NON-STANDARD PROFIT EFFICIENCY IN BANKING: A COPULA APPROACH

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Abstract

This paper estimates profit efficiency using a seemingly unrelated stochastic frontier model where both cost and revenue efficiencies are obtained simultaneously. This method implicitly accounts for and estimates the correlation between revenue and cost efficiencies and allows us to achieve consistent and more efficient estimates. We use a copula-based maximum likelihood method applied to a well specified non-standard profit function to generate the distribution of the composite error. We show the performance of this estimating method using a sample of US commercial banks to study the evolution of profit efficiency for U.S. community banks from 2004 to 2017. Our results show that banks are less efficient in cost, revenues and profit; we find the average in the efficiency of 76.6% in revenues, 75,2% in cost and 53.6% in profits. We obtain a positive and statistically significant correlation between the composite errors of the revenue and cost function, it means that banks, which are more efficient in cost, are efficient in revenues and profits.

Key Words: Banks, Stochastic frontier, Copula- based maximum likelihood, Efficiency.

1 Introduction

Measuring efficiency in banking has a vital importance, due to the impact of the financial sector in the country's macroeconomic stability (Berger and Humphrey, 1997). Technological advances, changes

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in financial market structures and regulation have led to the search for new methods to evaluate the performance and efficiency of financial institutions (de Abreu et al., 2018). Today banks play a fundamental role in resource allocation, macroeconomic policy, and economic growth; measure its efficiency and productivity is challenging and find better methods contribute to better measurements. Measuring efficiency is an input for Industry analysis (e.g., performance, industry structure, and industry dynamics), regulatory analysis (e.g., regulatory burden, bank supervision, bank closure) and macroeconomic studies (e.g., monetary policy transmission, financial stability, and credit allocation). Also is important to consider that the fast growth of banking industry, the accelerated growth of demand for banking services, the great impact of technologies and decreasing regulatory laws, allow a significant increase market competition and the search of efficiency by banks to be competitive.

The Non-Standard Profit Function (NSPF) approach by Humphrey and Pulley (1997) is the most used method in the literature to estimate profit efficiency in banking. Following these authors, Restrepo-Tobón and Kumbhakar (2017) show that the traditional approach used in the literature is based in a misspecified NSPF. To solve the problem, they propose a new method to estimate profit efficiency as a composite measure of cost and revenue efficiencies. They add two potential sources of inefficiency into the Non Standard Profit Function: input inefficiency in the cost function and output price inefficiency in the revenue function, with this alternative method they solve a misspecification problem in measure banks efficiency. The system of equations formed by the revenue and cost stochastic frontiers in Restrepo-Tobón and Kumbhakar (2017)'s approach can be seen as a system of seemingly unrelated stochastic frontiers, they estimate it equation-by-equation. The composite errors of both equations should be related as revenue and cost efficiencies for the same bank should be correlated. Thus, equation-by-equation estimation is inefficient and may lead to biased efficiency estimates. We can suppose that cost and revenue inefficiency are correlated because banks maximize their profits by setting outputs, prices and choosing input quantities. The goal of maximizing profits does not only need that the good or service have been produced at a minimum cost, is necessary to maximize the revenues too.

They find that the measures of profit efficiency are strongly correlated with estimates of costs and revenues efficiency in a way intuitively suggested by both theory and economic reasoning. Besides, their method allows for the estimation of other important measures such as returns to scale, technical progress and total factor productivity in a consistent framework. Also, they show that the profit maximization problem in this framework is equal to solving the nonstandard revenue maximization problem (Berger et al., 1996) and the standard cost minimization problem, separately. Then, revenue and cost efficiencies can be estimated separately, and the profit efficiency measure is composed by both. The efficiency measure estimated using an econometric model equation-by-equation is inefficient and may lead to biased efficiency estimates. In this paper, we propose to estimate a Non-standard profit function through a simultaneous estimation of Revenue and Cost functions using a maximum likelihood copula-based approach following Lai and Huang (2013) and Tsay et al. (2013). Tsay et al. (2013) estimate the cumulative distribution function of the composite error of a stochastic frontier model using maximum likelihood estimation with a censored dependent variable. The empirical results are consistent with the theory developed, specifically in the prediction of bias for the estimated coefficients. They use a Monte Carlo experiment to demonstrate that their proposed method is correct. Lai and Huang (2013) used a copula-based on maximum likelihood method to estimate a multiple stochastic frontier model with correlated composite errors; they showed that the joint estimation is different and significantly better than the separate estimation.

The revenue and cost efficiencies in the NSPF enter the system of stochastic frontiers with opposite signs. Thus, the copula approach proposed in Lai and Huang (2013) has to be modified since they only consider a system of stochastic frontiers were inefficiencies across equations enter with the same sign. In particular, their system of stochastic frontiers only considers cost frontiers. Likewise, Tsay et al. (2013) approach only considers production frontiers. In the system of stochastic frontiers of the NSPF framework inefficiencies across equations will have opposite signs: revenue inefficiency reduces revenues while cost inefficiencies increase costs.

The application of this method allow studying the evolution of profit efficiency for U.S. community banks from 2001 to 2017. Following Sealey and Lindley (1977), we model banks' activities using a balance-sheet approach This balance captures more completely the financial and activity structure in banking. We can find the liabilities formed by purchased funds and basic deposits, as well as labor and physical capital, which are the inputs in the production process that and debt, are used to invest in financial assets and generate loans, the assets are composed by commercial values and loans.

The data are from the Report of Conditions and Income (Call Reports) from the Federal Reserve Bank of Chicago, including all FDIC insured commercial banks between 2004 and 2017. We omit banks with negative values for input prices, assets and equity. The data were deflated using the Consumer Price Index of 2011 for all urban consumption. The paper is structured as follows: In section 2 we make a review of the most important literature of bank efficiency , Section 3 we review the main literature about a multiple stochastic frontier regressions model with correlated composite errors, and we explain the model to estimate revenue and cost inefficiencies. We present the econometric model; it consists of a copula-based joint maximum likelihood estimation of the frontier regressions. Then we describe the data to estimate the empirical model and Section 3 reports the empirical results and Section 4 outlines some concluding remarks.

2 Literature Review

The literature report different methodologies for bank efficiency; one of the most used is the Stochastic Frontier Analysis (SFA), it was developed by Aigner et al. (1977) and applied in banking sector by Ferrier and Lovell (1990). The basic idea of SFA consists in describe the potential maximum production that an economic unit can obtain with a vector of inputs, and efficiency term is defined as the ratio of the actual output to the maximum potential output (Kumbhakar and Lovell, 2000). The difference between the stochastic frontier and the observed production is the inefficiency, modeled as a random variable, with different distributions.(Greene, 2003).

