, although the heuristic parameters were optimized with I_{COE} as fitness function, the three optimized algorithms also performed satisfactorily well with the other explored fitness functions in the case study, namely $\overline{C_e}$, $Cost$, T_{sp}, MI_{COE} and MDI_{COE}.

The combination of the heuristic GA parameters was found to have a highly significant effect, on the mean fittest I_{COE} achieved per run, in the algorithms GAUniform (P-value $= 1.39E{-}12$), GAGauss (P-value $= 4.15E{-}13$) and GA-WA (P-value$<2.2E{-}16$). Furthermore, all heuristic parameters were found to independently have a significant effect, except for the *Mean Proportion of Individuals to Mutate* in elites (P-value = 0.077) and in non-elites (P-value = 0.061) for GA-WA. For the parameters *Proportion of Elites* and *Probability of Mutation in a given gene*, 0.1 was the value associated with the best performance in both GAUniform and GAGauss. Conversely, a value of 0.9 for these parameters was particularly detrimental because, too many elites or too much mutation, minimizes the genes exchange and the evolutionary process thus loses systematicity in its progress. Similarly, a 0.9 *Mean Proportion of Elites* was the common denominator among the worst performing combinations for GA-WA. The optimized parameters for GA-WA resulted in a 0.1 *Mean Proportion of Elites* (μn_e), among which a 0.5 mean proportion will mutate () in a 0.5 magnitude factor ($fMut_e$); and a 0.9 *Mean proportion of non-elites* defined to suffer mutations ($\mu pMut$) in a 0.1 magnitude factor ($fMut$). In other words, the best performance in GA-WA was obtained from a low proportion of elites with an intermediate occurrence of mutations and with half the magnitude of the average mutation of regular individuals; as well as a high occurrence of mutations in non-elites with a low mutation magnitude. Considering only the partially optimized configurations, GA-WA was found to have a significantly superior performance, on the mean fittest I_{COE} achieved per run, as compared to both reference algorithms GAUniform and GAGauss.

The three *Design Goals* in the case study were satisfactorily approached with GA-WA by using different fitness functions. As compared to a reference flat and fixed horizontal solar panel, the materials cost for a given energy need was reduced in 42.2% by using *Cost* as the fitness function and with the first population distributed around intuitive values. The daily energy, with restrictions in space and the number of solar cells, was increased in 133.4% by using $\overline{C_e}$ as the fitness function. Lastly, the occupied space needed to fulfill a given energy need was reduced in 99.98% by using T_{sp} as the fitness function and with the first population distributed around intuitive values. The solutions achieved through the interactive software, VTDesign, only surpassed GA-WA in the energy maximization goal due to the more complex constraints which were not all directly considered by the used fitness functions in GA-WA.

Both VTDesign and GA-WA were found to be effective, efficient and

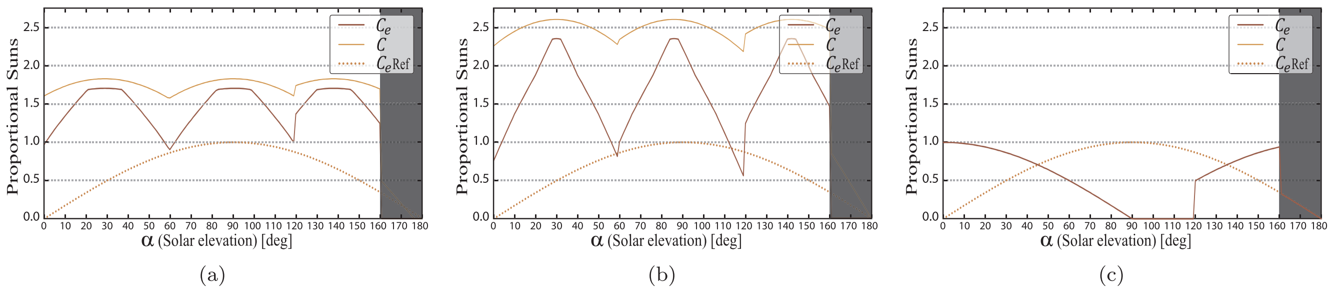

Fig. 13. Optical performance of the set-ups (a) $VT1_{icost}$, (b) $VT2_{VTD}$ and (c) $VT3_{itsp}$.

flexible tools in the problem of defining the parameters of a given solar V-Trough in a personalized scenario. The intuition and the more holistic exploration of a trained engineer with VTDesign can be complemented with the broader and less biased heuristic evolutionary optimization of GA-WA. Both tools can be implemented in parallel, to compare different solutions, or in series by distributing the first GA-WA population around the results achieved with VTDesign. On the other hand, the GA-WA results can be subsequently analyzed with VTDesign for a visual

