



NONLINEAR REGRESSION MODEL FOR RIDE ON RAILWAY

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ABSTRACT

The portable diagnosis system – SPD – evaluates the safety and ride quality aspects of the railway vehicles and the technical condition of the rail-vehicle interface. The objective of this article is to estimate the nonlinear regression model associated with the ride quality or motion behavior, by applying fuzzy clustering algorithms to the geometric data obtained from the technical condition of the railway-vehicle interface and measuring quasi-static lateral acceleration y_{gst} in different vehicles. The performance will be evaluated by comparing the measured acceleration y_{gst} with the acceleration calculated in our model y_{gstM} for 15 different vehicles. The obtained results will be then compared with the results of the multiple linear regression model used previously for the same purpose [16].

KEYWORDS: regression, models, fuzzy models, diagnostics.

Introduction

The ride quality of passenger railway vehicles, according to the UIC-518 norm from the International Union of Railways, is connected with the value of acceleration y_{gst} , this normative says that for ride quality it should have a limit value of $1,5 \text{ m/s}^2$ [7]. Testing and approval of railway vehicles from the point of view of their dynamic behavior includes: safety – fatigue of track – ride quality.

Due to the high cost involved in the measuring of the acceleration y_{gst} of each vehicle, it is necessary to obtain a tool (model) that allows to predict the behavior of the acceleration according to the measured 23 geometric variables which are routinely measured in the normal preventive maintenance routine of the railway, without the need of performing a y_{gst} measuring process. Many of the traditional method used to solve this problem are based on global models like Polynomials (ARMA, ARX, NARMX, NARMAX) [4,6,12], radial basis functions and neural network [1,2,3,5], fuzzy clustering [6,13], among others some of them are used in similar railway applications in the world [20,21].

The fuzzy clustering is to approximate a nonlinear regression problem by decomposing into several local linear models; this approach has advantages in comparison to global nonlinear models [13,16]. The model structure is easy to understand and interpret, both qualitatively and quantitatively. Besides, the approach has computational advantages and goes down to straightforward adaptive and learning algorithms. To show the feasibility of the approach, we will compare the obtained results using fuzzy clustering with the Babuska toolbox [13] with the results obtained with the multiple linear regression model used previously for the same purpose [16].

This article is part of the development of SPD (Portable Diagnostic System) [16,22,23,24, 17,18,19], which consists of the measurement of the vehicle's variables allowing the identification of the technical condition for the vehicle-railway interface.

The paper is organized as follows. In section 2, we will introduce the element for the regression used in the SPD system; in section 3 we will review the nonlinear regression; section 4 will detail the fuzzy clustering methodology; and sections 5 and 6 will show the results the comparison with NRL (Multiple nonlinear regression) [16] and conclusions respectively.

Study system

The Metro system of Medellin was created on the may 1979 by the Medellín Municipality and Antioquia Department, allowing the creation of the Metro de Medellín Company.

Description of the railroad (fig.1):

- Line A: parallel to the Medellín River and with the length of 23.2km, with 19 stations in North to South direction.
- Line B: it starts from the centre of the city in San Antonio B Station and goes westwards. It has the length of nearly 5.6km and has 7 stations.
- Linking Line: it connects the two lines described above and has the length of 3.2km.

- Line K: it is a cable transport system that connects the Acevedo Station. It consists of 4 stations and has the length of 2.4km.



Fig.1 . Metro System of Medellín

In order to extract the data, both estimators given by the UIC 518 standard and the geometric variables, the complete railroad of the train is taken and the measuring points were classified by sections, just as the standard UIC-518 recommends. The three zones proposed by the standard are considered: tangent tracks, large radius curve tracks and short radius curve tracks; however, the lengths of sections composing different zones were adapted according to the distribution of the Line A road of the Metro system. The considered lengths were:

- tangent track: 160 m,
- large radius curve track: 70 m,
- short radius curve track: 70 m.

Data acquisition

The Portable Diagnostic System – SPD – is a unique development for railway systems which, apart from evaluating safety, ride quality and monitoring the condition of geometric parameters of the track-vehicle interface, also allows to carry out the multidimensional monitoring of the condition and to determine the failures of passenger vehicles of the Metro [16,22,23,24].

To develop this diagnosis tool, different methodologies were used, grouping several modern and effective methods in diagnosis tasks, which go from the selection of measurement points, through the method of evaluation of the UIC-518 standard until the utilization of an optimized forecast method [7,16,22,23,24].

