Machine learning investment strategies and the liquidity premium for the Colombian yield curve.

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Abstract

This study provides empirical evidence in favor of the theory of liquidity risk premium theory for the Colombian government bonds in the period between 26/07/2006 and 02/08/2006. This theory states that the yield curve will have a positive slope even if no changes in interest rates are expected. This argument would partially invalidate the reasoning of the pure expectations theory, which implies that the interest rate in the curve (forward rate) is an optimal and unbiased predictor of the expectations of the future interest rate (Arosemena and Arango, 2002). We found that it is possible to predict a profit rate of return as a result of an arbitrage strategy. This study contributes by using a different validation method for the theory of liquidity risk premium such as decision trees and random forest, which are part of the supervised machine learning methods. Thus providing significant advantages, i.e.: these take into account nonlinear relationships between parameters; in addition, they can handle both numerical and categorical data and also multi-output problems. Additionally, these are simple to understand, interpret, and visualize.

Key words: Arbitrage strategy; Yield curve ; The Liquidity Premium Theory ; Term structure; Liquidity risk premium

JEL Classification: G190; G120.

1 INTRODUCTION

Some of the research on the yield curve or term structure has focused on the study of its shapes and evolution, the importance of the curve and its behavior lies in the possibility of, among other things, deducing if the monetary policy is being effective as indicated by Frankel and Lown (1994), to extract information on inflation expectations, interest rates, the product and the fiscal deficit (Arosemena and Arango, 2002), as well as for the management of fixed income portfolios, since as indicated by Phoa and Shearer (1998) it is used to compare the expected yields in investment strategies at different terms.

The Expectations Hypothesis and the Market Segmentation Theory (MST) are the principal explanations to the shape and evolution of the term structure on yield curve. The Market Segmentation Theory (Culbertson, 1957) establishes that investors adjust the maturity of their liabilities to the stretch of the curve in which they invest, lenders and borrowers decide the term according to the regulation, the cost of the information and other factors that make them indifferent to the risk premiums offered by the different financial assets.

For the development of this research The Expectations Hypothesis will be of great importance because the possibility of executing arbitrage strategies is theoretically based on it. The Expectation Hypothesis can be divided in Preferred Habitat Theory, The Pure Expectations Theory and The Liquidity Premium Theory, depending on the assumptions made. (Arosemena and Arango, 2002).

The Preferred Habitat Theory introduced by Modigliani and Sutch (1966) assumes that the agents will invest in the stretch of the curve that corresponds to the period of their liabilities as well as under the theory of market segmentation, with the difference that under this theory issuers will be able to offer higher rates for terms with low demand, incorporating a risk premium for agents to change their preferred habitat.

The Pure Expectations Theory introduced by Fisher (1896) and complemented by Kane and Malkiel (1967), assumes that the agents form their interest rate expectations rationally, so they do not incur in systematic errors in their predictions. The agents will make predictions on the interest rate making use of all the available information. This theory assumes that arbitrage opportunities are minimal and that future interest rates are unbiased estimates of future spot interest rates (Rueda and Arango, 2008).

Under the assumptions that the agents are risk neutral and indifferent to the liquidity of the securities, there is no market segmentation, no transaction costs, and the expectations about future interest rates are optimal and unbiased, the pure theory of expectations dictates that the long-term interest rate is determined by the mathematical average of the current and expected short-term rates (Arosemena and Arango, 2002). The expected future spot interest rate value, for a term s, which is negotiated at the time t+n-s, will be equal to the forward rate for this same period, which is negotiated at the time t (Rueda and Arango, 2008), this suggests that the interest rate of a 6-month security should be equal to the average of the current 3-month interest rate and the optimal forecast of the interest rate within three months (Forward Rate), so for an investor it will be indifferent to invest in the 6-month security or buy a 3-month security and then reinvest it in another 3-month security, all under securities of the same credit quality (Arosemena and Arango, 2002). The possibility of performing arbitration under this theory disappears, using the same example above, if at the moment zero the 3-month title is sold and the 6-month title is bought, three months later, after selling the 6-month title and paying the 3-month title, the net profit generated by the strategy will be zero.

