Exploring Undergraduate Students' Computational Modeling Abilities and Conceptual Understanding of Electric Circuits

Juan D. Ortega-Alvarez, William Sanchez, and Alejandra J. Magana

Abstract—*Contribution:* **This paper adds to existing literature on teaching basic concepts of electricity using computer-based instruction; findings suggest that students can develop an accurate understanding of electric circuits when they generate multiple and complementary representations that build toward computational models.**

Background: **Several studies have explored the efficacy of computer-based, multi-representational teaching of electric circuits for novice learners. Existing research has found that instructional use of computational models that move from abstract to concrete representations can foster students' comprehension of electric circuit concepts, but other features of effective instruction using computational models need further investigation.**

Research Questions: **1) Is there a correlation between students' representational fluency and their ability to reason qualitatively on electric circuits? and 2) Is the quality of student-generated computational representations correlated to their conceptual understanding of electric circuits?**

Methodology: **The study comprised two cases in which 51 sophomore-engineering students completed a voluntary assignment designed to assess their representational fluency and conceptual understanding of electric circuits. Qualitative insights from the first case informed the design of a scoring rubric that served as both the assessment and the data collection instrument.**

Findings: **The results suggest that a multi-representational approach aimed at the construction of computational models can foster conceptual understanding of electric circuits. The number and quality of students' representations showed a positive correlation with their conceptual understanding. In particular, the quality of the computational representations was found to be highly, and significantly, correlated with the correctness of students' answers to qualitative reasoning questions.**

Index Terms—**Electrical engineering, computer engineering, circuit analysis, mental models, simulation, rubric.**

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I. INTRODUCTION

THE CREATION and use of graphical representations are central to science and engineering discovery and innovation [1]. Practitioners use these representations as tools for communication and thinking [2]. In educational settings, computer-based instruction with external representations can be efficiently designed to support conceptual learning [3]. Representations can be particularly useful when concepts are abstract, invisible, and difficult to understand; one such abstract concept is electricity [4].

The study of electricity is central to science, technology, and engineering curricula at many levels of education. However, the concepts of electricity are abstract and complex, which makes them difficult to teach and learn effectively [5]–[7]. It is, therefore, not surprising that much educational research has focused on the teaching and learning of concepts related to electricity. The results of such research suggest that the learning process is challenging for both instructors and learners. For instance, some studies indicate that students often develop conceptions of electricity that can conflict with the formal conceptions held by experts in the field [8]. This inconsistency between students' and formal conceptions in scholarly literature is often referred to as misconceptions, naïve conceptions, or alternative conceptions [8], [9]. In the context of this study, misconceptions represent students' inability to employ—or inaccuracy in employing—formal concepts of electricity in the solution of problems related to direct current (DC) electric circuits.

Extensive research on misconceptions in DC electric circuits, carried out by [8], identified and grouped students' difficulties into three broad, non-exclusive categories. According to these researchers, students struggle to (a) understand the precise meaning of formal concepts and implement them into electric circuits; (b) relate formal and graphic representations to technical concepts, and (c) apply qualitative reasoning to the analysis of electric circuits. The first difficulty is related to conceptual understanding. The second comprises the ability to manipulate diagrams and mathematical equations, while at the same time connecting these representations to their conceptual meaning. The third difficulty is related to a higher-order skill that encompasses the previous two. The aim of this study to explore the relationships between students' conceptual understanding, as determined by their ability to qualitatively reason on electric circuits, and their use of multiple representations such as diagrams and equations, required to create computational models of simple electric circuits.

The role of representations in helping students correct misconceptions and improve their conceptual understanding of electric circuits is a fertile ground for research efforts. Many scholars have focused on finding instructional materials and practices that enhance the classroom experience [10]–[12]. Specifically, most of them have looked at educational interventions that involve the use of computers and simulations. While some of these studies focused on students' positive perceptions of such interventions [11], others also examined and found significant improvements in student performance. Often, this improvement was attributed to the use of schematics and the ability to measure, analyze, and compare computational models of circuits [13], [14]. With the aim of expanding insights into the role of computation in the instruction of electric circuits, the present study has two goals: (1) To explore potential correlations between the number and quality of student-generated representations during a problem-solving activity and their understanding of the concepts underlying the task. (2) To center the analysis on the affordances of computational representations, nowadays pervasive in science and engineering workplaces.

