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Luciano Angel[1], Myrian Tebaldi[2], Marcelo Trivi[2], Néstor Bolognini[2]

1 Departamento de Ciencias Básicas, Universidad EAFIT, Medellín, Colombia

2 Centro de Investigaciones Ópticas and UID OPTIMO (Fac. Ingeniería, UNLP) La Plata, Argentina.

ABSTRACT

In speckle photography the pupil aperture is usually not modified between exposures. In our work, the change of the pupil aperture scale between exposures is analysed on the basis of double-exposed image speckle, before and after a diffuser in-plane displacement is done. The apertures have the same shape but its scale is modified between exposures. Note that the relative position of the aperture is maintained. In particular, we analyse a simple case that uses a circular aperture whose diameter is modified for recording each image. The intensities in the image plane and the fringe visibility are evaluated, in terms of the geometric characteristics of the pupils.

Keywords: speckle, interferometry, metrology

1. INTRODUCTION

As it is well known, the high coherence of the laser light allows to observe interference effects in the light scattered out from diffuser surfaces [1, 2]. Concerning these properties, it appears non contacting techniques like speckle photography that permits to measure deformation, displacements, vibrations [3-5]. In speckle photography, two patterns are recorder in a photographic film. The diffuser object is rotated or displaced or deformed between exposures. In the analysis step, the coherent optical processing of the double exposure specklegram allows to measure the object deformation. The pupil aperture is usually not modified between exposure. In our work, the change of the pupil aperture scale between exposure is analysed. The apertures have the same shape but its scale is modified between exposures. Note that the relative position of the aperture is maintained. In particular, we analyse a simple case that uses a circular aperture whose diameter is modified for recording each image. The intensities in the image plane and the fringe visibility are evaluated, in terms of the geometric characteristics of the pupils.

2. ANALYSIS OF THE SCALED APERTURE PUPILS

The experimental set up is depicted in Figure 1. A random diffuser R is illuminated by means of a collimated laser beam of wavelength λ_W. An image of this input is formed by using a lens L_1. A pupil mask P with several apertures is located immediately in front of the lens. The resulting specklegram S recorded in a photographic film is reconstructed by illuminating the specklegram by a collimated laser beam of wavelength λ_R. Afterwards, the transmitted light is Fourier transformed by means of a lens L_2 of focal length f.

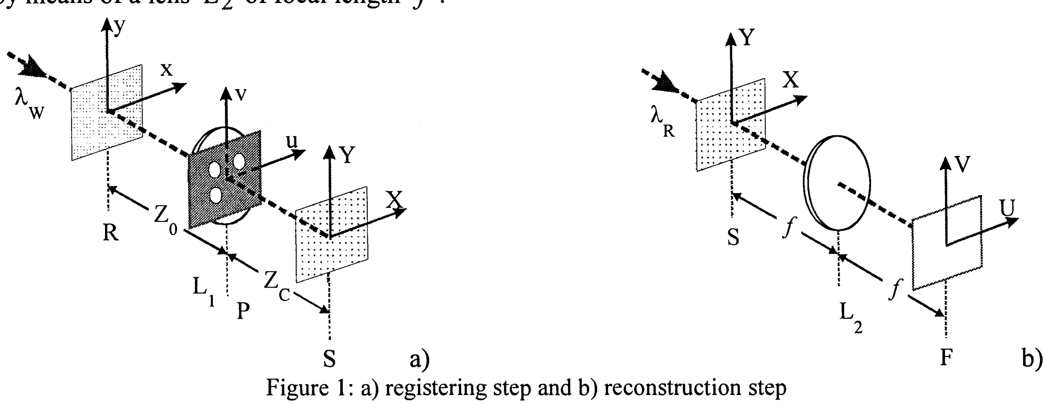

Figure 1: a) registering step and b) reconstruction step

In the following, it is analysed the effect of changing the pupil aperture scale between exposures. Let us consider a double exposure scheme. The pupil aperture used in the first and second exposure can be defined as:

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$$P^1(u,v)=\sum_{j=1}^{q} a\left(u-u_j, v-v_j\right) \qquad \text{and} \qquad P^2(u,v)=\sum_{j=1}^{q} a\left(\frac{u-u_j}{e_1}, \frac{v-v_j}{e_2}\right) \tag{1}$$

where e_1 and e_2 are positive constants which determine the scale change introduced in the respective axes, q is the number of apertures in the pupils, $a(u,v)$ is the aperture shape and $\left(u_j, v_j\right)$ are constant vectors for $j=1,2,\ldots q$.

In Ref. 6 is analysed a multiple exposure by using different pupil arrangements between exposures and it was demonstrated that the average intensity in the Fourier plane results:

$$\begin{aligned}\left\langle I_f(U,V)\right\rangle &= \sum_{k=1}^{N} \Im\left\{\left|\Im\left\{P^k(u,v)\right\}(X,Y)\right|^2\right\}(\vartheta U, \vartheta V)+ \\ &+2\sum_{\substack{k,l=1\\(k<l)}}^{N} \cos\left(\frac{2\pi}{\lambda_R f}\left[U\,\Delta X^{kl}+V\,\Delta Y^{kl}\right]\right)\Im\left\{\left|\Im\left\{P^{kl}(u,v)\right\}(X,Y)\right|^2\right\}(\vartheta U, \vartheta V)\end{aligned} \tag{2}$$

where the smoothed function: $\Im\left\{\left|\Im\left\{P^k(u,v)\right\}(X,Y)\right|^2\right\}(\vartheta U, \vartheta V)$ represents the diffraction halo corresponding to the pupil P^k [10, 12], $\Im$ denotes a two dimensional Fourier transform and $\vartheta=\left(\lambda_W Z_C / \lambda_R f\right)$. Furthermore, $\left(\Delta x^{kl}, \Delta y^{kl}\right)$ stands for the relative uniform in-plane displacement the diffuser undergoes between registered images. Then, $\left(\Delta X^{kl}, \Delta Y^{kl}\right)=-\frac{Z_C}{Z_0}\left(\Delta x^{kl}, \Delta y^{kl}\right)$ is the relative displacement of the respective image amplitude fields. The first term in eq. (2) describes the overlapping of all the smoothed diffraction halos, each one corresponding to an individual single-exposure recording. Note that this term does not contribute to fringe formation. In the second term of eq. (2) the factor $\Im\left\{\left|\Im\left\{P^{kl}\right\}\right|^2\right\}$ can be interpreted in terms of the halos associated with the common part of the pupils P^{kl}. The loci of the diffraction halo of P^{kl} is modulated by a fringe system associated to the relative displacement of the respective images.

To obtain the average intensity in the Fourier plane corresponding to the scaled pupil defined above, we evaluate:

$$\begin{aligned}\Im\left\{\left|\Im\left\{\left|P^2(u,v)\right|^2\right\}(X,Y)\right|^2\right\}(\vartheta U, \vartheta V) &= \left(e_1 e_2\right)^2\left\{\, q\,\Im\left\{\left|\Im\{a(u,v)\}\left(e_1 X, e_2 Y\right)\right|^2\right\}(\vartheta U, \vartheta V)\right. \\ &\left. +\Im\left\{\left|\Im\{a(u,v)\}\left(e_1 X, e_2 Y\right)\right|^2\right\}(\vartheta U, \vartheta V) * \sum_{\substack{k,l=1\\(k\neq l)}}^{q} \Im\left\{\exp\left[i2\pi\left(u_{kl}X+v_{kl}Y\right)\right]\right\}(\vartheta U, \vartheta V)\right\}= \\ &= q\, a\left(-\frac{\vartheta U}{e_1}, -\frac{\vartheta V}{e_2}\right) * a\left(\frac{\vartheta U}{e_1}, \frac{\vartheta V}{e_2}\right)+\sum_{\substack{k,l=1\\(k\neq l)}}^{q} a\left(-\frac{\vartheta U-u_{kl}}{e_1}, -\frac{\vartheta V-v_{kl}}{e_2}\right) * a\left(\frac{\vartheta U-u_{kl}}{e_1}, \frac{\vartheta V-v_{kl}}{e_2}\right)\end{aligned} \tag{3}$$