In The Pure Expectations Theory, two risks inherent to investments with the same credit risk but different negotiation terms are ignored. The price risk and the reinvestment risk, for the first of them Arosemena and Arango (2002) refers as the risk that the price is lower than expected and for the second as the risk that the interest rate at which the resources are reinvested may be different, due to this new proposals were presented that incorporated the uncertainty present in the market and the preference for liquidity.

The Liquidity Premium Theory introduced by John (1939) argues that in uncertainty and risk aversion agents prefer to invest in the short term than in the long term, the agents will only invest in the long term if these assets have a higher rate of return, such compensation is known as liquidity premium. The expected future spot interest rate value, for a term s, which is negotiated at the time t+n-s, will be higher than the forward rate for this same period, which is negotiated at the time t.

The liquidity premium theory states that the yield curve will have a positive slope even if no changes in interest rates are expected, which would partially invalidate the argument of the theory of pure expectations that the implied interest rate in the curve (forward rate) is an optimal and unbiased predictor of the expectations of the future interest rate (Arosemena and Arango, 2002), this suggests that the interest rate of a 6-month security will not be equal to the average of the current 3-month interest rate and the forecast of the interest rate within three months (Forward Rate).

Contrary to the theory of expectations, under the theory of liquidity premium is given the possibility of arbitration, if at the moment zero is sold the title of 3-month and buy the title of 6-month, three months later, after selling the title of 6-month and pay the title of 3-month, the net profit generated by the strategy will be positive, under the assumption that the real interest rate in month 3 is lower than the estimated (forward rate) in memento zero.

Empirical studies have sought information on future spot interest rates from two main sources, the first group of authors looks if the margin between the short and the long tranche of the curve contains information on future interest rate changes, the second group looks for whether the market uses all publicly available information, such as estimates of income, liquidity, among other variables, to form expectations of future rates.

In this paper, we will predict the profit of a strategy in the Colombian treasury bond market, by monitoring of the variables associated with its prediction. Many authors have estimated the liquidity premium in a moment t, under different methodologies both for the Colombian market and for the international market but none of them has emphasized about the arbitrage strategies derived from this hypothesis, finally, unlike the authors who have worked on the yield curve, our work seeks to identify for which sections of the curve the strategy generates a greater benefit.

Authors such as Hamburger and Platt (1975) study the short stretch of the curve in the Treasury bill market of the United States finding that the three-month forward rate for the 1960s is a poor predictor of the future long-term spot rate, Fama (1984) verifies the hypothesis of pure expectations, the author finds that when the forward rate is high, the spot rate does not grow in equal magnitude, demonstrating that the forward rate is not an unbiased estimator of the future spot rate, and therefore rejecting the hypothesis, Fama (2006) finds evidence of the predictive capacity of the forward rate over the future spot rate for horizons greater than one year in the U.S. Treasury bond market for the period 1952 to 2004, and that such predictive capacity increases as the term also increase, Campbell and Shiller (1991) argues that the rate differential does not correctly predict long-term rate movements but short-term interest movements for American Treasury bill, note and bond, measured over the period 1952 and 1987.

In Colombia, Rey (2005) found evidence against the hypothesis of pure expectations, using TES market data between 2000 and 2004; in this study is rejected the Pure Expectations Theory for all terms except 80 and 270 days. Rueda and Arango (2008) find evidence in favor of the liquidity premium theory, using a time series model based on the model of Fama (Fama, 2006), controlling by the persistence of rates and heteroscedasticity for the CDTs and TES market for the period 2002-2007, the authors also finds that the forward rate contains information on the future trends of the spot rate given that for all terms without exception the forward rate is positive and statistically significant.

The objective of the national government debt titles (TES) is the financing of the nation and the temporary management of treasury operations. Through time, they have become the most representative and used financial assets in the Colombian market, leveraging the capital markets and savings.