The system is composed of eight modules: sensors, signal processing, condition monitoring, condition testing, incipient failures detection in the wheel-rail interface, decisions support, forecast and presentation. In the fig. 2, the SPD module structure is shown.

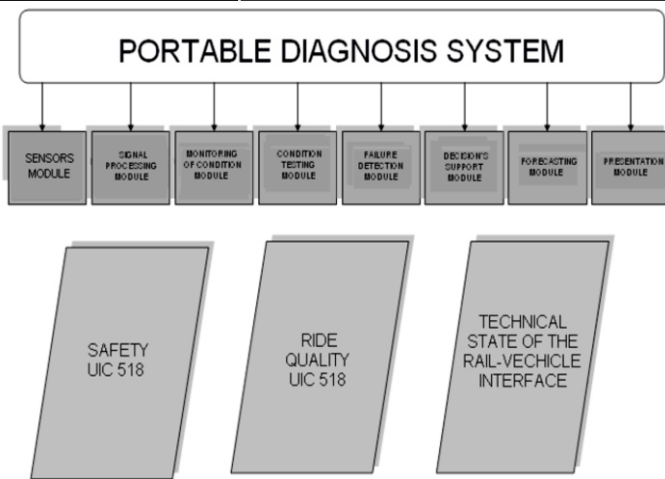


Fig.2. Module of the SPD

The signal obtained by the SPD allows calculating the lateral and longitudinal forces generated in the vehicle along the track which are necessary for safety evaluation.

The UIC-518 standard describes the experimental procedures to follow in order to carry out the motion tests and the analysis of the results, in terms of quality and rolling from the point of view of dynamic behavior in relation to safety, railroad wear and motion behavior (ride quality) with the purpose of an approval for the international railway traffic.

Table 1 presents the different estimators considered by the standard. It was necessary to acquire acceleration and forces signals in different parts of the train to calculate the estimators [5].

Table 1. Estimators for safety, ride quality, and track fatigue according to the UIC-518 Standard.

Estimator	Description	Units	Limit Value
SY2m	Sum of guiding forces for axle	kN	66.7
SY2m (99,85%)	Sum of guiding forces for axle, Percentile 99,85%	kN	66.7
SY2m (0,15%)	Sum of guiding forces for axle, Percentile 0.15%	kN	66.7
sSY	Weighted r.m.s of Sum of guiding forces por axle.	kN	33.3
Yqst	Quasi-static force between wheel and rail	m/s ²	60
y:*q	Lateral acceleration in the vehicle body	m/s ²	2.5
y:*q (99,85%)	Lateral acceleration in the vehicle body, Percentile 99.85% .	m/s ²	2.5
y:*q (0,15%)	Lateral acceleration in the vehicle body, Percentile 0.15%.	m/s ²	2.5
sy:*q	Weighted r.m.s of Lateral acceleration in the vehicle body.	m/s ²	0.5
y:*qst	Quasi-static acceleration in the vehicle body.	m/s ²	1.5
z:*q	Vertical acceleration in the vehicle body.	m/s ²	2.5
z:*q (99,85%)	Vertical acceleration in the vehicle body, Percentile 99.85%.	m/s ²	2.5
z:*q (0,15%)	Vertical acceleration in the vehicle body, Percentile 0.15%.	m/s ²	2.5
sz:*q	Weighted r.m.s of Vertical acceleration in the vehicle body.	m/s ²	0.75
I	Cant deficiency	mm	150

Because this article is limited to the ride quality evaluation, the estimator to use will be the acceleration y_{qst} . According to the UIC-518 standard [7], the limit value of this acceleration of $1,5m/s^2$, defines the ride quality or motion behavior of the vehicle.

This estimator is obtained from the lateral acceleration signal, taken form the passenger's box (vehicle). These measurements are filtered by a digital filter, topology Butterworth, order 8 and cutting frequency of 20Hz.

Geometric variables

Among the current maintenance routines of the railway system, different geometric variables that give an idea of the technical condition of the rail are measured. They are presented in Table 2.

Table 2. State condition of variables.