where $\left(u_{kl}, v_{kl}\right) \equiv \left(u_l-u_k, v_l-v_k\right)$. In particular, when $e_1=e_2=1$ equation (3) reduces to:

$$\Im\left\{\left|\Im\left\{\left|P^1(u,v)\right|^2\right\}(X,Y)\right|^2\right\}(\vartheta U, \vartheta V)=q\, a(-\vartheta U,-\vartheta V) * a(\vartheta U, \vartheta V)+\sum_{\substack{k,l=1\\(k\neq l)}}^{q} a\left(-\vartheta U+u_{kl},-\vartheta V+v_{kl}\right) * a\left(\vartheta U-u_{kl}, \vartheta V-v_{kl}\right) \tag{4}$$

Also

$$P^{12}(u,v) \equiv P^1(u,v)\left[P^2(u,v)\right]^* = \sum_{j=1}^{q} a\left(u-u_j, v-v_j\right) a\left(\frac{u-u_j}{e_1}, \frac{v-v_j}{e_2}\right) = \sum_{j=1}^{q} a\left(\frac{u-u_j}{e_1'}, \frac{v-v_j}{e_2'}\right) \tag{5}$$

where $e_k' = \min\left\{1, e_k\right\} = \begin{cases} 1 & \text{si } e_k \geq 1 \\ e_k & \text{si } e_k < 1 \end{cases}$ $k=1,2$. Let us operate on P^{12}, by following the same procedure as to obtain equation (3). Then, it results:

$$\Im\left\{\left|\Im\left\{P^{12}(u,v)\right\}(X,Y)\right|^{2}\right\}(\vartheta U,\vartheta V)=q\,a\left(-\frac{\vartheta U}{e_1'},-\frac{\vartheta V}{e_2'}\right)*a\left(\frac{\vartheta U}{e_1'},\frac{\vartheta V}{e_2'}\right)$$
$$+\sum_{\substack{k,l=1\\(k\neq l)}}^{q}a\left(-\frac{\vartheta U-u_{kl}}{e_1'},-\frac{\vartheta V-v_{kl}}{e_2'}\right)*a\left(\frac{\vartheta U-u_{kl}}{e_1'},\frac{\vartheta V-v_{kl}}{e_2'}\right) \tag{6}$$

We consider that the pupils $P^1(u,v)$ and $P^2(u,v)$ have one circular aperture ($q=1$) of diameter D_1 and D_2, respectively. Then, $P^k(u,v)=\mathrm{cyl}\left(r/\mathrm{D}_k\right)$ where $r=\sqrt{u^2+v^2}$ is the radial coordinate. In our case $e_1=e_2=\mathrm{D}_2/\mathrm{D}_1\equiv e$ and $e_1'=e_2'=1$ and $\mathrm{D}_2>\mathrm{D}_1$. By using equations (3), (4) and (6), the average intensity (see equation (2)) results:

$$\left\langle I_f(U,V)\right\rangle=\mathrm{C_8AI_R}\left\{\aleph\left(\frac{\rho}{\mathrm{D}_2'}\right)+\aleph\left(\frac{\rho}{\mathrm{D}_1'}\right)\left[1+2\cos\left(\frac{2\pi}{\lambda_{\mathrm{R}}f}\left(U\Delta X^{12}+V\Delta Y^{12}\right)\right)\right]\right\} \tag{7}$$