To estimate profit efficiency, the classic way of obtaining it in the literature, is based on the neoclassical profit function, in which we can observe that prices of inputs and prices of final products are given, it means that the entrepreneur chooses how much input quantities need to produce and prices are established in the market by supply and demand mechanism. NSPF approach of Humphrey and

Pulley (1997) is the main used method to estimate profit efficiency in the literature abput bank efficiency. (Restrepo-Tobón and Kumbhakar, 2017). In this framework banks do not determine the output quantities, the demand of clients gives these, the banks set the prices of the outputs (interest rate) to which they demand loans. NSPF is an alternative profit function and is a widely used to estimate efficiency in banks, for example, Akhavein et al. (1997) and Casu and Girardone (2002).

Many authors find in their empirical applications a different average of banks efficiency, and correlation between cost, revenue, and profit efficiencies. Berger and Humphrey (1997) find a statistically significant negative correlation between cost efficiency and two profit efficiencies. Rogers (1998) explains that a better estimator of bank efficiency regards how cheaply a bank produces output and how effectively it sells its output, knowing and linking the information from cost and revenue efficiency; also they find a positive correlation between revenue and profit efficiency and a negative correlation between cost and revenue inefficiencies. Maudos et al. (2002) find high levels of efficiency in costs and lower levels in profits, their estimation shows a low but positive correlation between the rankings of cost and profit efficiency, they find that banks with higher costs compensate its inefficiency getting higher revenues than its competitors, using in a better way its vector of production or by benefiting from greater market power. Restrepo-Tobón and Kumbhakar (2017) identify that revenue and cost efficiencies are negatively correlated, and both are correlated positively with profit efficiency; they obtain this results estimating equation by equation. Recently many research focus in the use of simultaneous stochastic frontier model, it consists in estimating at the same time the cost frontier and the revenue frontier and the regression equations contain composed errors, to find the joint probability density function of the composed many author use copulas, for example: Huang and Kao (2006) use a multiple output cost frontier, to examine input-oriented technical efficiencies and production risk simultaneously. Furthermore Shi and Zhang (2011) introduced a copula regression model to construct the stochastic panel frontier, after that they estimate firm efficiency and correlations with performance measures. also, Huang et al. (2017) and Huang et al. (2018) use a multiple stochastic frontier to estimate the relationship between marginal cost and the Lerner index.

3 Econometric Model

3.1 Cost and revenue simultaneous modeling

Profit efficiency takes into account the effects of the cost function minimization and revenue function maximization. Given the input and output price vectors; banks maximize profits using inputs and outputs. Restrepo-Tobón and Kumbhakar (2017) estimate revenue and cost inefficiencies with a two-step procedure introducing two potential sources of inefficiency in the revenue function. They demonstrated that the profit function could be maximized by maximizing the revenue and cost function separately if the correlation between them is not important. Post-estimation, they find that revenue and cost efficiencies are negatively correlated, and both are correlated positively with profit efficiency. They explain that this negative correlation between both efficiencies signify that the less cost efficient banks offset their related losses with higher revenue efficiency and less efficient banks provoke higher costs in getting higher quality outputs that lead in higher revenues.

The revenue function of a bank depends on the output vector y and the price of outputs p:

$$R = \sum p_m y_m = \frac{1}{\eta} \sum p^*_m y_m = \frac{1}{\eta} R^* \leq R^* \tag{1}$$

The cost of a bank depends on the input quantity vector x and the price of inputs w.

$$C = \sum w_j x_j = \frac{1}{\theta} \sum w_j x^*_j = \frac{1}{\theta} C^* \geq C^* \tag{2}$$

Assuming the econometric specification of a typical SFA model, the revenue function (equation 3) and cost function (equation 4) we estimate the revenue and cost frontiers using translog functional forms:

$$\ln R = R^*(\ln w, \ln y, t) + \upsilon - \ln \eta \tag{3}$$

$$\ln C = C^*(\ln w, \ln y, t) + \varepsilon - \ln \theta \tag{4}$$

where η and θ are the inefficiency of revenues and cost successively. If outputs are given, the presence of revenue inefficiency means that observed output prices are lower than optimal prices affecting the optimal revenue, while cost inefficiency (θ) implies that realized cost are higher than optimal cost. Following the SFA, the random error term μ_{1t} and μ_{2t} are correlated because the revenue and cost function appertain to the same bank, and are affected for common characteristics. Across the stochastic function, ν_{1t} and ν_{2t} suppose to be correlated for the common stochastic shocks.

The SFA regressions has the form $y_i = f(x_i, \beta) + \nu + \mu$, this function gives the maximum possible output as a function of certain inputs; y_i is the maximum output obtainable from x_i, a vector of (non-stochastic) inputs, and β is an unknown parameter vector to be estimated (Aigner et al., 1977); ν represents the statistical noise and $\mu \geq 0$ is the managerial inefficiency. An important assumption in single stochastic frontier model framework is that that ν and μ are mutually independent (Pal and Sengupta, 1999). Many researchers have extended the single stochastic frontier model methodology to the simultaneous stochastic frontier model. The extension to the system of SF regressions is analogous to the classical generalization to the system of seemingly unrelated regressions developed by Zellner (1962); they found that the regression coefficient estimator obtained are at least asymptotically more efficient than those obtained by an equation-by-equation application of least squares when the disturbance terms in differential equations are highly correlated. Following the most general notation in this framework:

$$\begin{cases} Y_{1j} = X_{1j}^T \beta_1 + \nu_{1j} - \mu_{1j} \\ Y_{2j} = X_{2j}^T \beta_2 + \nu_{2j} + \mu_{2j} \end{cases} \tag{5}$$

where y_{ij} and x_{ij} denote the log output and the log inputs of the stochastic frontier regressions, $\nu_{ij} \sim N(0, \sigma_{\nu j}^2)$ is the noise component and $\mu_{ij} \sim N^+(0, \sigma_{uj}^2)$ is the inefficiency term, in the model is assumed that ν_{ij} and μ_{ij} are independent.

Lai and Huang (2013) show in their study that joint estimation is significantly different from separate estimation without considering the correlated composite errors in the two divisions and they show that when stronger is the correlation between the two SF regressions, the more estimation efficiency is lost in separate estimations. Huang et al. (2018) estimate the market power and cost efficiency in a single step jointly. They use the copula approach, to incorporate dependence between market power and cost efficiency. In this paper, they show the advantages of this methodology in an empirical study on the banking sectors. Also Huang et al. (2017) proposes a simultaneous stochastic frontier model to study the competitive conditions of Taiwan's banking industry.

When estimating the single stochastic frontier model generally in the literature is done by maximum likelihood, assuming that the noise follows a normal distribution and the inefficiency component follows a normal or truncated distribution. The derivation of the probability density function of the composite error $\varepsilon = \nu - \mu$ and the maximum likelihood estimator, in this framework is relatively easy to do. But when we have the joint estimation, this derivation is so complicated to estimate in the analytic form, if we do not assume simplified assumptions (Lai and Huang, 2013).