Geometric Variable	Description	Units	Limit Value
X1	Equivalent conicity with standard deviation of 1.25 under the UK method.	N/A	
X2	Equivalent conicity with standard deviation of 2.5 under the UK method.	N/A	
X3	Equivalent conicity with standard deviation of 3.75 under the UK method.	N/A	
X4	Maximum speed vehicle	km/h	80
X5	Standard deviation of the vertical alignment.	mm	2.3
X6	Standard deviation of the horizontal alignment.	mm	1.5
X7	Cant deficiency	mm	150
X8	Curve radius	m	0
X9	Horizontal alignments	mm	3
X10	Diferencia de altura entre la cabeza del hilo alto y bajo.	mm	3
X11	Vertical alignments	mm	10
X12	Gap between the internal rail faces.	mm	3
X13	Synthetic coefficient of the railroad quality.	mm	0
X14	Vertical wear of the head rail for the high rail (east-south)	mm	12
X15	Vertical wear of the head rail for the high rail (west-north)	mm	12
X16	r.m.s of the corrugation for the high rail for a wave lenght between 30 and 100 mm.	mm	10
X17	Excess percentage for the high rail for a wave lenght between 30 and 100 mm.	%	50
X18	r.m.s of the corrugation for the high rail for a wave lenght between 100 and 300 mm.	mm	20
X19	Excess percentage for the high rail for a wave lenght between 100 and 300 mm.	%	50
X20	r.m.s of the corrugation for the low rail for a wave lenght between 30 and 100 mm.	mm	10
X21	Excess percentage for the low rail for a wave lenght between 30 and 100 mm.	%	50
X23	r.m.s of the corrugation for the low rail for a wave lenght between 100 and 300 mm.	mm	20
X24	Excess percentage for the low rail for a wave lenght between 100 and 300 mm.	%	50

Principles of regression

Generally, fuzzy systems are approximations of functions. Because of this, they can also be used in nonlinear regression problems. The nonlinear regression is a modeling of static dependence of the response of a variable called regressor, where:

$y \in Y \subset \mathfrak{R}$, is a regression vector, $x = [x_1, x_2, \dots, x_p]^T$, over the $X \subset \mathfrak{R}^p$ domain. The elements of the regression vector can be called regressors and the X domain can be called regressor space. The system generated by the data can be described by:

$$y \approx f(x) \tag{1}$$

The deterministic function $f(\cdot)$ captures the dependence of y in x , and the symbol \approx reflects the characteristics of that are not exact in function of f . The objective of the regression is to use the data in order to build a function $F(x)$ as an approximation to $f(x)$ not only because of the data, but because of the domain itself. The definition of a reasonable approximation depends on the purpose for which the model is built. If the objective of the model is to obtain predictions of y , the accuracy must be the most relevant criteria. The accuracy insufficiency is usually known as the integral error over the domain.

$$I = \int_x \|f(x) - F(x)\| dx \tag{2}$$

Generally, this error can not be computed, since the value of f is only known with the availability of the data. However, the average of the error prediction of the available data is often used

$$J = \frac{1}{N} \sum_{i=1}^N \|f(x_i) - F(x_i)\| \quad (3)$$

where N is the number of data in the sample.

Apart from the prediction accuracy, the objective can also be to obtain a model which can be used in order to analyze and understand the real properties of the data generator system. The potential of fuzzy models is that they describe systems as the collection of simple local sub-models expressed by rules. The rules can be formulated using a natural language which is more understandable than a mathematical language. The rules can also be combinations of analytical models commonly used in the control field of engineering, like the local linear models in Takagi-Sugeno [12].

Nonlinear regression model

The input of our model are 23 geometric variables of the rail state, and with them, the modeled acceleration Y_{optM} is calculated.

An arrangement is conformed having a line for each of the n geometric variables measured for each section, and a column for each of the N sections. This arrangement is called the matrix of observation X (regresor space).

$$Z = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1N} \\ z_{21} & z_{22} & \dots & z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & \dots & z_{nN} \end{bmatrix} \quad (4)$$

Traditionally, the clustering terminology defines the columns of the matrix of observation X as characteristics or attributes, while the lines are called patterns or objects.

Fuzzy clustering logics

It is defined as cluster, the sub-set of data which are more similar between them than with other data from another sub-set.

There are different types of data association or clustering, one of the most popular is the "Hard clustering" which refers to grouping data in specific clusters mutually exclusive (see fig.3), meaning that the data belongs only to one cluster and not to several clusters at the same time. In Figure 3, the data z_3 could belong to both clusters c_1 and c_2 , this data is not taken into account when using the Hard cluster.