II. Conceptual Framework

A. Previous Research

Much educational research on students' use of, and ability to work with, representations in different disciplines explores either students' ability to translate between visually different yet equivalent representations [15], [16], or the affordances of particular types of representations [17]. Some scholars also examine the evolution of students' basic representations toward more complete and accurate models that effectively incorporate specific concepts [18], [19]. In the domain of basic electric circuits, the study of the benefits of abstract vs. contextualized representations has been of particular interest for the design of computer-based instruction [20]–[24]. Many of these studies suggest that novice learners develop better near-and far-transfer problem-solving skills when their instruction relies on abstract representations [20], or moves from abstract to contextualized representations [21]–[23]. From a cognitive perspective, abstract representations help students focus on the task and identify elements transferable to similar situations [21], [22]. On the other hand, contextualized representations allow students to draw and build upon their prior knowledge and access their long-term memory, at the risk of becoming too distracting when irrelevant details capture students' attention [22].

Instead of looking at the design features of computer-based instruction of basic concepts of electricity, this study focuses on investigating students' ability to create complementary representations that build toward a computational model of a realistic problem with electric circuits. Moreover, it explores how this ability is related to the conceptual understanding required to solve the problem successfully and qualitatively reason on it. To that aim, two interrelated themes informed the design and execution of the study: models and modeling perspective (MMP), and model-eliciting activities (MEAs). Both concepts are briefly introduced in this section.

B. Models and Modeling Perspective

Although sometimes differentiated in scholarly literature, the terms *representations* and *models* are used as synonyms for the purposes of this paper. MMP studies the development of models used to represent specific concepts. Specifically, MMP focuses on the development of multiple representations that individuals use to produce conceptual tools. In MMP, models are "*conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notations systems, and that are used to construct, describe, or explain the behavior of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently.*" [24, p. 10]. Models both reside in the mind of the students and are embodied in the diagrams, mathematical depictions, and other representational media that students and problem solvers use. In other words, models are a way to represent concepts internally or externally in order to reason with those concepts [25]. The ability to effectively express, use, and think with models has been termed *representational competence* [19].

Modeling is the process of creating representations (models) with a particular aim in a specific situation [26]. Some scholars suggest that modeling depends on the ability to translate between different representations, called *representational fluency* [18]. Educational research suggests that representational fluency—also referred to as representational transformation or representational literacy—can be used to build, describe, and measure students' conceptual understanding [18], [27], [28].

C. Model-Eliciting Activities

MMP assumes that individuals use internal conceptual systems to interpret their experiences, by selecting, filtering, organizing, transforming, or inferring patterns from information [26]. Moreover, MPP maintains that individuals must express conceptual systems that are relevant to a real-life problem in a variety of interacting media in order to have appropriate tools to deal with the problem. Such media include diagrams, spoken and written language, metaphors, and computer-aided simulations [29], [30]. MEAs allow for the uncovering of the thought processes individuals undertake when using internal conceptual systems and various interacting media to solve problems. In other words, MEAs are thought-revealing activities that can be used to demonstrate student modeling abilities, representational competence, and representational fluency [31].

MEAs usually take the form of open-ended problems based on real-life scenarios; these are often client-driven situations that stimulate teamwork and interdisciplinary work over short periods. These problems require students to construct, describe, or explain different representations in order to find a solution [18], [31]. Although born in the context of math education, MEAs have been adapted into engineering education [32], [33] with several proven benefits: the enhancement of conceptual understanding, the improvement of problem-solving and conceptual skills, and the development of ethical reasoning. In the same vein, [34] suggested that MEAs have three major affordances in helping students learn engineering concepts: (a) reinforcement of concepts being studied; (b) integration of previous knowledge with new information; and (c) discovery of concepts before they are formally introduced. Moreover, [35] reported on the successful use of MEAs to address student's misconceptions in mechanics and thermal sciences.

The design of an MEA should follow six guiding principles, in that they should: (1) present a real-life, contextualized problem; (2) require the construction of a model; (3) require students to document their reasoning when solving the problem; (4) provide opportunities for students to self-assess their proposed solution; (5) allow the model and thought process be transferred to similar problems; and (6) support the generalization and adaptation of the solutions and models generated [36], [37]. As discussed in the next section, the first three of these MEA principles informed the design of the assignment that served both as the learning experience, and as the data collection instrument, in this study [38].

III. Methods

This research study comprises two cases. The first took place in the fall of 2014 and the second in the spring of 2016. In both cases, data was collected from a voluntary, extra credit homework assignment in a circuit analysis course at a large Midwestern university. This section describes the full extent of the study, including both cases. The detailed results from the first case have been published elsewhere [38] and will be summarized in Section IV.

A. Participants

The participants of the two case studies were 25 and 26 sophomore engineering students enrolled in a linear circuits analysis class in Fall 2014 and Spring 2016, respectively. The classes were offered by two different instructors from the Department of Electrical and Computer Engineering. Both instructors used a traditional (i.e., not active) approach, and delivered the same materials and exams. The participating students had taken a previous class in programming applications for engineers. Both classes had a majority of male students, most of them electrical engineering and computer engineering majors taking the class for the first time.