To estimate the correlation between the inefficiencies of the revenue and cost function we follow Lai and Huang (2013), they propose a copula approach to construct the joint probability of ε_{1t} and ε_{2t} in equation (5). To estimate The copula-based likelihood function of the joint SF regressions is necessary to compute the cumulative distribution function (CDF) of the composite error. The process to obtain the mathematical approximation to the Normal CDF is complicated, and we need numerical methods to process it. Tsay et al. (2013) derive an analytic closed-form formula for the cumulative distribution function of the composite error of the SFA model.

3.2 Copula-based joint Probability Distribution Function and Maximum Likelihood estimation

The model by Lai and Huang (2013) consists in a seemingly non-related stochastic frontier regression with correlated composite errors. The model focuses on the correlation among a set of individual Stochastic Frontier regressions and they take into account the level of mutual dependency among this regressions. In the context of a system of frontier equations, researchers use a flexible multivariate distribution for the inefficiency error term. To derivate the joint probability distribution function is necessary and possible to use copula methods (Carta and Steel, 2012).

The copula approach is often employed in multivariate analysis and recently extended to the area of productivity and efficiency analysis. That is the case of Smith (2008), Carta and Steel (2012) and Amsler et al. (2014). The most important result of copula theory is the Sklar (1959) theorem, it explains how the copula function let to establish relationships between multivariate distribution functions and their univariate margins. A copula lets separate the dependence structure of the marginal distributions (Carta and Steel, 2012), it support to model the dependence between the inefficiencies of cost and revenues functions. The copula has a parameter that measures the dependence; we use ρ Spearman Myers and Sirois (2006), which is invariant respect to the margins.

The joint cumulative distribution function of ε_{1t} and ε_{2t} can be expressed as a function of its one-dimensional margins as:

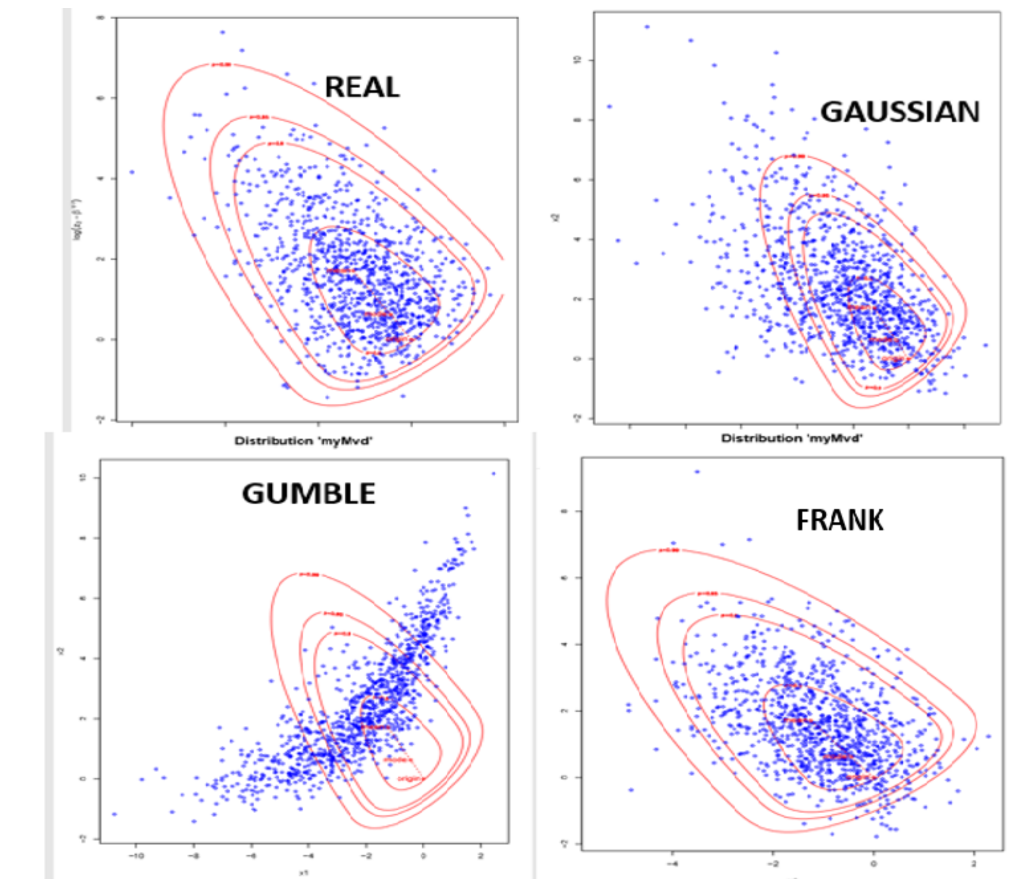
$$F(\varepsilon_{1t}, \varepsilon_{2t}) = C(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}), \rho) \tag{6}$$

where $C(\cdot)$ is the copula function of ε_{1t} and ε_{2t} which is unique, and is $F_1(\varepsilon_{1t})$ and $F_2(\varepsilon_{1t})$ are continuous, the dependence between the cumulative distribution functions is measure by ρ, the Spearman parameter. The joint Probability Distribution Function of (2) is:

$$f(\varepsilon_{1t}, \varepsilon_{2t}) = c(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \rho) \cdot \prod_{j=1}^{2} f_j(\varepsilon_{jit}) \tag{7}$$

where $c(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \rho) = \frac{\partial^2 C(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \rho)}{\partial F_1(\varepsilon_{1t}) \partial F_2(\varepsilon_{2t})}$ is the copula density and $f_j(\varepsilon_{jit})$ is the marginal Probability Distribution Function. In the literature, we can find the application of several multivariate copulas, like the multivariate Student's t copula, Archimedean copula, Gumble n-copula, and Clayton n-copula. We can see authors like Cherubini et al. (2004) who address the mathematics of copula functions illustrated with finance applications. As Lai and Huang (2013) we use the Gaussian copula to derive the bivariate distribution function of (2). To select the best copula, we simulate a generating process data, from two skew normal distribution; after that we generate different types of copula, two symmetric: (Gaussian and Frank) and one asymmetric (Gumble) with both skew normal marginal distribution, we plot the same contours as for the Multivariate Skew Normal additional to the random numbers sampled from the Copula created. Figure I show that the symmetric copulas (Gaussian and Frank) are better adjusting to the generated data.