On the other hand many authors have used different types of models and variables to explain the behavior of the future spot rates. Dewachter et al. (2014) uses Macro-finance models to estimate the term premium dynamics, finding that long term bonds are affected by all macro shocks, in particular with long-run inflation shocks and additionally finding that movements in the term premium are associated with financial shocks. Yun (2019) investigates the liquidity premium for the Korean government bonds, finding that, similar to the Unites States bond market, the liquidity premium is affected by domestic expected inflation; but additionally, he found that liquidity premium is also affected by global liquidity factors such as S&P 500 option-implied volatility, bank capital flows and the leverage of global banks. Furthermore, Baele et al. (2010) used a dynamic factor model where stocks and bond returns depend on a series of economic state variables where additional to interest rate, inflation, output growth and cash flow growth, the model includes macro-economic uncertainty measures derived from survey data on inflation and GDP growth.

2 INVERSION STRATEGY

As reviewed above, the authors Rey (2005) and Rueda and Arango (2008) found evidence in favor of the liquidity premium in the Colombian market, meaning the yield curve will have a positive slope and forward rate is not an unbiased estimate of the future real interest rate. So the expected value of the spot interest rate, for a term n-s, negotiated in t+s, will be less than the forward rate negotiated in time t for the same periods of time:

$$E(y_{t+s,n-s}) < f^{t}_{t+s,n-s} \tag{2.1}$$

Liquidity theory makes it possible to execute arbitrage strategies by taking short position at $y_{t,s}$ and long position at $y_{t,n}$ and then sell the long position at $y_{t+s,n-s}$ and pay the short position in the period s.[1]

The profit and loss statement (P&L) of the long position will be represented by $\frac{100\ e^{(n)\ y_{t,n}}}{e^{\ (n-s)\ y_{t+s,n-s}}}$ and of the short by $100\ e^{(s)\ y_{t,s}}$.

So the final P&L of the strategy will be given by the result of the equation.

$$P\&L = \frac{100\ e^{(n)\ y_{t,n}}}{e^{\ (n-s)\ y_{t+s,n-s}}} - 100\ e^{(s)\ y_{t,s}} \tag{2.2}$$

As we know,

$$n\ y_{t,n} = (s)\ y_{t,s} + (n-s)\ f^{t}_{t+s,n-s} \tag{2.3}$$

And therefore the equation (2.3) can be rewritten as.

$$P\&L = \frac{100\ e^{(s)\ y_{t,s}+(n-s)\ f^{t}_{t+s,n-s}}}{e^{\ (n-s)\ y_{t+s,n-s}}} - 100\ e^{(s)\ y_{t,s}} \tag{2.4}$$

1

The rate in the period s for a period n-s is $y_{t+s,n-s}$.

Assuming that the rates are continuous compounding, The Forward rate is $f^{t}_{t+s,n-s} = e^{((\frac{n}{360})y_{t,n}\ -\ (\frac{s}{360})y_{t,s})\frac{360}{n-s}}$. Where, The rate in the period t for a period n is $y_{t,n}$, and the rate in the period t for a period s is $y_{t,s}$, By simplifying the equation, and assuming that the rates are continuous compounding.

$$P\&L(\%) = (\ e^{(s/360)\ y_{t,s}}\ (e^{((n-s)/360)\ (f^{t}_{t+s,n-s}\ -\ y_{t+s,n-s})} - 1))\frac{360}{s} \tag{2.5}$$

Using this equation we calculated the rate of the profit and losses of the strategy for the periods t+s, n-s = (90,90 ; 180,180 ; 360,360 ; 720,360 ; 1080,360 ; 1440,360 ; 1800,360 ; 2160,360 ; 2520,360 ; 2880,360 ; 3240,360) between 28/04/2006 and 22/02/2019.

For example, if we want to execute the strategy for the periods t+s, n-s = (1080,360), assuming t=0, we have to take a short position today at 1080 days at a rate of $y_{0,1080}$ and a long position at $y_{0,2160}$, and then sell the long position at $y_{1080,360}$ and pay the short position in the period 1080.