B. Data Collection Method

A homework assignment was created to assess students' representational competence and fluency, and their conceptual understanding through qualitative reasoning. The assignment needed to be simple and engaging, so students would voluntarily participate, and complex enough to allow for the emergence of multiple approaches to a solution. To that aim, the design of this assignment leveraged three principles of MEAs: First, it presented students with a problem contextualized in a realistic scenario. Second, it asked students to construct different representations when trying to find a solution [38]. Third, although the assignment did not explicitly ask students to document their thinking process, further analysis and categorization allowed the authors to make inferences regarding students' reasoning (Section IV-A). In addition, the assignment covered two learning outcomes of the course, namely to develop the abilities to (a) analyze linear resistive circuits, and (b) analyze first-order linear circuits with sources and passive elements. The authors selected this design approach to overcome the inadequacy of traditional book-type quantitative problems for reliably eliciting students' thinking process and assessing their conceptual understanding [8, p. 995].

The two-part assignment, adapted from [39, p. 142], posed the problem of starting a car whose battery is dead using a second battery. In the first part (Task 1), students received the written description of the problem and a diagram of the circuit and were required to develop two representations: a mathematical representation (circuit variables and equations), and a computational representation (i.e., a MATLAB or equivalent simulation model). In the second part (Task 2) students were asked to optimize the configuration and design a circuit able to start the car. For this part, the students needed to create the diagram first and then develop the other two representations mentioned above. At the end of both tasks, students were required to interpret their results and make qualitative inferences based on them, which allowed the authors to assess their conceptual understanding. The authors used a rubric with a performance scale from one (below basic) to four (advanced) to assess students' responses to each question of the assignment. The development of this rubric—briefly presented along with the rubric itself in [38]—drew upon the data from the first case and will be discussed in the next subsection.

Students in the first case (Fall 2014) received partial credit for incomplete assignments. That is to say, they were allowed to choose not to develop every representation, which resulted in a significant number of "no response". Students in the second case (Spring 2016) received credit only if they turned in complete assignments with all the representations and the conceptual understanding part. Whereas students in the first case received only complete points, students in the second case received partial scoring (half points) for the representations they developed and their answers to conceptual understanding questions that could not be mapped precisely to one of the four performance levels of the rubric. For instance, in the second case a score of 3.5 was given to answers that were above a 3-point score but not complete enough to earn a 4-point score.

C. Data Analysis Method

After collecting the data from the first case of this study, one of the authors qualitatively analyzed the representations and answers produced by the students. The author looked at every question independently, compared and contrasted the answers, and sorted them into categories according to their completeness and solution patterns. This process of intra-rater agreement was iterated four times until the resulting categories seemed comprehensive and consistent. A rubric with four levels of performance was developed and revised based on the categorization that resulted for each different representation and conceptual questions [38], [40]. The rubric scoring ranged from one to four, where scores below 1.6 were considered as below basic performance, scores between 1.6 and 2.5 were considered as basic performance, scores between 2.6 and 3.5 were considered as proficient, and scores over 3.5 were considered as advanced performance. In the first case only, an additional grading level of "no response" was included to account for instances where a student gave no answer.

That author, and a second researcher with a degree in electrical engineering, used the rubric to independently rate the entire set of students' answers to representation problems and conceptual understanding questions for both cases. Gwet's AC2 analysis of inter-rater reliability with a weighting coefficient of 0.8 for adjacent ratings yielded a substantial agreement of 0.64 and 0.71 for case one and case two respectively [41, p. 166]. This level of agreement was deemed appropriate given the multiple categories of the rubric and the fact that the second rater evaluated the whole data set for both cases. Therefore, the scores assigned by the researcher more familiar with the data set were kept and analyzed via descriptive statistics and correlation analyses using Pearson product-moment coefficient. Correlations were considered strong for values of 0.5 or higher, moderate-to-strong for values of 0.4 or higher, moderate for values of 0.25 or higher, and weak for values lower than 0.2 [42].

Finally, the conceptual understanding scores were tested for significant differences between students who performed at or below the median of the computational representation scores and those who performed above it. This comparison was performed with a Mann-Whitney/Wilcoxon (MWW) non-parametric test, which accounted for the non-normality and skewness of the data. Each task within each case was independently analyzed.

IV. RESULTS

The results section is divided into three subsections. First, the qualitative results that informed the creation of the rubric are presented. Second, the relevant results of the first case are summarized from [38]. Finally, the descriptive statistics and correlational analyses of the second case are presented. The results of the MWW test are included at the end of the third subsection. Together, these results respond to the two goals of this study: (1) to explore potential correlations between the number and quality of student-generated representations during a problem-solving activity and their understanding of the concepts underlying the task; and (2) to explore correlations between students' performance in computational representations and their qualitative reasoning on simple electric circuits.