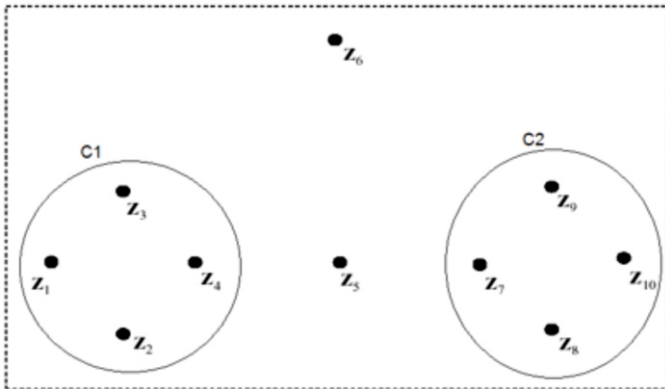


Fig. 3. Data set

(Delft center for System, TU Delft, BABUSKA. R)

It is reasonable to think that on the border of two clusters c_1 and c_2 there are some points which have a degree of belonging to both clusters. The algorithm *c-means* (Bezdek in Jang, 1997) allows that each point belongs to a cluster with a certain degree of belonging, so each point belongs to several clusters. This makes the fuzzy clustering, in some real situations, to be more natural than the Hard clustering.

Partition Fuzzy

The objective of clustering is to divide the data set $Z = \{z_1, z_2, \dots, z_N\}$ in c clusters ($2 \leq c \leq N$), that partition $U = [u_{ik}]$, where u_{ik} is the degree of belonging of i point to the cluster k . U represents a fuzzy partition if the points meet the following conditions:

$$u_{ik} \in [0,1] \quad 1 \leq i \leq c, \quad 1 \leq k \leq N, \quad (5)$$

$$\sum_{i=1}^c u_{ik} = 1 \quad 1 \leq k \leq N \quad (6)$$

$$0 < \sum_{k=1}^N u_{ik} < N \quad 1 \leq i \leq c, \quad (7)$$

Defining the fuzzy partition space as:

$$M_{fc} = \left\{ U \in \mathfrak{R}^{c \times N} \mid u_{ik} \in [0,1], \forall i, k; \sum_{i=1}^c u_{ik} = 1, \forall k; 0 < \sum_{k=1}^N u_{ik} < N, \forall i \right\}$$

Algorithm for Fuzzy C-Means

There are different algorithms for fuzzy clustering, the most used is the "C-Mean" algorithm. This algorithm makes the data partition one can of minimize the objective function [6]:

$$J(Z; U, V) = \sum_{i=1}^c \sum_{k=1}^N u_{ik}^m d_{ik}^2 \quad (8)$$

where:

$$Z = \{z_1, z_2, \dots, z_N\} \quad (9)$$

is the data set to classify.

$$U = [u_{ik}] \in M_{fc} \quad (10)$$

is the partition matrix Z .

$$V = [v_1, v_2, \dots, v_c], \quad v_i \in \mathfrak{R}^n \quad (11)$$

is the centre vector (clusters) to find.

$$d_{ik}^2 = \|z_k - v_i\|^2 \quad (12)$$

is the Euclidian norm, distance from the data to the center of the cluster.

$$m \in [1, \infty) \quad (13)$$

is an exponent that determines the fuzziness of the obtained clusters.

The steps of the algorithm are:

- to select a belonging matrix,
- to start the number of clusters,
- to calculate the centroid of the clusters,

$$v_i = \frac{\sum_{k=1}^N u_{i,k}^m z_k}{\sum_{k=1}^N u_{i,k}^m} \quad (14)$$

- to calculate the Euclidean distance,

$$d_{ik} = (z_k - v_i)^T (z_k - v_i) \quad (15)$$

- to update the belonging matrix,

$$u_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}} \right)^{\frac{1}{m-1}}} \quad (16)$$

The equation (14) gives the value v_i which is the weighted average of the data belonging to a cluster, where the weights are the belonging functions. This algorithm presents the following disadvantages:

- the final results depend on the final partition,
- the number of clusters is defined at the beginning of the algorithm,
- the Euclidean distance method allows detecting only spherical clusters.

This very last disadvantage is a drawback because the most ideal shape of data grouping is given by an ellipse (fig.4), so the most appropriate algorithm is one called "Gustafson-kessel" because this one looks for hyper ellipsoids clusters, which very well detect the quasi-linear behavior of data.