In Table 1 we can see some descriptive statistics for the profitability of the strategy which is expressed in annual continuous compounding.

Table 1

	90 - 90	180 - 180	360 - 360	720 - 360	1080 - 360	1440 - 360	1800 - 360	2160 - 360	2520 - 360	2880 - 360	3240 - 360
Min	-2.76%	-2.45%	-2.19%	-1.13%	-0.85%	-0.14%	-9.62%	-5.75%	0.07%	-0.04%	-0.02%
Max	**3.44%**	**4.95%**	**9.35%**	**7.63%**	**7.69%**	**7.21%**	**4.97%**	**9.93%**	**9.42%**	**3.25%**	**4.28%**
Average	**0.08%**	**0.45%**	**1.30%**	**1.48%**	**1.94%**	**1.75%**	**0.76%**	**0.07%**	**2.47%**	**1.17%**	**1.18%**
P10	-0.72%	-0.73%	-0.78%	-0.29%	**0.59%**	**0.42%**	-0.18%	-1.40%	**1.55%**	**0.76%**	**0.56%**
P20	-0.43%	-0.28%	-0.13%	**0.07%**	**1.00%**	**1.04%**	-0.06%	-1.00%	**1.75%**	**0.94%**	**0.65%**
P30	-0.21%	-0.08%	**0.40%**	**0.57%**	**1.15%**	**1.33%**	**0.03%**	-0.52%	**1.89%**	**1.03%**	**0.81%**
P40	-0.06%	**0.12%**	**0.70%**	**0.97%**	**1.52%**	**1.55%**	**0.17%**	-0.33%	**2.06%**	**1.11%**	**0.94%**
P50	**0.07%**	**0.35%**	**1.03%**	**1.35%**	**1.97%**	**1.73%**	**0.50%**	-0.17%	**2.24%**	**1.17%**	**1.06%**
P60	**0.18%**	**0.55%**	**1.29%**	**1.62%**	**2.42%**	**1.89%**	**0.94%**	**0.12%**	**2.51%**	**1.23%**	**1.27%**
P70	**0.31%**	**0.74%**	**1.62%**	**1.94%**	**2.71%**	**2.15%**	**1.24%**	**0.77%**	**2.86%**	**1.29%**	**1.42%**
P80	**0.45%**	**1.02%**	**2.27%**	**2.42%**	**2.94%**	**2.53%**	**1.58%**	**1.31%**	**3.30%**	**1.37%**	**1.54%**
P90	**0.75%**	**1.45%**	**3.61%**	**3.73%**	**3.33%**	**2.92%**	**2.10%**	**1.69%**	**3.65%**	**1.55%**	**1.95%**
N	3346	3281	3150	2889	2628	2366	2106	1846	1585	1323	1062

As we can see from the 50th percentile, the strategies for all periods except the period (2160, 360) show a positive return, this means that more than half of the strategies executed for each period had a positive return, also in periods such as (1080, 360) (1440, 360) (2520, 360) (3240,360) it is evident that more than 90% of the periods analyzed are positive. This brings a first great conclusion, demonstrating that in the majority of cases the strategy gives a positive result and in some specific periods 90% are positive.

For the period (2520, 360) it is observed that in the 1846 times that the strategy was executed, the result of this was positive, being the minimum profitability reached 0.07% acc and the maximum 9.42% acc which was only surpassed by the maximum profitability reached in the period (2160, 360) with a value of 9.93%.

Now, it is important to understand that the number of data will change depending of the period of the strategy, the period (360, 360) uses the data from any day and the data a year later, while the period (90, 90) uses the data 90 days later. In Table 1 we can see that the difference between the number of data we have for the period (360, 360) and (90, 90) is 196, which is equal to one year less one quarter in working days. In consequence as t+s increases, the number of variables available for prediction decreases.

3 Prediction of the Profit Rate

In this paper we used a method of a specific sub-field of machine learning called predictive modeling to predict the profitability of the different strategies, the method is known as Regression Tree, we selected the 10-fold cross validation as a test to identify how good is our method, using the mean squared error as a metric of evaluation, this gave a gross idea of how wrong all predictions are (0 is perfect).