where $\rho=\sqrt{U^2+V^2}$, $\mathrm{D}_k'=2\mathrm{D}_k\vartheta=2\mathrm{D}_k\lambda_{\mathrm{R}}f/\lambda_{\mathrm{W}}\mathrm{Z_C}$, D_1' and D_2' represent the diameters of the diffraction halo of $P^1(u,v)$ and $P^2(u,v)$, respectively, and

$$\aleph\left(\frac{\rho}{\mathrm{D}_k'}\right)=P^k(-\vartheta U,-\vartheta V)*P^k(\vartheta U,\vartheta V)=\frac{1}{2}\mathrm{D}_k{}^2\,\mathrm{cyl}\left(\frac{\rho}{\mathrm{D}_k'}\right)\left[\cos^{-1}\left(\frac{2\rho}{\mathrm{D}_k'}\right)-\left(\frac{2\rho}{\mathrm{D}_k'}\right)\left(1-\left(\frac{2\rho}{\mathrm{D}_k'}\right)^2\right)^{1/2}\right] \tag{8}$$

The function $\aleph\left(\rho/\mathrm{D}_k'\right)$ is proportional to the diffraction halo associated to the pupil $P^k(u,v)$ [4]. From equation (7) is apparent that the interference fringes are only obtained in the Fourier plane region that corresponds to the smallest pupil halo, in our case $P^1(u,v)$. Therefore, in the region where $\rho\geq\mathrm{D}_1'/2$, the average intensity coincides with the halo of the second exposure $\left(P^2(u,v)\right)$ as can be confirmed by observing Fig. 2. There, $\left\langle I_f(U,V)\right\rangle$ is represented along an axis which is perpendicular to the interferometric fringes as defined by equations (7) and (8). The parameters are: $\lambda_{\mathrm{R}}=633\,\mathrm{nm}$, $\lambda_{\mathrm{W}}=514\,\mathrm{nm}$, $\mathrm{Z_C}=485\,\mathrm{mm}$, $f=100\,\mathrm{mm}$, $\mathrm{D}_1=12\,\mathrm{mm}$, $\mathrm{D}_2=20\,\mathrm{mm}$ and $\Delta X^{12}=80\,\mu\mathrm{m}$. By observing Fig. 2, it can be inferred that the average intensity is restringed to a region between the envelopes:

$$\left\langle I_f(U,V)\right\rangle_{max}=\mathrm{C_8AI_R}\left[\aleph\left(\frac{\rho}{\mathrm{D}_2'}\right)+3\,\aleph\left(\frac{\rho}{\mathrm{D}_1'}\right)\right]\quad\text{and}\quad\left\langle I_f(U,V)\right\rangle_{min}=\mathrm{C_8AI_R}\left[\aleph\left(\frac{\rho}{\mathrm{D}_2'}\right)-\aleph\left(\frac{\rho}{\mathrm{D}_1'}\right)\right] \tag{9}$$

If $\rho\geq\mathrm{D}_1'/2$ both envelopes coincide, and also they coincide with the second exposure halo. By using the defined envelopes, the fringe visibility results [5]:

$$\mathbf{V}(U,V)\equiv\frac{\left\langle I_f(U,V)\right\rangle_{max}-\left\langle I_f(U,V)\right\rangle_{min}}{\left\langle I_f(U,V)\right\rangle_{max}+\left\langle I_f(U,V)\right\rangle_{min}}=\frac{2\aleph(\rho/\mathrm{D}_1')}{\aleph(\rho/\mathrm{D}_1')+\aleph(\rho/\mathrm{D}_2')} \tag{10}$$

Figure 3 displays the visibility in the Fourier plane in terms of the distances to the coordinate origin for several values of the scale parameter $e=\mathrm{D}_2/\mathrm{D}_1$. Note that in all cases in the coordinate origin ($\rho=0$) the visibility reaches its maximum value $2\left(1+e^2\right)^{-1}$. If ρ increases, the visibility decreases $\rho=\mathrm{D}_1'/2$ and if $\rho>\mathrm{D}_1'/2$ the visibility is null. Note that the fringe visibility is determined by the correlation properties of the image field, which depends on the scaled factor *e*.