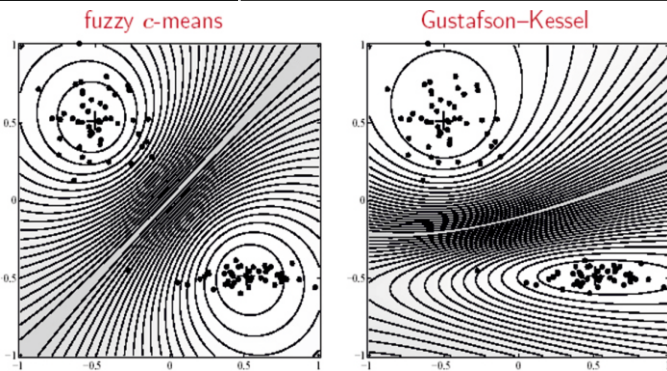


Fig. 4. Clusters of different shape (Delft centre for System, TU Delft, BABUSKA R)

Gustafson-Kessel (GK) algorithm

This algorithm is found among the adaptive distance algorithms. This one extends the fuzzy c-means by choosing a different norm B_i for each cluster instead of keeping it constant.

$$d_{ikB}^2 = (z_k - v_i)^T B_i (z_k - v_i) \tag{17}$$

where: B_i are the possible optimization matrixes of the objective function, and correspond to the covariance of each cluster.

Then, the objective function is defined as follows:

$$J(Z; U, V) = \sum_{i=1}^c \sum_{k=1}^k u_{ik} d_{ikB_i}^2 \tag{18}$$

In order to obtain a viable solution, B_i must be somehow limited. In this case, we will keep its volume constant by fixing the determinant of B_i .

$$B_i = [\rho_i \det(F_i)]^{\frac{1}{2}} F_i^{-1} \tag{19}$$

where: F_i is the covariance matrix for each cluster.

The GK algorithm fits the purpose of identification because it has the following characteristics:

- the cluster dimension becomes limited by measuring the distance and by the definition of the clusters prototype as a point;
- in comparison to other algorithms, GK is relatively insensitive to the initialization of the partition matrix.

Once we have the groups of data, the next step is to derive the inference rules which identify a fuzzy model. To achieve that, there are different types like:

- Mandami: fuzzy rules with fuzzy antecedents and fuzzy consequents.
- Takagi-Sugeno (TS): fuzzy rules with fuzzy antecedents and consequents that could be expressed in a simple way like the first order linear model [12].

Because the TS fuzzy model is an effective tool for the approximation of nonlinear systems based on the information of inputs and outputs through the interpolation of local linear models, which for this case are determined by the cluster, we use this TS model in the solution of the identification of the model we are looking for.

The solution consists of projecting the belonging of the obtained cluster in the desired space (fig.5), thus obtaining belonging functions from the cluster.

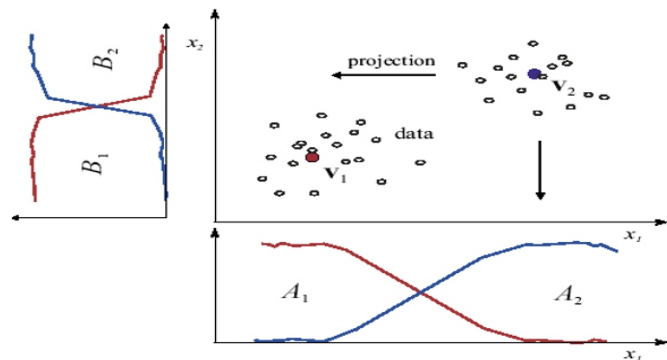


Fig. 5. Extraction of rules by fuzzy clustering (Delft centre for System, TU Delft, BABUSKA R)

Takagi-Sugeno Model

In the Takagi-Sugeno model, the consequent rules are function of the inputs:

$$R_i: \text{If } x \text{ is } A_i \text{ Then, } y_i = f_i(x), i = 1, 2, \dots, K. \tag{20}$$

where: $x \in \mathbb{R}$ is the input variable (antecedent), A_i is a multidimensional fuzzy set (cluster), y_i is the output variable (consequent), R_i is the its rule and K is the number of rules of the rules set.

The consequent function can be linearly expressed as:

$$y_i = a_i^T x + b_i \tag{21}$$

Substituting (21) in (20) we get:

$$R_i: \text{if } x \text{ is } A_i \text{ then } y_i = a_i^T x + b_i \tag{22}$$

Given the outputs of the individual consequents y_i , the global output and the Takagi-Sugeno model is calculated by:

$$y = \frac{\sum_{i=1}^K \beta_i(x) y_i}{\sum_{i=1}^K \beta_i(x)} \tag{23}$$

where: β_i is the commitment degree of the antecedent of its rule, calculated as the belonging degree of x in the interior of the A_i cluster:

$$\beta_i(x) = \mu_i(x) \tag{24}$$

normalizing,

$$h_i(x) = \frac{w_i(x)}{\sum_j w_j(x)} \tag{25}$$

so the TS model could be interpreted as a quasi-linear model with dependence on the input x parameter.

$$y = \sum_{i=1}^r h_i(x) \cdot (a_i^T \cdot x + b_i) \tag{26}$$

Fig. 6 and 7 shows an example of a function $y=f(x)$, represented by four TS rules.