The main data source is the zero-coupon curve (Collected from Bloomberg), because as we saw before with this, it can be calculated the liquidity premiums for the different terms, we also used the implicit inflation between the zero coupon curve in COP and the zero coupon curve UVR for 1, 3, 5, 7, 10 and 20 years using the parameters provided by the central bank of Colombia, finally we included market variables such as the market rate of return (Colcap) and THe exchange rate yield (TRM) (Collected from Bloomberg).

To understand the importance of the model that we are going to use, first let's review the method of Decision tree for classification.

3.1 Decision Tree

A Decision Tree is a series of yes-no questions about our regressors variables, this questions eventually leads to a classification or continuous value.

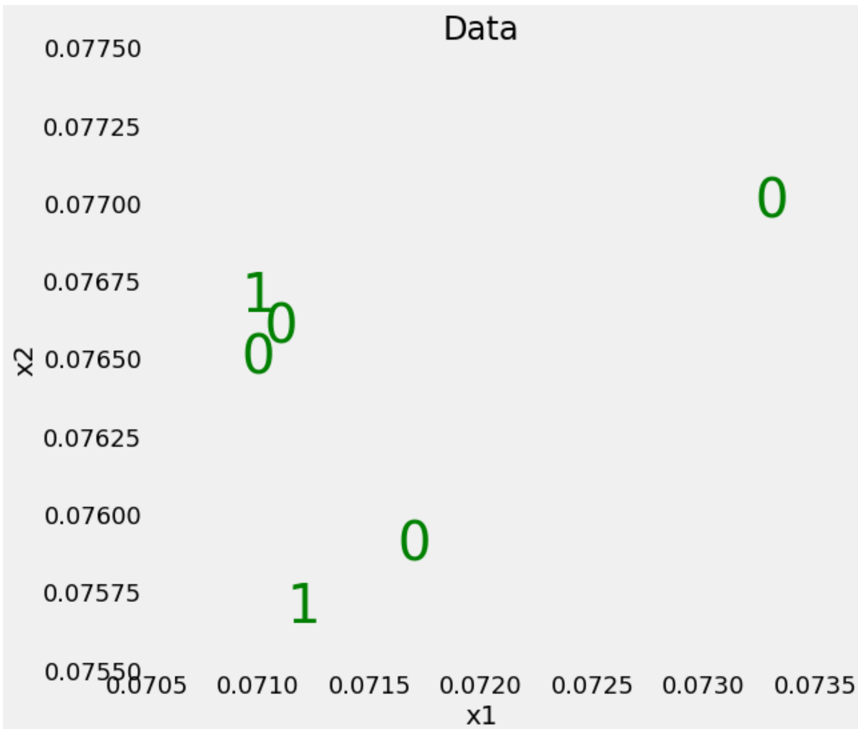
3.1.1 Basic Problem

We made an example using a part of our data to make easy to understand the method; we used the profits and losses for the period (90,90) on the dates between 26/07/2006 and 02/08/2006 as a labels y, but we made a simple transformation, if the profit and loss was negative we put 0 for the value of the variable and if the variable was positive we put a 1. Finally, for the features x we used the rates of the zero-coupon curve for the nodes 180 and 360 days.

Table 2

(90,90)	Transformation (90,90)	180	360
-0.34%	0	0.0733	0.0770
-0.11%	0	0.0717	0.0759
0.01%	1	0.0712	0.0757
-0.26%	0	0.0710	0.0767
0.16%	1	0.0710	0.0765
0.30%	1	0.0711	0.0766

To get an idea of the data, we can plot the data points with the number shown on the label.