Figure I: Simulated copulas from two skew normal marginal distributions



The Gaussian copula has the form:

$$C(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \omega) = \phi_2(\phi^{-1}(F_1(\varepsilon_{1t})), \phi^{-1}(F_2(\varepsilon_{2t})); \omega) \quad (8)$$

$$= \int_{-\infty}^{\phi^{-1}(F_1(\varepsilon_{1t}))} \int_{-\infty}^{\phi^{-1}(F_2(\varepsilon_{2t}))} \frac{1}{2\pi|\omega|^{\frac{1}{2}}} exp[\frac{-1}{2} Z' \omega^{-1} Z] dZ_1 dZ_2$$

where $\phi^{-}1(\cdot)$ is the inverse of the cumulative distribution function of the standard univariate normal distribution and $\phi_2(.)$ is the standardized bivariate normal distribution function of the random variables with the off-diagonal elements of ω where we can find the correlation coefficients between both variables. We can derive the Gaussian copula density of (4) as:

$$C(F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \omega) = \frac{1}{|\omega|^{\frac{1}{2}}} exp(-\frac{1}{2} \zeta_{it}' (\omega^{-1} - I_2) \zeta_{it}) \quad (9)$$

where $\zeta_i = [\phi^{-1}(F_1(\varepsilon_{1t}) \phi^{-1} F_2(\varepsilon_{2t})]'$ and I_2 is a $2x2$ identity matrix.
The joint Probability Distribution Function of the composite errors is:

$$f(\varepsilon_{1it}, \varepsilon_{2it}) = C(F_1(\varepsilon_{1it}), F_2(\varepsilon_{2it}); \omega) \cdot \prod_{j=1}^{2} f_j(\varepsilon_{jit}) \quad (10)$$

$$= \frac{1}{|\omega|^{\frac{1}{2}}} exp(-\frac{1}{2} \zeta_{it}' (\omega^{-1} - I_2) \zeta_{it}) \cdot \prod_{j=1}^{2} f_j(\varepsilon_{jit}) \quad (11)$$

The maximum likelihood estimation requires to generate the probability distribution function of the composite errors which computation is relatively easy and routine (Lai and Huang, 2013) and the inverse of the distribution function.The computation of the cumulative distribution function is no easy and routine because does not exist its closed for evaluation, it requires numerical integration procedures. Tsay et al. (2013) and Lai and Huang (2013) develop an mathematical approximation function to obtain the closed form of the cumulative distribution function.

The log-likelihood function of the multiple SF regressions of (1) is:

$$lnL(\theta) = \sum_{i=1}^{N} lnf(\varepsilon_{1i}, \varepsilon_{2i}) = \sum_{i=1}^{N} lnC(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}; \omega)) + \sum_{i=1}^{N}\sum_{j=1}^{N} lnf_j(\varepsilon_{12}) \quad (12)$$

where the $\theta = (\theta_1^T, \theta_2^T, \rho^T))$ and θ_1 and θ_2 are vector of unknown parameters.
We estimate the technical efficiency of the half model of Revenue and Cost; following Kumbhakar and Lovell (2000) we define:

$$TE_i = exp(-\mu_{*i} + \frac{1}{2}\sigma_*^2)\frac{\Phi(\frac{\mu_{*i}}{\sigma_*} - \sigma_*)}{\Phi(\frac{\mu_{*i}}{\sigma_*})} \quad (13)$$

where $\mu_{*i} = \frac{-\sigma_u^2 * \varepsilon_i}{\sigma_v^2 + \sigma_u^2}$ and $\sigma_*^2 = \frac{\sigma_v^2 \sigma_u^2}{\sigma_v^2 + \sigma_u^2}$

3.3 Iterative profile maximum likelihood estimation

To maximize the likelihood function is difficult because the large number of parameters to be estimated, because some of the specifications involve up to 320 parameters. Therefore, this research Oberhofer and Kmenta (1974) uses this procedure maximizing the (log)likelihood function by an iterative procedure. The iterative strategy can be thought as an initial step in finding the most suitable initial parameter vector to start the process of the maximum likelihood estimation.

The maximum likelihood function of the model can be written as $L(\Theta, X)$; where Θ is a vector of parameters to be estimated and X is a matrix containing the data for estimation. The vector Θ can be partitioned into two set of parameters $\Theta = \{\theta_1, \theta_2\}$. The iterative procedure implies four steps:

1. Maximizing $L(\theta_1, \theta_2, X)$ with respect to θ_1 taking θ_2 as given, starting with a feasible set of initial values for both θ_1 and θ_2.

2. Maximizing $L(\theta_1, \theta_2, X)$ with respect to θ_2 taking θ_1 equal to the set of estimated parameters $\hat{\theta}_1$ in the first step given a feasible set of initial values for θ_2.

3. Repeat steps 1 and 2 many times until an arbitrary convergence criterion is satisfied. For in-

stance, until the (log)likelihood function changes little from one step to the other.

4. Take the estimates of $\hat{\theta}_1$ and $\hat{\theta}_2$ in the last iteration of step 3 and maximize $L(\Theta, X)$ with respect to θ_1 and θ_2 simultaneously.

Under general conditions, Oberhofer and Kmenta (1974) demonstrated that this procedure gives the maximum likelihood estimates of $L(\Theta, X)$ as if we were maximizing it in one step. They recommend using this procedure when maximizing $L(\Theta, X)$ is difficult.

In our application θ_1 is a set of parameters to be estimated corresponding to the parameters of the revenue and cost function, while θ_2 is the set of parameters to be determined corresponding to five different equations: the equation for the variance of the inefficiency term of revenue function and the inefficiency term of cost function, the equation for the variance of the error term of both functions, an the equation of the correlation between the revenue and cost function.

3.4 Data

Following Restrepo-Tobón and Kumbhakar (2017) we extend the database used in their research to study the evolution of profit efficiency for U.S. community big banks from 2004 to 2017. We study the case of big banks (assets > 1 billion) to know the behavior of efficiency average measures of it kind of banks. Following Sealey and Lindley (1977), we model banks' activities using a balance-sheet approach. The data set is an unbalanced panel with information for 480 banks. The data is from the Report of Conditions and Income (Call Reports) from the Federal Reserve Bank of Chicago and include all FDIC insured commercial banks with data between 2004 and 2017. Table 1 shows the summary statistics for outputs, inputs, prices, cost, revenues, and profits. Also, we include variables that can affect bank efficiency: Total Assets, Leverage, measure as the total Liabilities/Assets, Offbal as non-interest income/total income, Int. Ratio as Interest income/Total income and Equity as Total equity.