A. Qualitative Analysis of Student Answers

The four levels of performance of the rubric were defined using the data collected in the first case. The four subtasks of the assignment (one for each representation and a set of three conceptual questions, per task) constituted the four criteria of the rubric. Each criterion is presented below with the description of the advanced level of performance and one example of student performance at any of the four levels.

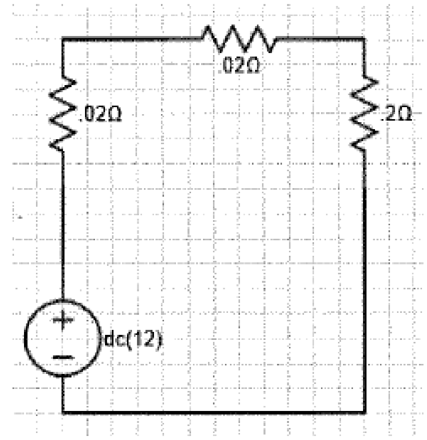

Fig. 1. Diagrammatic representation, proficient level example. The circuit accurately represents only one particular instance of the task, because the dead battery was not included.

1) Diagrammatic Representation: To achieve advanced performance in the diagrammatic representation, the students needed to create an accurate diagram taking into account all the circuit components in the context of the task (e.g., batteries and their internal resistors should not be separated in the diagram because they are not separate in a real battery). Fig. 1 shows an example of proficient performance (one level below the highest possible) in the diagrammatic representation. In this example, the student did not include the dead battery, presumably because it does not add voltage to the circuit. In the case of a dead battery, the circuit response would not change, but the battery should have been included to have a fully accurate diagram.

2) Mathematical Representation: For the mathematical representation, advanced performance entailed developing an accurate set of equations that described the circuit without conceptual issues or calculation errors. To that aim, students could employ any solution method, such as nodal analysis using Kirchhoff's current law, mesh analysis using Kirchhoff's voltage law, superposition theorem, and source equivalences such as Thévenin's theorem and Norton's theorem. The method employed did not affect the score. Fig. 2 depicts an example of performance at the basic level (one above the lowest possible). At this level, students usually carried out a correct circuit analysis but incurred modeling errors due to conceptual understanding issues. In this example, the resistor's voltage is only depending on the voltage of the closest battery and not on the circuit itself. The voltage should depend on the voltage difference between both terminals of the resistor. This conceptual error affected the subsequent mathematical representation and compromised the accuracy of the answer.

3) Computational Representation: Students were asked to apply their programming skills and create a computer model with the mathematical equations they produced. Although they were encouraged to create the model using MATLAB, the students had the option of using any software appropriate for the

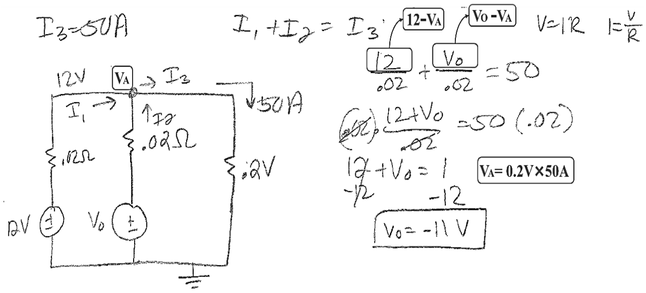

Fig. 2. Mathematic representation, basic level example. Errors are outlined in boxes. V_A represents the voltage in the point indicated on the circuit diagram.

The code is given below

```
V_0 = (50-(12-(0.2*50))/0.02)*0.02+10
V_0 = 0:1:12
I = (V_0-10)/0.02 + (12-10)/0.02
plot(V_0,I)
title('Plot of Current through load resistance (I) and
Voltage (V_0)')
ylabel('Current through Load resistance (I) [in amperes]')
xlabel('Voltage (V_0) [in volts]')
grid
```

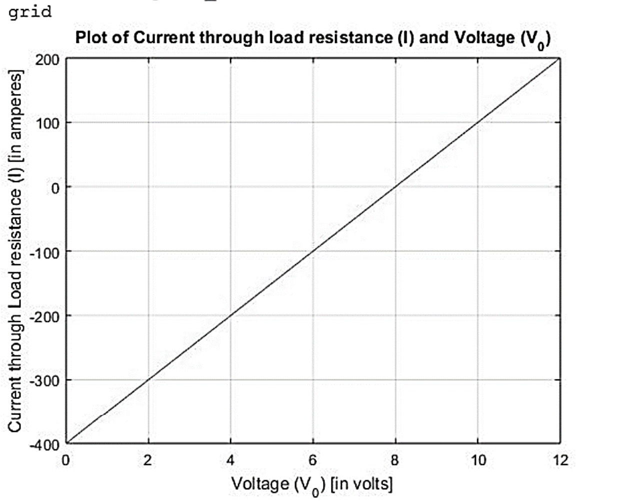

Fig. 3. Computational representation, basic level example. The box indicates an error in the code that makes the model reach extreme, incorrect values.

solution method they chose. Performance in this representation depended on the correctness of the answer and not on the method employed. Students' responses at the advanced level included fully accurate computational models that solved the problem and generated the requested graphs.