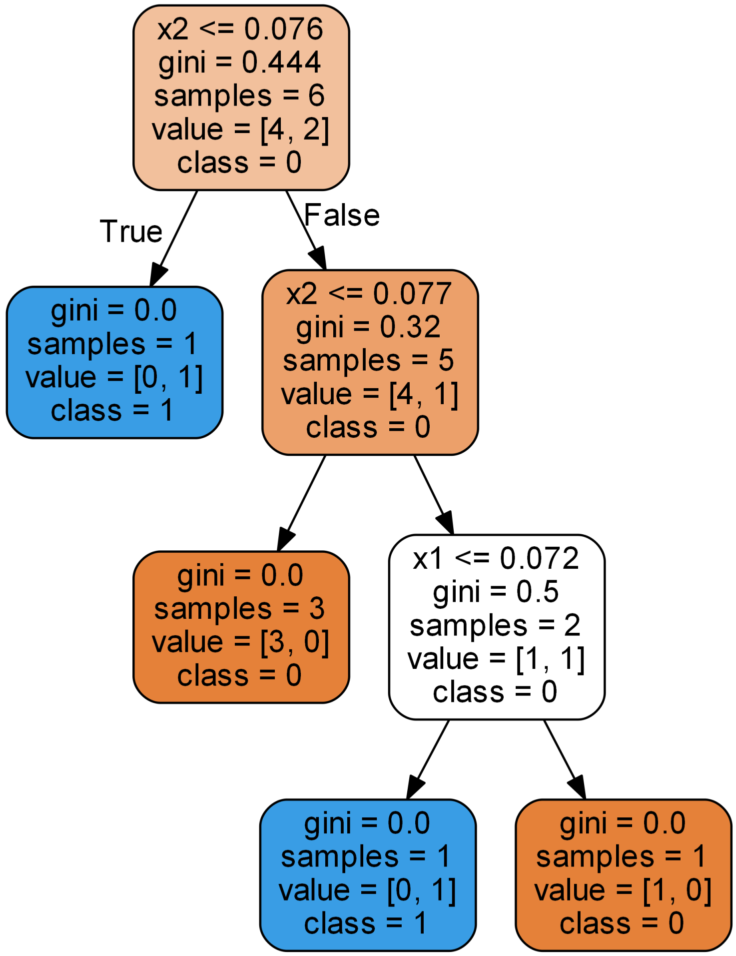
As we can see this is a linearly inseparable problem, a simple line would not be able to separate the points. The single decision tree will be able to completely separate the points because it essentially draws many repeated linear limits between the points.

3.2 Single Decision Tree

The tree will learn how to separate the points, by building a diagram of questions based on the values of the features. At each stage, the decision tree is split to minimize the Gini impurity. This is a problem because it leads to overfitting, in practice we have to limit the depth of the tree so it made well in general data.

If we train our tree without restricting the depth, the tree it would have seven nodes and three of depth. Like we said, the accuracy of this tree it would be of 1[2].

Let's take advantage of the small structure of the tree to understand how it thinks.



The five rows in each node (Except the final nodes) represents.

- Question asked about the futures, in our case about the value of the interest rate in the nodes 180 and 360.
- **gini:** Gini Impurity of the node.
- **samples:** The number of training observations in the node.
- **value:** Number of samples in each class [0,1].

2

The gini Impurity is the probability that a randomly selected sample from the node will be incorrectly classified. The decision tree make the splits base on the gini impurity, it splits the sample base on the value of the future that reduce the gini impurity, eventually the gini impurity goes to 0 if the tree hasn't limit of depth.

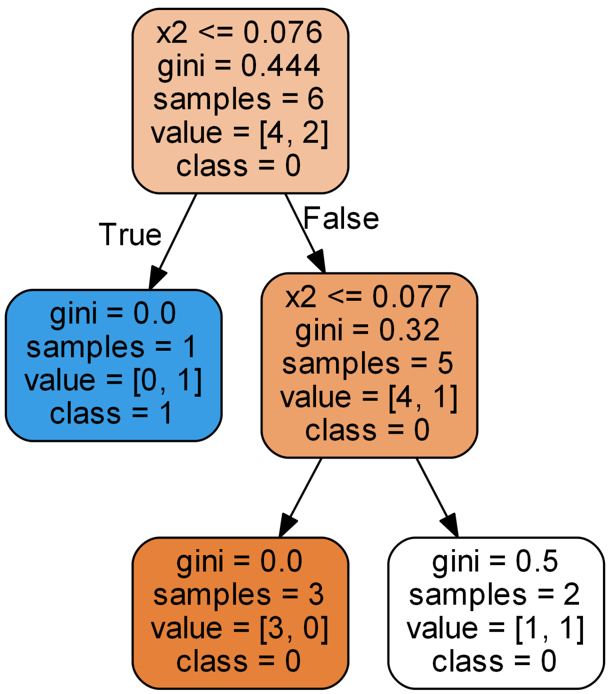
- **class:** The predicted class for all the sample of the node.

