Spectral-based mesh segmentation

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Abstract

In design and manufacturing, mesh segmentation is required for FACE construction in boundary representation (B-Rep), which in turn is central for feature-based design, machining, parametric CAD and reverse engineering, among others. Although mesh segmentation is dictated by geometry and topology, this article focuses on the topological aspect (graph spectrum), as we consider that this tool has not been fully exploited. We pre-process the mesh to obtain a edge-length homogeneous triangle set and its Graph Laplacian is calculated. We then produce a monotonically increasing permutation of the Fiedler vector (2nd eigenvector of Graph Laplacian) for encoding the connectivity among part feature sub-meshes. Within the mutated vector, discontinuities larger than a threshold (interactively set by a human) determine the partition of the original mesh. We present tests of our method on large complex meshes, which show results which mostly adjust to B-Rep FACE partition. The achieved segmentations properly locate most manufacturing features, although it requires human interaction to avoid over segmentation. Future work includes an iterative application of this algorithm to progressively sever features of the mesh left from previous sub-mesh removals.

Keywords

Fiedler vector Mesh Laplacian Mesh segmentation Spectral analysis

Abbreviations

${\varvec{M}}$
Triangular mesh of a connected 2-manifold embedded in ${\mathbb {R}}^3$ composed by the set of points $X = \{x_1,x_2,\ldots ,x_n\}$ and the set of triangles ${\mathcal {T}} = \{\tau _1,\tau _2,\ldots ,\tau _p\}$.

${\varvec{E}}$
Set of the edges $\{e_1,e_2,\ldots ,e_n\}$ of all the triangles ${\mathcal {T}}$ describing the connectivity of *M*.

${\varvec{G}}$
Graph representation of *M* consisting of the pair (*X*, *E*).

${\varvec{W}}$
Weighted adjacency matrix of *G* of size $n \times n$.

${\varvec{D}}$
$n \times n$ diagonal matrix where D_{ii} is equal to the degree (weighted neighborhood size) of the vertex x_i.

${\varvec{L}}$
Laplacian matrix of *G* defined as $L = D-W$.

$\lambda _i$
$i^{\hbox {th}}$ eigenvalue of the matrix *L* (sorted in ascending order).

${\varvec{u_i}}$
Corresponding eigenvector of $\lambda _i$.

${\varvec{u'_2}}$
Second eigenvector of *L* sorted in ascending order.

${\varvec{V}}$
Indices of the vertices of *G* in concordance with ${\varvec{u'_2}}$ (re-labeling).

${\varvec{d}}$
Second differences of ${\varvec{u'_2}}$ with respect to *V*.

${\varvec{d'}}$
Filtered version of *d*.

${\varvec{t_i}}$
$i^{\hbox {th}}$ local maximum of the set of all local maxima of d' sorted in descending order.

${\varvec{{\mathcal {M}}}}$
A connected and oriented Riemannian 2-manifold embedded in ${\mathbb {R}}^3$.

${\varvec{\frac{\partial }{\partial y^i}}}$
Tangent vectors defining a local coordinate system at a point $p \in {\mathcal {M}}$.

${\varvec{g}}$
Metric tensor which defines an inner product on ${\mathcal {M}}$ where g_{ij} is the local inner product between the coordinates ${\varvec{\frac{\partial }{\partial y^i}}}$ and \({\varvec{\frac{\partial }{\partial

y^j}}}\) at a point \(p \in {\mathcal {M}}\).

\({\varvec{\nabla }}\)
Gradient operator acting on the surface defined by \({\mathcal {M}}\).

div
Divergence operator acting on the surface defined by \({\mathcal {M}}\).

\({\varvec{\varDelta }}\)
Laplace-Beltrami operator on manifolds defined as \(\varDelta = -(\hbox {div} \circ \nabla)\).

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