While calculation errors alone would lower the assessment to the proficient level, the basic level was assigned to answers also containing errors indicative of conceptual understanding deficiencies. An exemplar of basic performance in this representation is shown in Fig. 3. As indicated in the box, the student assumed that the voltage in the common node of the batteries for the first configuration is constant. The voltage in this node is actually changing as the voltage of the batteries changes, so it should be modeled through an equation instead of just a constant.

The data collection method of the study did not allow the authors to identify with certainty the underlying difficulties that lowered students' scores in the representations. However, analysis of common mistakes students incurred in their computational representations suggested three patterns. The most frequent of these was a lack of an overall strategy to solve the problem. The following pattern dealt with deficiencies in generating a good mathematical representation in the first place. Less frequent were the arithmetic and coding mistakes, sometimes transferred from the mathematical representation.

4) Conceptual Understanding: Students were confronted at the end of Task 1 with two reasoning questions on the analysis of the circuit, and, after Task 2, with the same two questions plus an additional one. A response received full score if it had the correct answer and a proper interpretation. The total score for conceptual understanding was assigned depending on the number of correct responses and their completeness. The first question read: "How does current change as the voltage of the dead battery increases from 0V to 12V? Why?" An example of an answer to this question with room for improvement is:

> "Linear relationship V↑ C↑ [if the voltage increases the current increases]."

Although accurate, this answer needed elaboration and therefore was rated proficient. The second question read: "Based on the last question and the graph generated in [the computational representation] for this [the base or the optimized] circuit, does the current I always achieve the goal of starting the engine for each value of voltage from 0V to 12V? Why?" A student provided an example of a complete and accurate answer (advanced performance):

> "[The circuit] Achieves [the] goal [of starting the car] when the current is greater than or equal to 50A which is when the voltage of [the] dead battery is at least 9V."

Lastly, the third question asked: "Please discuss possible downsides to this optimized design and explain why." Accurate answers to this question read:

> "• The "dead" battery might be an open circuit if it was damaged.
>
> • The "dead" battery won't get charged by the good battery in this case since there is no voltage moving from the positive to [the] negative terminals of the dead [battery] and therefore no energy is being added to the dead battery.
>
> • Circuit elements along the given path might not be designed to survive voltages higher than 12V, which could cause serious damage to the vehicle at a higher cost than a new battery, or alternatively, the current produced by the higher voltage might be too large."

Answers to these open-ended questions provided clues about students' thought-process and the clarity and accuracy of the concepts they applied to the solution of the problem. In other words, these qualitative reasoning questions proved to be suitable probes for conceptual understanding.

B. Quantitative Results From the First Case

These results were published in [38], and only the descriptive statistics and the most relevant correlations are summarized here. Table I presents the overall descriptive statistics

TABLE I
DESCRIPTIVE STATISTICS FOR THE FIRST CASE

Task	Representation	N	M	SD	Min	Max
1 Basic Circuit	Mathematical	25	3.53	1.07	1	4
	Computational	13	1.59	1.58	1	4
	Conceptual U.	17	3.29	0.98	2	4
2 Optimized Circuit	Diagrammatic	25	3.08	1.04	1	4
	Mathematical	9	1.16	1.67	2	4
	Computational	13	1.08	1.47	1	4
	Conceptual U.	17	1.80	1.71	1	4

"Min" and "Max" refer to the minimum and maximum students' scores for a representation. "Conceptual U." stands for conceptual understanding scores.

TABLE II
DESCRIPTIVE STATISTICS FOR COMPLETE RESPONSES TO THE FIRST CASE

Task	Representation	N	M	SD	Min	Max
1 Basic Circuit	Mathematical	12	3.75	0.87	1	4
	Computational	12	2.33	1.50	1	4
	Conceptual U.	12	3.50	0.90	2	4
2 Optimized Circuit	Diagrammatic	5	3.08	0.45	3	4
	Mathematical	5	3.40	0.89	2	4
	Computational	5	3.40	1.41	1	4
	Conceptual U.	5	3.6	0.89	2	4

"Min" and "Max" refer to the minimum and maximum students' scores for a representation. "Conceptual U." stands for conceptual understanding scores.

for this case, and Table II exhibits the descriptive statistics for students who completed all the representations and answered all the qualitative reasoning questions on the basic (Task 1) and the optimized circuit (Task 2) respectively.