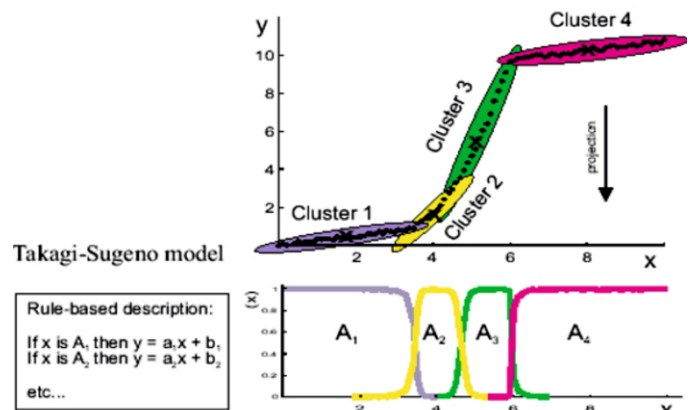


Fig. 6. Takagi-Sugeno fuzzy clustering (Delft centre for System, TU Delft, BABUSKA R)

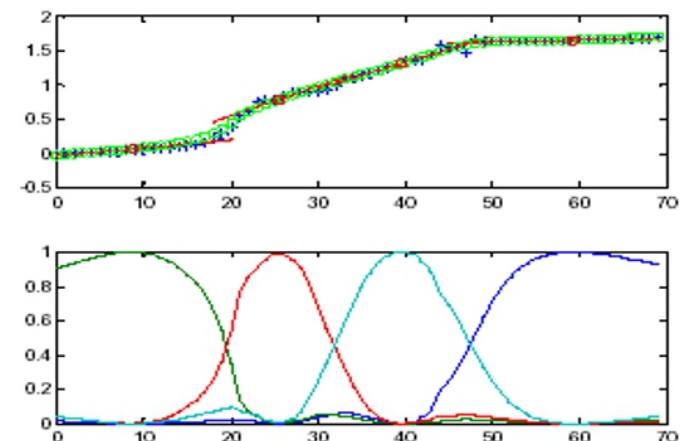


Fig.7. GK y TS fuzzy clustering

The antecedent of each rule defines a valid zone (fuzzy) for the correspondent linear model of the consequent. The global output function is calculated through weighting the local linear models.

Numerical result

During this work, the toolbox developed by Babuska [13] from the Delft Centre for System and Control was used. This tool was developed to be worked on MATLAB. Once the information is collected from the matrix of observations X , it is taken to pre-processing that consists in centering the matrix (fig.8).

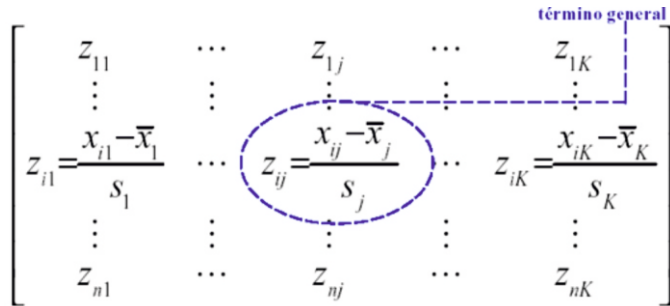


Fig.8. Normalization of a matrix

Where: \bar{x} is the measure of each variable

S is the standard deviation.

The quality of the model is evaluated by calculating the average error, its equivalent in the used toolbox corresponds to the percentile variance accounted (VAF) [13], between the real and the estimated data.

This coefficient is obtained between two signals:

$$VAF = 100\% \cdot \left[1 - \frac{\text{var}(y1 - y2)}{\text{var}(y1)} \right] \quad (27)$$

where the value of VAF will be 100% if both signals are equal. If the values are quite different, the value of VAF will tend to zero.

For each vehicle, the following procedure was followed:

- the matrix is normalized;
- the matrix is divided into two sections, one for the identification of the model and the other to carry out the verification of the obtained model;
- the real acceleration with the obtained model by multiple linear regression was plotted [14,15]. This was also used to determine the ride quality model [16], therefore it will be a reference to validate our results and the obtained result by the fuzzy clustering method;
- the VAF coefficient is calculated to determine the accuracy of the model compared with the real data.