To estimate the revenue and cost function, we define the input and output variables. The variables for each year are computed as the quarterly average of balance-sheet nominal (stock) values. The output variables are household and individual loans (y1), real estate loans (y2), business loans and to other institutions (y3), Federal funds sold and securities purchased under agreements to resell (y4), and other assets (y5) (Restrepo-Tobón and Kumbhakar, 2017). The input variables are: Labor, it is the number of full-time equivalent employees in a period (x1), premises and fixed assets, capitalized leases (x2), purchased federal funds and securities sold under agreements to repurchase (x3), non-transaction accounts (x4) and interest-bearing transaction accounts. The total costs are equal to the sum of expenses for five inputs; total revenues are equal to the sum of revenues for each output category; and, profits are equal to total revenues minus total costs. All nominal quantities of the data are deflated by the 2011 Consumer Price Index for all urban consumption which is published by the Bureau of Labor Statistic (Restrepo-Tobón and Kumbhakar, 2017).

Table I: Descriptive statistics

Variables	**N**	**Mean**	**SD**	5^{TH}	25^{TH}	50^{TH}	75^{TH}	95^{TH}
Household and individual loans	6373	886,540	6,386,526	1230	10,455	34,946	129,907	1,180,588
Real state loans	6373	3,814,302	19,100,000	394,596	648,459	991,018	2,005,173	10100000
Business loans	6373	2,733,036	21,600,000	76,818	190,894	339,211	739,593	5,304,660
Federal funds and securities	6373	3,117,315	28,500,000	87,647	229,009	413,895	902,263	6,009,148
Other assets	6373	661,359	4,936,549	21,522	40,712	75,624	182,548	1,259,249
Labor	6373	1,946	11,395	157	278	433	827	4,403
Premises and fixed assets	6373	103,531	481,357	5,697	17,549	30,120	55,526	297,356
Purchased funds	6373	2,089,584	18,800,000	11,676	74,456	173,283	437,795	4394925
Interest bearing trans accounts	6373	1,952,260	5,740,845	9,310	43,118	724,876	1,780,980	7644613
Non/trans accounts	6373	7,394,882	43,000,000	714,681	1,042,764	1,582,289	3,278,188	20,000,000
w1	6373	72.229	26.530	42.530	55.000	67.207	83.213	117.718
w2	6373	0.450	3.673	0.112	0.170	0.245	0.389	0.977
w3	6373	0.130	3.259	0.002	0.012	0.025	0.037	0.057
w5	6373	0.013	0.011	0.001	0.003	0.008	0.020	0.035
Profit	6373	350,782	2,165,892	21,682	36,511	59,377	130,144	882,594
Revenue	6373	712,007	4,453,237	52,471	77,066	122,614	249,847	1,637,077
Cost	6373	361,224	2,537,410	23,724	39317	62242	122673	727580
Total Assets	6373	11,800,000	77,500,000	1,053,000	1,337,000	2,064,000	4,406,000	29,700,000
Leverage	6373	0.890	0.041	0.829	0.879	0.898	0.911	0.930
Offbal	6373	0.133	0.114	0.021	0.062	0.103	0.165	0.359
Loan/Asset	6373	0.708	0.126	0.475	0.642	0.723	0.792	0.884
Int.Ratio	6373	0.673	0.140	0.415	0.600	0.691	0.766	0.868
Equity	5819	454,373	1,062,717	90,568	129,275	195,338	392,304	1,449,046

This table shows the average (Mean), standard deviation (SD), the 5, 25, 50, 75, and 95th percentiles. The data used for estimation include 6373 year-bank observations for 480 banks with information for at least 14 years between 2004 and 2017.

3.5 A Montecarlo simulation

We conduct a Montecarlo simulation, with two stochastic frontier regressions taking into account the change in signs in the composite errors. The objective of the simulation is to analyze the estimation of the separate and joint models, and to understand the consequence of make estimations ignoring the correlation between the composite errors. We simulate the stochastic frontier of two regressions:

$$y_1 = \beta_{10} + \beta_{11}x_1 + \beta_{12}x_2 + \beta_{13}x_3 + \varepsilon_1 \tag{14}$$

$$y_2 = \beta_{20} + \beta_{21}x_1 + \beta_{22}x_2 + \beta_{23}x_3 + \varepsilon_2 \tag{15}$$

The number of replications was 1000, and the used sample was n=500 and n=1000, we applied this procedure to various degrees of correlation between Equation 14 and Equation 15. Following the computation issue by Lai and Huang (2013), we draw two independent random variables, z_1 and z_2, from N(0, 1).After that we compute $\gamma_1 = \phi(\zeta_1)$ and $\gamma_2 = \Phi(\zeta_2)$ where γ_1 and γ_2 are the probability integral transformations of ζ_1 and ζ_2. The composite errors are ε_1 and ε_2 generated by the inverse CDF.

Given the parameters of equation 14: $\beta_{10} = 1$, $\beta_{11} = 0.3$, $\beta_{12} = 0.45$, $\beta_{13} = 0.25$, $\sigma^2_{v1} = 0.6$, $\sigma^2_{u1} = 0.4$; and the parameters of equation 15: $\beta_{20} = 1$, $\beta_{21} = 0.2$, $\beta_{22} = 0.5$ and $\beta_{23} = 0.3$, $\sigma^2_{v2} = 5.55$, $\sigma^2_{u2} = 0.5$ Table 3 shows the separate and joint estimation of equation 14 and equation 15 for a simulation with n=500 and n=1000, and a dependence between the two regressions $\rho = 0.3$ We can observe that almost all standard errors are greater in the separate estimation.

We can observe in the estimation, (Table II) the significant gain in efficiency in the estimators when we use the joint method. For that reason ignoring the dependence between both equations will also cause severe bias in estimating the technical efficiencies, sub estimating or overestimating the inefficiency in revenues and cost.

Table II: Separate and Joint ML estimation (ρ=0.3)

n	500		1000	
VARIABLES	Separate estimation	Joint estimation	Separate estimation	Joint estimation
Equation 1: ($\varepsilon_1 = \eta - \mu$)				
x_{11}	0.363***	0.296***	0.205***	0.305***
	(0.331)	(0.067)	(0.071)	(0.013)
x_{12}	0.448***	0.460***	0.394***	0.459***
	(0.097)	(0.019)	(0.073)	(0.013)
x_{13}	0.313***	0.256***	0.242***	0.240***
	(0.097)	(0.019)	(0.071)	(0.013)
Constant	0.534***	1.886***	1.351***	1.877***
	(0.454)	(0.071)	(0.219)	(0.052)
σ_v^2	5.259***	0.109***	0.574	0.116
	(0.655)	(0.102)	(0.045)	(0.012)
σ_u^2	3.713	0.087	0.554	0.077
	(2.510)	(0.060)	(0.131)	(0.049)
Equation 2: ($\varepsilon_2 = \eta + \mu$)				
x_{21}	0.346	0.204***	1.350**	0.204***
	(0.883)	(0.053)	(0.657)	(0.053)
x_{22}	-1.861*	0.376***	0.161	0.376***
	(0.890)	(0.053)	(0.646)	(0.053)
x_{23}	0.611	0.271***	1.264**	0.271***
	(0.911)	(0.055)	(0.646)	(0.055)
Constant	3.320***	-2.170***	2.291	1.738***
	(3.301)	(0.266)	(2.15)	(0.266)
σ_v^2	5.259	0.337***	5.326	0.337
	(0.655)	(0.040)	(0.432)	(0.040)
σ_u^2	3.713	0.133	4.084	0.133
	(2.510)	(0.271)	(1.527)	(0.271)
Observations	500	500	1000	1000