The results of the first case (both tasks, n = 25) suggest that the number and quality of students' representations are correlated to their conceptual understanding scores. This finding is consistent with the results of previous studies in the same domain of electric circuits [23]. In particular, a high correlation was found between the number of representations students developed and their mean scores in the conceptual understanding part ($r(23) = 0.53$, $p = 0.006$). Similarly, there was also a moderate, borderline significant correlation between the mean scores for representations and conceptual understanding ($r(23) = 0.37$, $p = 0.060$). This correlation was significant when looking only at conceptual understanding scores of students who provided complete answers in both tasks (n=14, $r(12) = 0.92$, $p < 0.001$).

Looking only at the data from students who answered the conceptual understanding questions on Task 1 (n = 12), the results suggest a moderate correlation between the scores of computational representations and the mean score of conceptual understanding, although above the significance threshold ($r(10) = 0.54$, $p = 0.072$). For Task 2 (n = 5), this correlation was stronger and significant ($r(3) = 1$, $p < 0.001$). In both tasks, the scores of the diagrammatic and mathematical representations also exhibited strong, significant correlations with conceptual understanding. However, as shown in Table II,

TABLE III
DESCRIPTIVE STATISTICS FOR THE SECOND CASE (N=26)

Task	Representation	M	SD	Min	Max
1 Basic Circuit	Mathematical	3.15	0.99	1	4
	Computational	2.52	1.15	1	4
	Conceptual U.	3.04	1.16	1	4
2 Optimized Circuit	Diagrammatic	2.98	1.22	1	4
	Mathematical	2.92	1.23	1	4
	Computational	2.92	1.19	1	4
	Conceptual U.	2.83	0.95	1	4

"Min" and "Max" refer to the minimum and maximum students' scores for a representation. "Conceptual U." stands for conceptual understanding scores.

the sample sizes were very small due to non-response. This limitation led to students being required, in the second case, to complete all the representations and qualitative reasoning questions for extra credit.

C. Quantitative Results From the Second Case

Students in the second case showed a moderate level of achievement, as observed through their average score for representations ($M = 2.89$, $SD = 0.83$). Their conceptual understanding ranked moderate as well ($M = 2.93$, $SD = 0.82$). These results suggest a positive impact of requiring students to work through all representations in the second case. Table III presents the descriptive statistics for this case discriminated by task and type of representation.

As mentioned before, participants in this case developed all five representations (two for Task 1 and three for Task 2) and answered the conceptual understanding questions for both tasks. Therefore, it was meaningless to look at any correlation with the number of representations developed. On the other hand, correlating the mean scores of all representations with that of conceptual understanding yielded a high, significant value ($r(24) = 0.65$, $p < 0.001$). This finding holds when looking at Task 1 ($r(24) = 0.67$, $p < 0.001$) and Task 2 ($r(24) = 0.56$, $p = 0.003$) individually. Contrasting these values with the one obtained for the same correlation from case one ($r(23) = 0.37$, $p = 0.060$), suggests a positive effect of working through all five representations in fostering conceptual understanding. This inference is consistent with the high correlation found from the first case between the number of representations students developed and their scores for conceptual understanding.

Tables IV and V present the relevant correlations estimated with the data disaggregated by task and type of representation. Values obtained from both tasks suggest a high, significant correlation between the mathematical representation and the computational one, and between these two and the conceptual understanding. Notably, diagrammatic representation scores for Task 2 seemed to be unrelated to those of computational representation and conceptual understanding, and only moderately correlated with those of mathematical representation. Note that the analysis of Task 1 (Table IV) does not include

TABLE IV
CORRELATION BETWEEN REPRESENTATIONS AND CONCEPTUAL UNDERSTANDING FOR TASK 1, SECOND CASE (N=26)

	Representation	1	2
1	Mathematical		
2	Computational	0.71***	
3	Conceptual U.	0.67***	0.58**

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

TABLE V
CORRELATION BETWEEN REPRESENTATIONS AND CONCEPTUAL UNDERSTANDING FOR TASK 2, SECOND CASE (N=26)

	Representation	1	2	3
1	Diagrammatic			
2	Mathematical	0.47*		
3	Computational	0.30	0.82***	
4	Conceptual U.	0.24	0.47*	0.69***

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$. N = 26.