The best results were obtained using the toolbox with the following parameters:

- FM.c = 12; % number of clusters
- FM.m = 2.8; % fuzziness parameter
- FM.tol = 0.1; % termination criterion
- FM.ante = 2; % 2 - projected MFS
- FM.cons = 2; % 2 - weighted LS

where FM is the defined structure by Babuska in MATLAB for the parameters of the toolbox.

In Table 3 present the obtained results with the fuzzy nonlinear regression model and the obtained results with the multiple regression model [16].

Table 3. Results

Item	Unidad	Regresión Lineal(%VAF)	Fuzzy Clustering(%VAF)
1	05	76.16	100
2	09	97.52	100
3	10	80.01	95.53
4	12	86.89	100
5	13	87.03	99.17
6	15	78.69	100
7	17	87.19	100
8	19	92.66	100

9	22	86.81	100
10	24	94.7	100
11	34	84.93	100
12	35	70.86	99.07
13	38	77.83	100
14	40	94.15	92.28
15	41	92.5	92.6

Fig.9 shows the measured data of the acceleration and the model output data obtained by fuzzy clustering for the vehicle 05.

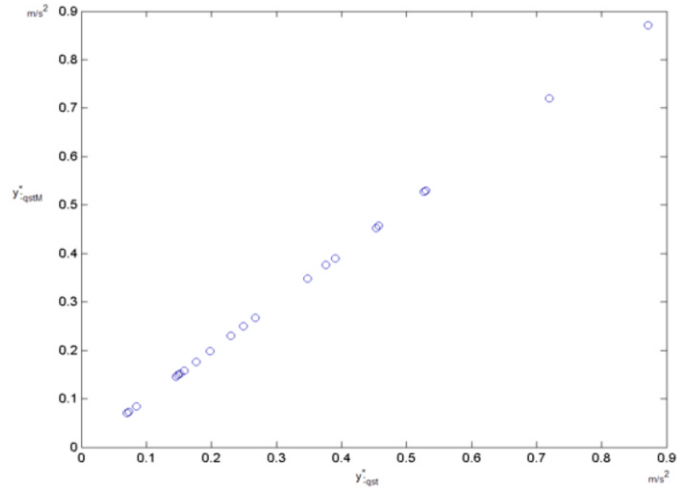


Fig.9. Plot Yqst vs. output fuzzy model vehicle 05

It is noticeable that in the table of results there is a VAF of 100%, which corresponds to the line at 45° of the figure and besides faithfully produced acceleration, as shown in the fig.10.

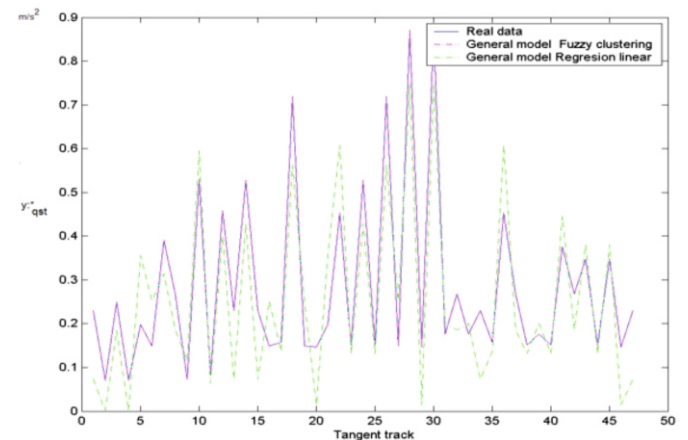


Fig.10 . Real acceleration curve vs. curve vehicle 05

In Table 3, it is observed that the worst VAF coefficient corresponds to the vehicle 40, with a VAF of 92.28. If we plot the data (fig.11), it can be clearly observed which model did not estimate the data well, leaving it out of the comparative graphic of 45° (fig.12).

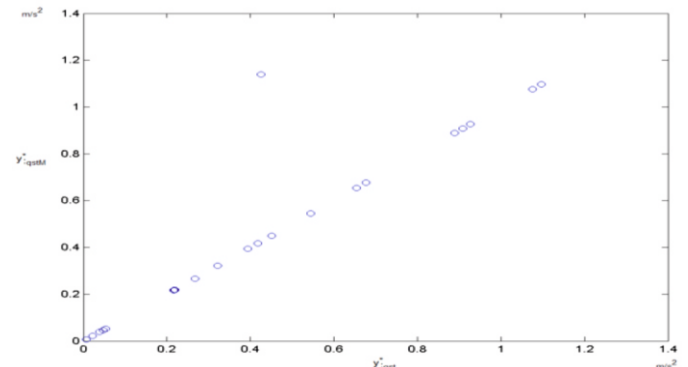


Fig. 11. Plot Yqst vs. fuzzy model output Vehicle 40

This could be due to tuning problems of the model, either in the sensors installation, different geometric conditions of the rail or the equipment capacity, etc.