Only in the final nodes the tree will take a decision about the class of the sample.

3.3 Limit Maximum Depth

In practice, we want to limit the maximum depth so the tree can do better with all the data and not only with the training set. So let's make an example to see how it works if we limit the maximum depth of the tree to 2.

If we limit the depth of the tree to two, the accuracy of it will be 0.666, let's visualize the tree to understand the changes.



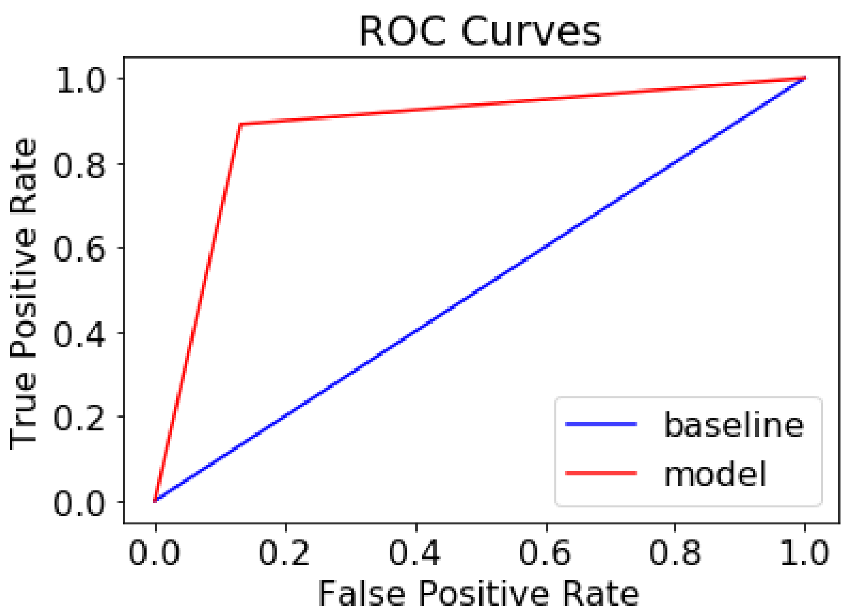
As we can see our tree has no more a perfect accuracy in the training data but it probably do better on the testing set. The bias - variance tradeoff said a model with high variance learn the training data very well but it can't do as the same with the points on the testing set. On the other hand, a model with high bias has not learned the training data very well because the lack of complexity, but it can generalized the results very good to the points in the testing set. Our example proves that a way to improve the bias is limiting the depth of the tree, but other option is using an entire forest of trees, the idea is to train each tree with different sub sample of the training data to finally takes the average of each individual tree to arrive at a classification.

Now lets use all our data set with the same transformation that we used before, where the variable of profit takes the value of 0 if this one is negative and 1 if is positive, this exercises will help us to predict with all our variables and for all our strategies when the result of it will be positive or negative.