TABLE VI
CHARACTERISTICS OF THE SAMPLES OBTAINED FOR THE MWW TEST

Case	Task	Median CR score	N (lp)	N (hp)
1	1. Basic Circuit	1.5	6	6
	2. Optimized Circuit	2	5	5
2	1. Basic Circuit	2	14	12
	2. Optimized Circuit	3.5	14	12

"CR" stands for computational representation; "lp" and "hp" stand for low performers and high performers respectively.

diagrammatic representations since they were provided to the students as prompts in this task.

Across tasks, the mean scores of the computational representation had the highest correlation with those of conceptual understanding ($r(24) = 0.67$, $p < 0.001$), followed by the correlation between mathematical representation and conceptual understanding mean scores ($r(24) = 0.61$, $p < 0.001$). The strong correlation between computational representation and conceptual understanding scores was further probed using the MWW test. Each task within each case was analyzed as a separate sample to acknowledge the requirement of independent observations of the MWW test and the methodological differences between cases. Using the computational representation scores, each sample was split into high (above the median) and low (at or below the median) performers. The characteristics of the resulting samples and subsamples are presented in table VI.

For the first case, the difference in conceptual understanding scores between high and low performers was not statistically significant ($U = 9.0$, $p = 0.07$) for Task 1. For Task 2 this difference was highly significant, although the calculation of the MWW test statistic was affected by the invariability of the above-median scores, which were all fours ($U = 0$, $p = 0.007$). The small subsample sizes caused by the non-response, as presented in Table IV, posed a great limitation for case one. On the other hand, tasks one and two yielded statistically significant differences in the second case with larger subsamples ($U = 25.5$, $p = 0.002$; $U = 30.0$, $p = 0.005$).

```
I_L = -1.05*(600+(V_o/0.02))+(600+(V_o/0.02))

plot(-I_L,V_o)
```

Fig. 4. Calculation error later corrected by the student to plot appropriately.

```
for i = 1:13
    I_load_s = subs( I_load_eq, V_0, V_0_graph(i));
    I_load_fin(i) = int32(I_load_s);
end
```

Fig. 5. Coding error: A continuous result was stored into an integer variable.

V. DISCUSSION

The qualitative analysis of students' representations and answers to the qualitative reasoning questions allowed the authors to create a consistent rubric for scoring purposes. Moreover, this analysis made it possible to observe solution patterns and infer students' reasoning. Similarly, when forming categories of performance, the authors identified some similarities in the type of errors and misconceptions students incurred. The majority of errors were associated with students' deficiencies in generating a mathematical model to represent the electric circuit, which in turn affected negatively the construction of the computational representation. Occasionally, arithmetic and calculation mistakes led to errors in the mathematical representation. Fig. 4 depicts one such case, with an interesting outcome. In the mathematical model, the student added an unnecessary negative sign at the beginning of the calculation of the current (I_L). Later, probably after plotting the result, the student realized that the graph had the wrong slope and added another negative sign in front of the variable I_L. In other words, the result of the computational representation prompted the student to identify a calculation mistake. However, instead of going back and correcting the mathematical model, the student changed the code to portray a more senseful result with the computational representation.

Errors directly related to coding were even less frequent, perhaps due to the simplicity of the task. In most cases, students were able to translate accurately their mathematical representation into a computational one using either a numerical or a literal approach. The latter approach, arguably more advanced, also gave more room for mistakes. For instance, one student did not assign the appropriate variable type to a calculated variable. As shown in Fig. 5, the student used an integer variable to store a continuous magnitude. As a result, the plot generated resembles a tooth saw or a staircase, with abrupt increments instead of a constant slope (see Fig. 6). Interestingly, in this case the graphical result did not prompt the student to double check the code. It stands to reason that differences in conceptual understanding influenced students' decision of whether to revise their computational or mathematical representations based on the outcome.

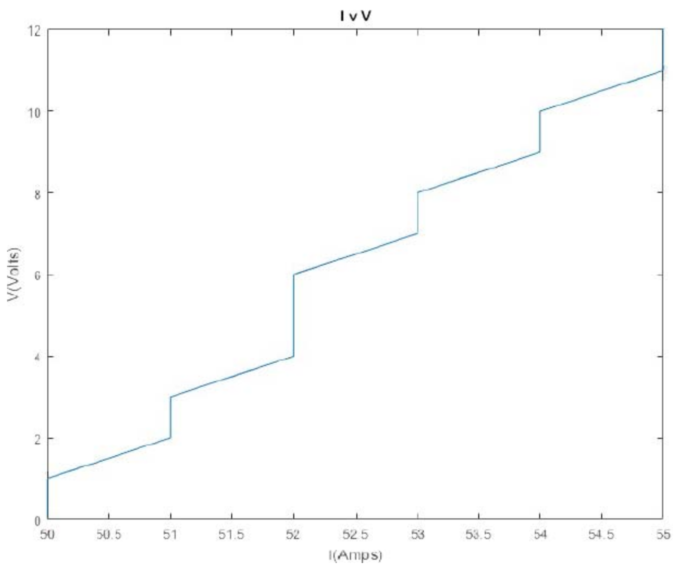

Fig. 6. Calculated current using an integer variable to store the result.