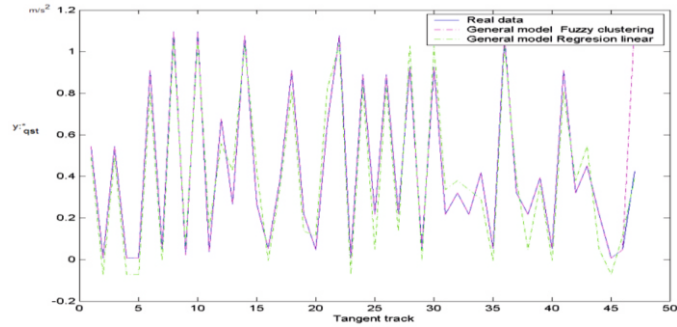


Fig.12. Real acceleration curve vs. vehicle 40

To obtain a general model, all the samples of the 15 vehicles in the matrix were taken, and then a pre-processing consisting of interchanging the files randomly was performed. Afterwards, the same process on each vehicle individually was performed and the results were a VAF of 97.35. It can be graphically observed in figures 13 and 14.

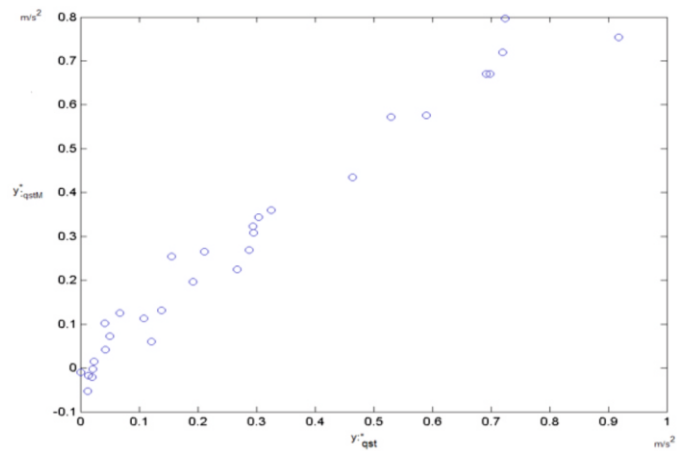


Fig. 13. Plot Yqst vs. Fuzzy general model output

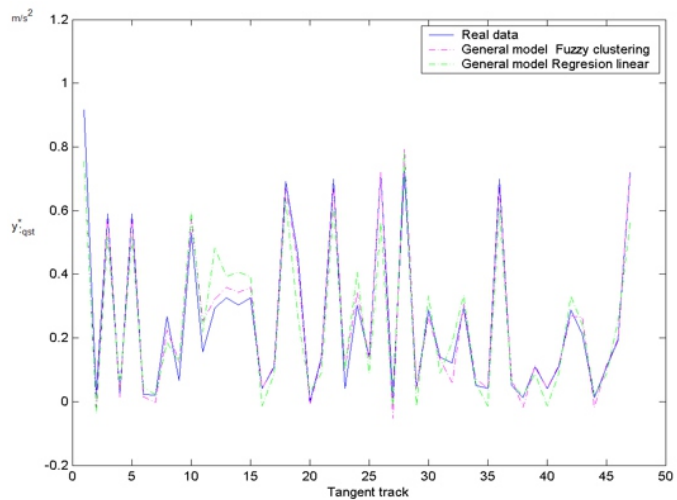


Fig.14. Real acceleration general model curve

Conclusions

In this article, the main Fuzzy clustering aspects for the model identification were revised. Although the obtained results with the linear multiple regression are satisfactory, comparing the obtained results, we find that the quality of the fuzzy model is better in 14 out of 15 analyzed vehicles, and only one vehicle of the model of linear multiple regression is better with the fuzzy model.

We showed that Fuzzy clustering is a good tool to approximate nonlinear functions, especially the Takagi-Sugeno model.

This regression model can be integrated into the process for decision support in the maintenance of rail-vehicle interface to reduce the cost associated with the maintenance work, human resources and the increase of system reliability.

Due to the reasons explained before, when it comes to identifying a nonlinear model, we recommend the fuzzy model to be used in future implementations among the SPD.

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