The previous analysis can be confirmed by observing Table 1. In the second column are shown the speckle pattern corresponding to the first and second exposure of the double exposure specklegram depicted in the third column with a scaled factor detailed in the first column. D_k (k = 1,2) indicates the aperture diameter. In the recording and reconstruction processes the experimental set-ups of Figure 1 are employed. An He-Ne laser is employed in the write-in and read-out processes. The parameters are: $Z_0 = 135\,\text{mm}$, $Z_C = 485\,\text{mm}$, $f = 100\,\text{mm}$, $\Delta x^{12} = 40\mu\text{m}$ and an holographic film (Agfa-Gevaert 10E-75) is employed.

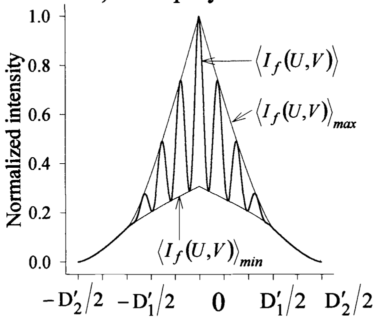


Figure 2: Computed average intensity profile in the Fourier plane.

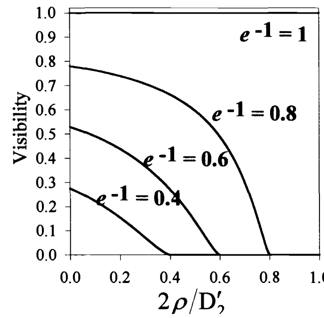
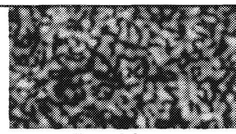
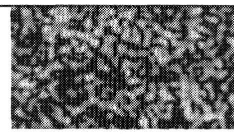
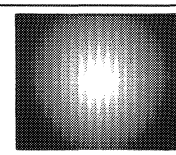
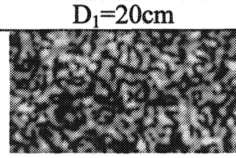
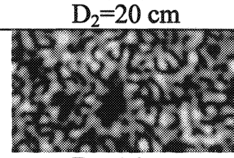
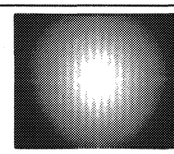
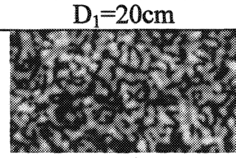
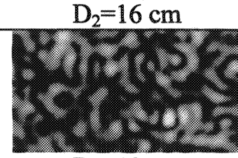
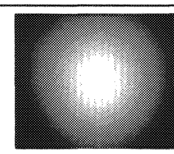


Figure 3: Visibility curves for different values of the scaled parameter e^{-1}.

SCALE FACTOR	MAGNIFIED SPECKLE INTENSITY DISTRIBUTIONS		DOUBLE EXPOSURE SPECKLEGRAM
	FIRST EXPOSURE	SECOND EXPOSURE	
$e = 1$	 D_1=20cm	 D_2=20 cm	
$e = 1.25$	 D_1=20cm	 D_2=16 cm	
$e = 1.67$	 D_1=20cm	 D_2=12 cm	

3. CONCLUSIONS

In this paper is analyzed the effect of the modification of the pupil aperture scale between exposures in speckle photography. It is obtained a general expression for the average intensity of the spectra and the fringe visibility in terms of the scale parameter. In particular, we studied circular aperture pupils. It is established that the fringes are obtained in the common region of the diffraction patterns belonging to each pupil. The fringe visibility depends on the scale ratio. It reaches its maximum value in the centre of the common diffraction region and is null outside the non-common diffraction halos. Besides, the visibility decreases gradually in between.

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