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

4 Empirical Results

Table III and Table IV provide the results of the estimation of the cost and revenue function. The coefficients estimated in our new aproach are more efficiently than the separate method and most of the dependent variables are significant, it indicate that the dependence between the revenue and cost exists and is important, being the simultaneous model more efficient than the separate equation model, eliminate the correlation may cause inconsistent and inefficient estimated parameters, and the wrong measures of technical efficiency. The econometric efficiency gain from using the copula method estimation procedure is confirmed. It is essential emphasizing that the estimated dependence parameter ρ in the Gaussian copula is statistically significant, for that reason, the dependence between the cost and revenue exists and is important. Our method exhibit an estimated correlation between composed error from revenue function and cost function of 0.73. Rogers (1998) and Berger and Mester (1997) using a misspecified econometric model, show a negative correlation of the rankings between cost and revenue frontiers and between cost and profit frontiers, a resultf for which researchers have no compelling arguments. Our results show that banks which are efficients in cost tend to have more efficiency in revenues and profits. Some studies through different methodologies analyzes the relationship between cost, revenue and profit efficiencies Ferrier and Lovell (1990) found statistically insignificant spearman correlation coefficients of 0.0138. Resti (1997) find a high and significant correlation between estimated inefficiencies of 0.98. Also, Restrepo-Tobón and Kumbhakar (2017) find that while revenue and cost efficiencies tend to be negatively correlated, both correlate positively with profit, we contrast with this results, obtaining that revenues and cost efficiencies are positive correlated, this authors find high levels of cost and revenues efficiencies, revenue efficiency average of 0.853 for big banks, and cost revenue average of 0.883, in our approach is evidently that banks are more inefficients.

The measures of efficiency estimates in Table V and Table VI show that the separate equation model leads to quite a high average efficiency in revenues of 0.878, in cost 0.821 and profit efficiency average is 0.681. Our average efficiency in the copula method shows a reduction in average efficiency in revenues with 0.766, cost efficiency of 0.752 and profit efficiency of 0.536. These results show that when taking into account the correlation between the composed errors of both equations, the right levels of efficiencies are lower, this agrees with the results obtained when applying these methodologies, in previous studies like Huang et al. (2017) and Huang et al. (2018). Therefore, we find that the actual levels of banks' profitability are lower and affected by the relation between cost inefficiency and revenue inefficiency.

Table VII shows the sample correlation coefficients between the revenue efficiency and cost efficiency from the joint and single methods. The new method shows a higher correlation between

Table III: The separate and joint ML estimation results-Revenue function

	Joint estimation		*Separate estimation*	
Variables	**Coefficients**	**Std. Err.**	**Coefficients**	**Std. Err.**
lw1sq	-0.1072883***	(0.0250997)	-0.3217791***	(0.0592154)
lw2sq	-0.0388180***	(0.0064280)	0.0179910	(0.0146825)
lw3sq	0.0004572	(0.0016486)	-0.0004637	(0.0019956)
lw4sq	-0.0051278**	(0.0028017)	0.0011075	(0.0031028)
lw5sq	0.0383825***	(0.0106220)	0.0755828***	(0.0193382)
lw x 1w2	0.0148945*	(0.0109176)	-0.0481998**	(0.0195894)
lw1 x lw3	-0.0307899***	(0.0084118)	-0.0166275	(0.0193310)
lw1 x lw4	-0.0014000	(0.0067070)	0.0204208	(0.0126610)
lw1 x lw5	-0.0428377***	(0.0126775)	-0.0060881	(0.0300952)
lw2 x lw3	0.0115229***	(0.0042540)	0.0085009	(0.0072413)
lw2 x lw4	0.0145394***	(0.0035489)	0.0092369*	(0.0050088)
lw2 x w5	-0.0404665***	(0.0062929)	-0.0454396**	(0.0155225)
lw3 x w4	0.0045438**	(0.0023038)	0.0022444	(0.0028859)
lw3 x w5	-0.0057703	(0.0043635)	-0.0036198	(0.0068726)
lw4 x w5	0.0001500	(0.0040042)	0.0024363	(0.0056208)
ly2 x y3r	-0.0012297	(0.0036523)	0.0743981***	(0.0059291)
ly2 x y4r	-0.0148592***	(0.0027705)	0.0774837***	(0.0088813)
ly2 x y5r	-0.0591127***	(0.0041636)	0.0686649***	(0.0060399)
ly3 x y4r	-0.0228548***	(0.0036325)	0.1207477***	(0.0147867)
ly3 x y5r	-0.0367017***	(0.0043667)	-0.0068984	(0.0064623)
ly4 x y5r	-0.0411085***	(0.0045473)	-0.0074836	(0.0063428)
ly2r x t	0.0007394	(0.0019005)	-0.0067252	(0.0092986)
ly3r x t	0.0063009***	(0.0021161)	0.0253372	(0.0101385)
ly4r x t	-0.0058327***	(0.0017187)	-0.0221388	(0.0083543)
ly5rt	-0.0019230	(0.0024725)	-0.0024237	(0.0131998)
lw1 x t	-0.0059955*	(0.0044330)	-0.0067653	(0.0082278)
lw2 x t	-0.0035370*	(0.0022656)	-0.0053121	(0.0040569)
lw3 x t	0.0001412	(0.0016329)	-0.0008655	(0.0020972)
lw4 x t	-0.0000284	(0.0013657)	0.0005675	(0.0017398)
lw5 x t	-0.0099824***	(0.0029646)	-0.0001749	(0.0053232)
σ_u^2	0.0256410***	(0.1664011)	0.0280385	(0.0959260)
σ_v^2	0.0346142***	(0.0460575)	0.0294888	(0.2435703)

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Table IV: The separate and joint ML estimation results-Cost function