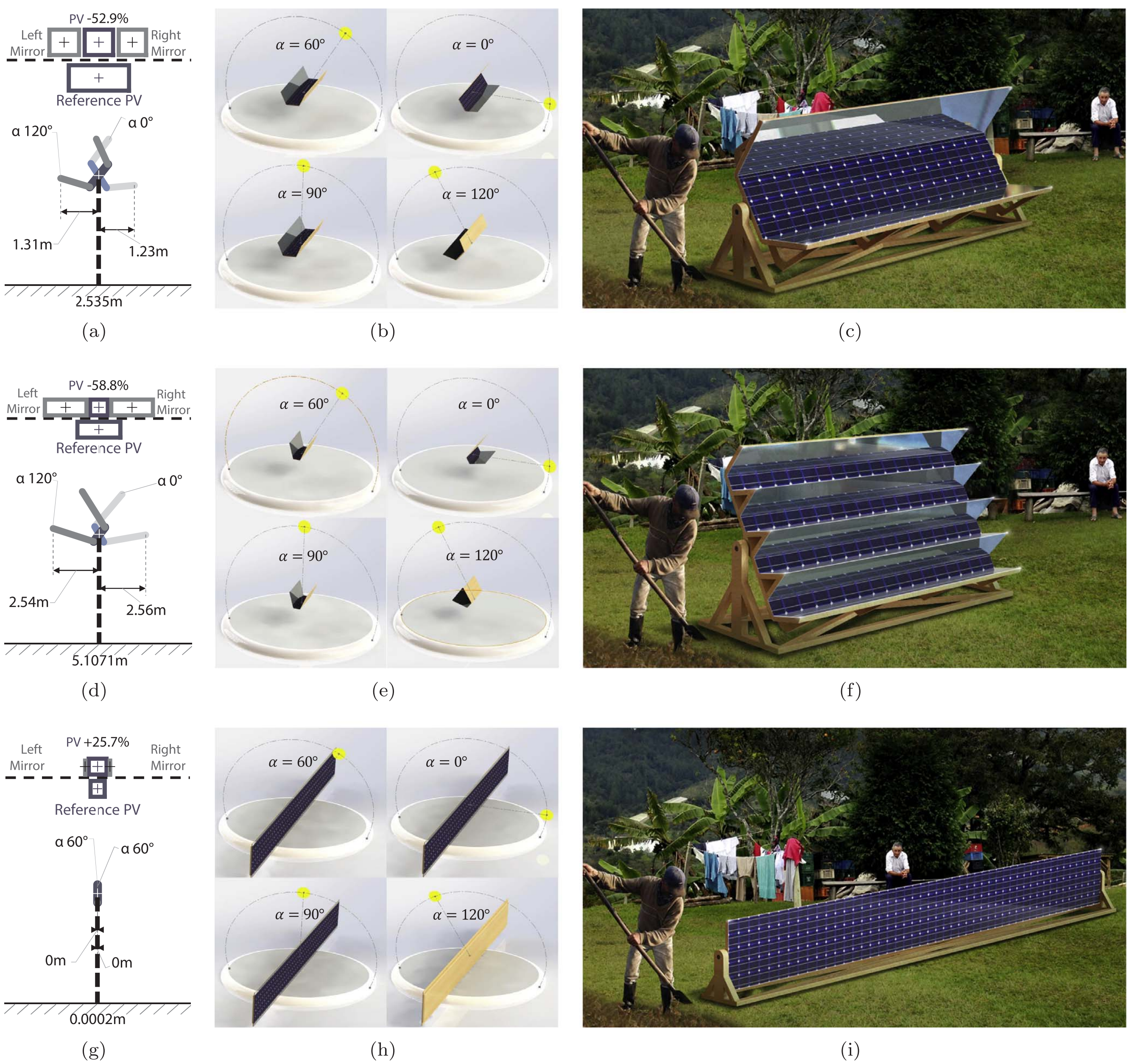

Fig. 14. Practical implications of the selected set-ups: (a–c) $VT1_{icost}$, (d–f) $VT2_{VTD}$, and (g–i) $VT3_{itsp}$.

understanding of the set-ups and a further interactive exploration.

The new heuristic processes of GA-WA were motivated by the solar V-Trough design problem. However, this new genetic algorithm can be set with other fitness functions for it to be used in a broad variety of engineering problems. GA-WA is of special interest when a more detailed control of the random processes is desired or when the genetic exploration is desired to be performed with a bias towards a pre-existing set of genes.

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