In spite of these interesting findings, the authors did not deem it appropriate to speculate on the causes behind these errors. The design of the study did not afford classifying, characterizing, and mapping students' errors to an existing framework of problematic misconceptions in electric circuits. Future qualitative studies using think-aloud sessions and interviews could allow characterizing and mapping error patterns to misconceptions identified in electricity [8] or to common students' difficulties when coding.

Results from the quantitative analysis of both cases suggest that the quantity and quality of students' representations of electric circuits were positively and significantly correlated with their conceptual understanding of the topic. Specifically, working through all five representations had a positive effect on students' conceptual understanding scores. These results are consistent with the findings of [23], which not only support the correlations found but also suggest that the use of multiple representations in electric circuit analysis fosters near- and far-transfer problem-solving skills. It is, therefore, reasonable to conclude that working with multiple representations that build progressively toward computational models fosters conceptual understanding of electric circuit concepts. On the other hand, the present study used a realistic situation to pose a contextualized problem, which seems to contradict previous findings that favor an instructional strategy from abstract to concrete [20]–[23]. However, this contradiction exists only at the surface. First, although students were given a realistic problem to begin with, the multiple representation approach moved from abstract, simple representations to more complex ones. Second, the present study had a slightly different focus than most of the studies referenced. That is, while previous literature directly explored the affordances of representations in computer-based instruction [4], [5], [20]–[23], this study looked at a formative assessment piece presented to the students to assess and reinforce classroom instruction.

In the design of the assignment, the multiple representations are linked and should be done in order. That is to say, the diagrammatic and mathematical representations should contribute to the creation of the computational model. This study showed that the mathematical and computational representations were highly correlated with each other and with conceptual understanding scores. More importantly, a high correlation was observed between the scores of computational representations and those of conceptual understanding. These findings suggest that computational representations can further contribute to students' qualitative reasoning when they are preceded by skillful handling of the mathematical representations. This interpretation is consistent with previous literature suggesting that algebra can be used to induct students into electric circuits problem-solving [4].

The correlation between the scores of different representations that complement each other is consistent with previous work that identified that individuals require the ability to produce, read, manipulate, interpret, and reinterpret models to use representations effectively [43]. Moreover, individuals must comprehend equivalences in different modes of expression, and be able to learn, transform, and apply information from one representation to another [44]. The results for the diagrammatic representation did not conform to this premise. The authors believe this deviation was observed because the assessment of the diagrammatic representation accounted for issues relevant in representing a real-life scenario, but that did not result in wrong calculations when translated to the mathematical or computational models. For instance, omitting a dead battery in the diagram resulted in a poorly-assessed diagrammatic representation, but carried no consequences to the calculation model.

Future work will explore more deeply the benefits of computational representations alone, and attempt to discern the actual effects of the multi-representational approach. Recent research has shown that students can benefit as much from a computer-based instruction session with computational representations as they would from a traditional lecture on the same topic [46].

The authors are aware of three major limitations of the present study. First, the instrument used for data collection, although rationally designed and drawing upon relevant literature, was not validated to measure conceptual understanding. Second, the small sample sizes can be problematic for statistical inference purposes. The results have been presented independently, and the analysis contrasts and compares these independent results, instead of merging data from two slightly different studies for the sake of a larger sample. Finally, the present study did not account for the variability in the results that might have been introduced by differences in the demographic characteristics of the groups (e.g., proportions of new students and retakers, domestic and international students, EE and CE majors) or dissimilar characteristics of the instructors.

Despite the simple context, a detailed analysis allowed the authors to identify remarkable differences in students' performance with computational representations of basic electric circuits. Future work could expand and test the results of this study by exploring these differences and their impact on the conceptual understanding of more complex topics.

VI. Conclusion

A multi-representational approach to teach electric circuits that builds toward the construction of computational models can support conceptual understanding of the subject matter. In fact, preliminary studies conducted by engineering education researchers suggest that the integration of discipline-based computation learning activities can foster not only the acquisition of computing concepts and procedures but also the acquisition of disciplinary concepts and their application to the solution of engineering problems [45].

The authors believe that it is worth exploring how different levels of performance in representational competence and fluency could be mapped to misconceptions identified by educational researchers. Such a study, in turn, could inform the design of activities to address particular difficulties students might have when learning electric circuit analysis and optimization, particularly activities involving the use of computing tools.

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