	Joint estimation		*Separate estimation*	
Variables	**Coefficients**	**Std.Err.**	**Coefficients**	**Std.Err.**
lw3rsq	-0.0005684	(0.0019319)	0.0015472	(0.0031797)
lw4rsq	-0.0025005	(0.0032503)	-0.0032332	(0.0038402)
lw5rsq	0.0759817***	(0.0115368)	0.0964557***	(0.0170232)
lw2w3r	0.0081045***	(0.0048493)	0.0056456	(0.0096745)
lw2w4r	0.0193332***	(0.0041722)	0.0185007***	(0.0066285)
lw2w5r	-0.0055347	(0.0071916)	-0.0263024**	(0.0136583)
lw3w4r	0.0035515	(0.0026838)	0.0032817	(0.0040012)
lw3w5r	0.010153**	(0.0050048)	0.0146602**	(0.0079008)
lw4w5r	-0.0099407**	(0.0046618)	-0.0092867	(0.0063413)
ly1	-0.8975996***	(0.1563357)	-0.8865834***	(0.2081791)
ly2	-0.4821073	(0.3494233)	-0.857061	(0.6108690)
ly3	0.5027578	(0.3484200)	0.4761506	(0.4314414)
ly4	0.052897	(0.2804795)	-0.1726561	(0.3485743)
ly5	2.103601***	(0.4167448)	2.726574***	(0.6882771)
ly1 x ly2	-0.0413574***	(0.0046441)	-0.0301625***	(0.0084937)
ly1 x ly3	-0.0078401***	(0.0022820)	-0.0074464**	(0.0033789)
ly1 x ly4	-0.0201241***	(0.0053808)	-0.0263219***	(0.0088200)
ly1 x ly5	-0.0410796***	(0.0084889)	-0.0354909***	(0.0143685)
ly2 x ly3	0.0005916	(0.0047736)	-0.0100639	(0.0082740)
ly2 x ly4	-0.0254707***	(0.0035240)	-0.0213157***	(0.0061398)
ly3 x ly4	-0.0070132	(0.0044449)	-0.001641	(0.0064237)
ly3 x ly5	-0.0130379*	(0.0050961)	-0.0060927	(0.0065845)
ly1 x t	0.0031302***	(0.0011289)	0.0035755**	(0.0015380)
ly2 x t	-0.0029414	(0.0023213)	-0.0088081**	(0.0040239)
ly3 x t	0.0084408***	(0.0025003)	0.008679***	(0.0030993)
ly4 x t	0.0011462	(0.0020285)	0.0011929	(0.0025677)
ly5 x t	-0.0102991***	(0.0028887)	-0.005911	(0.0048272)
lw2r x t	0.0034197	(0.0026106)	-0.0012299	(0.0043775)
lw3r x t	0.0043821**	(0.0018826)	0.0040139	(0.0028730)
lw4r x t	-0.0040167**	(0.0015976)	-0.0033747**	(0.0018649)
lw5r x t	-0.0042424	(0.0033336)	0.0008289	(0.0047177)
σ_u^2	0.00055148***	(0.7867979)	0.03456872***	(0.0753202)
σ_v^2	0.0609788***	(0.0059795)	0.07137098***	(0.1079363)
ρ	0.7311245***	(0.0057968)		

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Figure II: Histogram and density plots for efficiency measures.

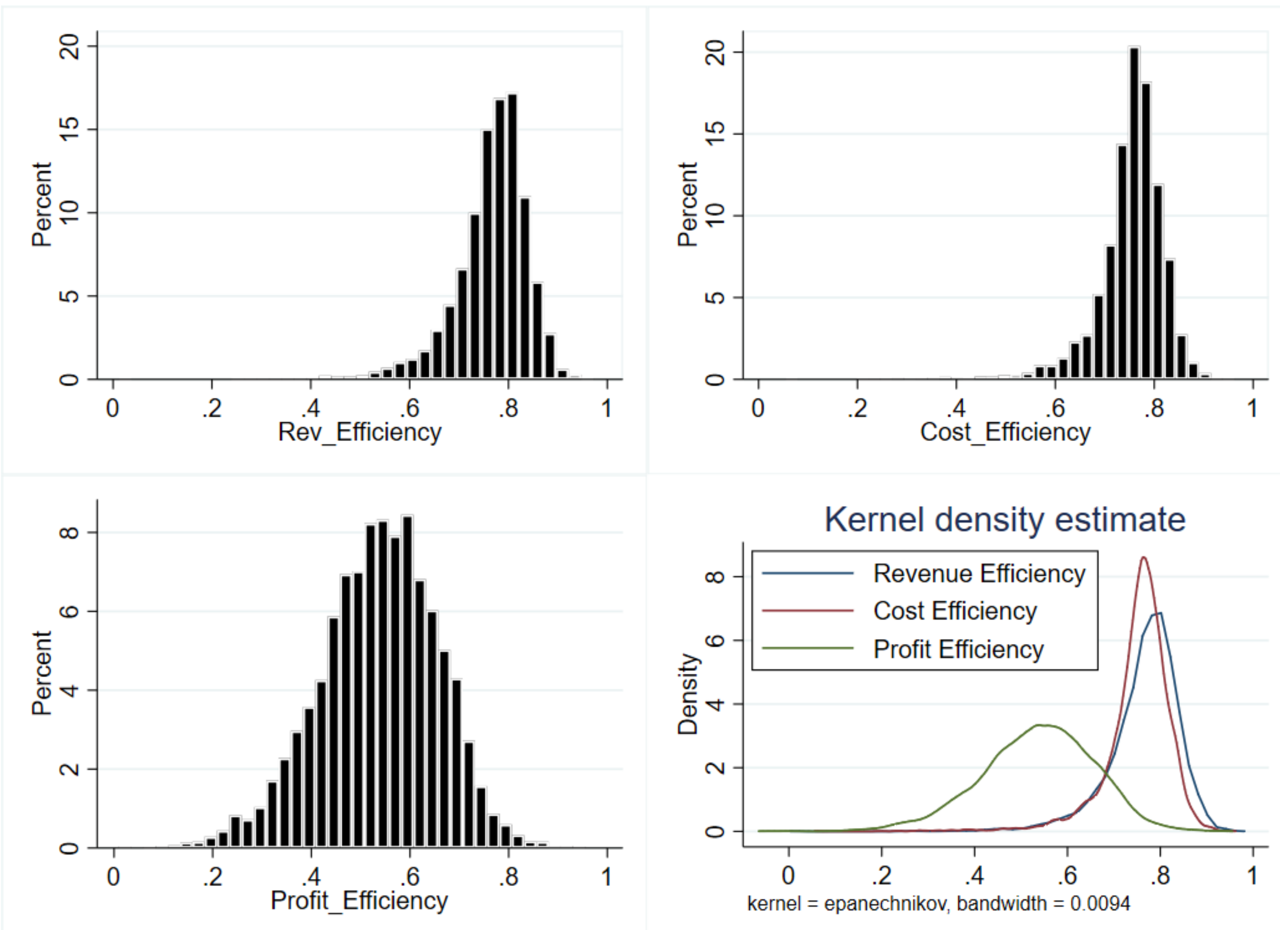


Table V: Summary Statistics for Estimated Efficiencies (Joint ML estimation)

Efficiencies	**Mean**	**SD**	5^{TH}	25^{TH}	50^{TH}	75^{TH}	95^{TH}
Revenue	0.766	0.0760	0.627	0.732	0.777	0.813	0.864
Cost	0.752	0.0715	0.629	0.727	0.762	0.793	0.840
Profit	0.536	0.122	0.327	0.460	0.542	0.620	0.720

This table shows the average (Mean), standard deviation (SD), the 5, 25, 50, 75, and 95th percentiles for the estimated efficiencies measures for bigs banks (assets > $1 billion$), estimated by the joint ML method.

Table VI: Summary Statistics for Estimated Efficiencies (Separate ML estimation)

Efficiencies	**Mean**	**SD**	5^{TH}	25^{TH}	50^{TH}	75^{TH}	95^{TH}
Revenue	0.878	0.0472	0.795	0.861	0.886	0.905	0.931
Cost	0.821	0.0809	0.662	0.793	0.837	0.871	0.915
Profit	0.681	0.109	0.488	0.622	0.693	0.756	0.829

This table shows the average (Mean), standard deviation (SD), the 5, 25, 50, 75, and 95th percentiles for the estimated efficiencies measures for bigs banks (assets > $1 billion$), estimated by the separate ML method.

efficiencies (0.9638), and the correlation coefficient in the single method is 0.4775, it means that the separate method underestimate the correlated relationship between cost and revenues in banking, for that reason is important to use the joint method when the functions are correlated.

We present summary statistics of the measure of revenue efficiency, cost efficiency, and profit efficiency by years in table VIII. We can observe that is not a clear patron of increase or decrease of efficiency levels, the lowest values in profit efficiency coincide in the period 2007-2010, in the financial crisis.

We compute the change in returns on equity (ROE) and returns on assets (ROA), for the joint estimated model and separate estimated model. The results are reported in IX. We find that a 1% increase in revenue and cost efficiencies would yield a 1.43% increase in ROE in the joint model and 0.45% increase in the single model. Also, we can observe in the ROA case that a 1% increase in profit efficiency would yield a 0.0287% increase in ROE in the joint model and 0.0135% in the separate model. This result implies that the effect in changes in ROA and ROE from changes in banks

Table VII: The correlation coefficients of the predicted efficiencies under separate and joint estimations

	RE (Joint)	CE (Joint)	RE(Sep)	CE (Sep)	PE(Joint)	PE(Sep)
RE(Joint)	1					
CE (Joint)	**0.7053**	1				
RE (Sep)	0.8921	0.4959	1			
CE (Sep)	0.6646	0.9638	**0.4775**	1		
PE(Joint)	**0.8081**	**0.4326**			1	
PE(Sep)			**-0.0749**	**0.0260**		1

Revenue Efficiency (RE), Cost Efficiency (CE) and Profit Efficiency (PE) denote the predicted efficiencies of the Revenue function, Cost function and Non Standard Profit Function; parentheses denote separate and joint estimation, respectively.

Table VIII: Mean Efficiency Measures by year

	Revenue		*Cost*		*Profit*	
Year	**Mean**	%Δ	**Mean**	%Δ	**Mean**	%Δ
2004	0.760		0.760		0.542	
2005	0.766	0.6	0.752	-0.8	0.518	-2.4
2006	0.774	0.8	0.753	0.1	0.484	-3.4
2007	0.778	0.4	0.756	0.3	0.463	-2.1
2008	0.765	0	0.756	-0.6	0.476	1.3
2009	0.760	-1.3	0.750	0.9	0.508	3.2
2010	0.768	-0.5	0.759	-1.8	0.549	4.1
2011	0.758	-1.0	0.741	0.8	0.551	0.2
2012	0.772	1.4	0.753	1.2	0.574	2.3
2013	0.760	-1.2	0.754	0.1	0.560	-1.4
2015	0.749	-1.1	0.748	-0.6	0.556	-0.4
2016	0.676	-7.3	0.783	3.5	0.652	9.6
2017	0.770	9.4	0.743	-4	0.603	-4.9
Total	0.766	0.076	0.752	0.071	0.536	0.122

This Table shows annual means of the estimated efficiency measures.

Table IX: Percentage Points Change in ROE due to a 1% Change in Revenue and Cost Efficiencies

Variables	Mean	SD	5^{TH}	25^{TH}	50^{TH}	75^{TH}	95^{TH}
ROE (Joint)	1.433	38.75	0.0432	0.0696	0.109	0.231	1.613
ROE (Separate)	0.448	3.503	0.0231	0.0365	0.0575	0.120	0.794
ROA (Joint)	0.0287	0.221	0.0136	0.0191	0.0243	0.0309	0.0419
ROA (Separate)	0.0135	0.00680	0.00709	0.00998	0.0126	0.0160	0.0218

This Table presents summary statistics of the increase in ROE when, revenue and cost efficiency estimates are shifted up by 1% The data used for estimation include 6373 year-bank observations for 480 big banks (assets > $1billion$).Efficiencies measures for Joint and separate estimation method.

efficiency, is higher if we take into account the existing correlation between the composite error of both function; it let us conclude that the traditional estimation, sub-estimate the effects of changes in efficiency in the banks rent-ability.

5 Conclusions

This paper proposes an econometric framework for the joint estimation of bank-level revenue and cost efficiency using copula methods. The copula approach proposed in Lai and Huang (2013) has to be modified since they only consider a system of stochastic frontiers were inefficiencies across equations enter with the same sign. The model simulation with two stochastic frontier regressions taking into account the change in signs in the composite errors demonstrates that joint estimation provides more efficient estimators than the separate estimation

Restrepo-Tobón and Kumbhakar (2017) find that revenue and cost efficiencies tend to be negatively correlated, both correlate positively with profit efficiency, they obtain this results estimating equation by equation. The separate method is likely to underestimate the joint interrelated between the cost and revenue efficiencies; the results show a reduction in the standard errors of the estimated frontier coefficients β as compared with those of a separate estimation. The novelty of our approach lies in to apply a new methodology to estimate profit efficiency in banking; results show significant changes in the measure of average efficiency and our model exhibit better econometric behavior. We apply the new approach to estimate profit efficiency for U.S. community and non-community banks from 2004 to 2017. The estimated parameters in the joint method are statistically significant. We find that banks are less efficient in cost, revenues and profit, we find the average in efficiency of 76.6% in revenues, 75,2% in cost and 53.6% in profits, contrasting with the literature where Restrepo-Tobón and Kumbhakar (2017) find higher levels of efficiency in banking. The empirical results show a positive correlation between the composite errors of the revenue and cost functions, it means that levels of bank's profit efficiency are positively correlated with cost and revenues efficiencies, for that reason we conclude that banks more efficient in cost tend to be efficient in revenues and profits.

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