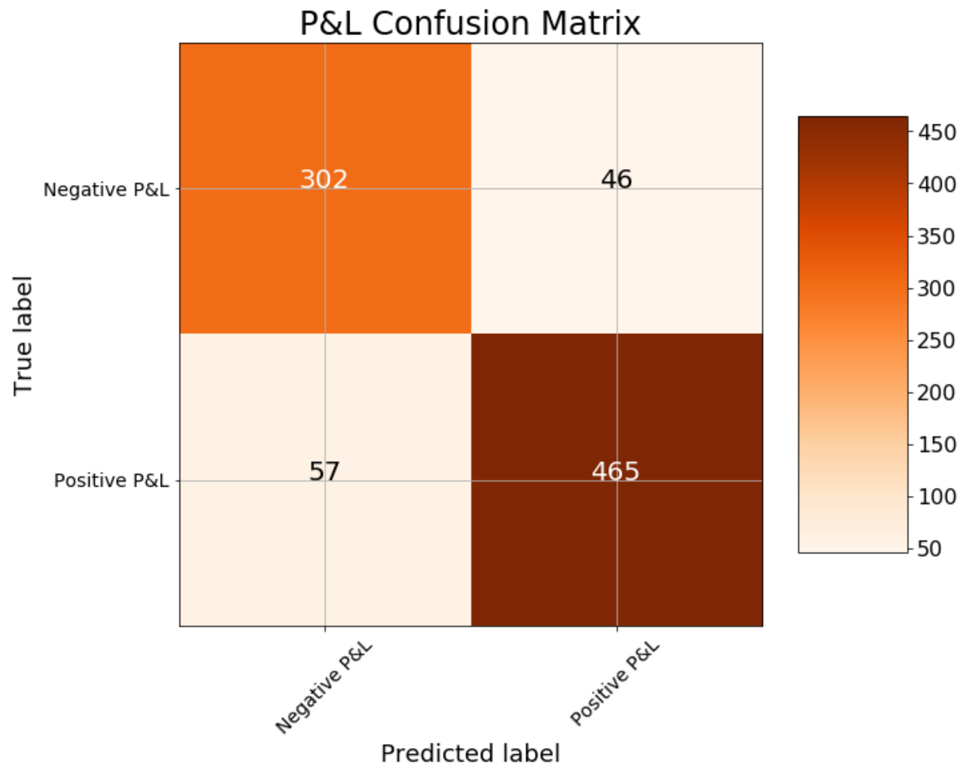
First we'll try with the data for the strategy for the periods (90, 90), if we don't restrict the depth of the tree we'll have 299 nodes with maximum depth of 15, as we could see before, it is expect this tree has overfit in the training data, meaning the performance of it is better in the training data than in the testing data. The score of the ROC AUC in the training data is 1.00, and in the testing data is 0.88 as it can be see in table 3.

Table 3

	Baseline	**Test**	**Train**
Recall	1	0.89	1
Presicion	0.6	0.92	1
ROC	0.5	0.88	1



Now we can see the overfitting in the training data with this results, finally let's see the confusion matrix, to complete the result analysis.



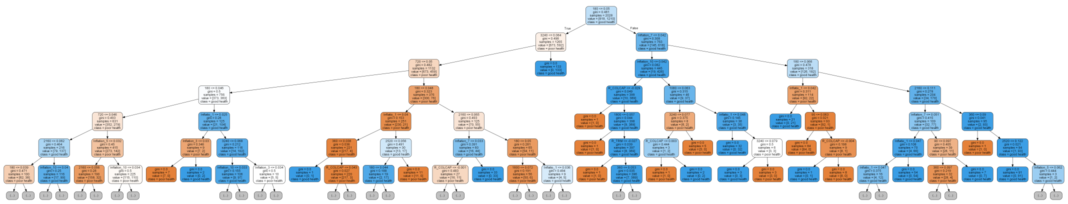
The number of errors in classifying samples that were positive when in fact they were negative is 46, on the other hand, the number of errors in classifying samples that were negatives when in fact they were positive is 57. Also, we can determine what are the most important futures for the tree, by determining the reduction in gini impurity over all the nodes in which the future is used.

Table 4

Feature	Importance
Node 180	0.289819
Node 3240	0.104953
1 year Inflation	0.093712
7 year Inflation	0.091384
Node 720	0.081656

The most important future of the tree is the interest rate of the node 180 in the zero coupon curve because it is the one that contributes the most in the reduction of the gini impurity.

By not restricting the depth of the tree you can show the amount of nodes it has, making evident the variance that finally makes our model has an overfitting in the training set.



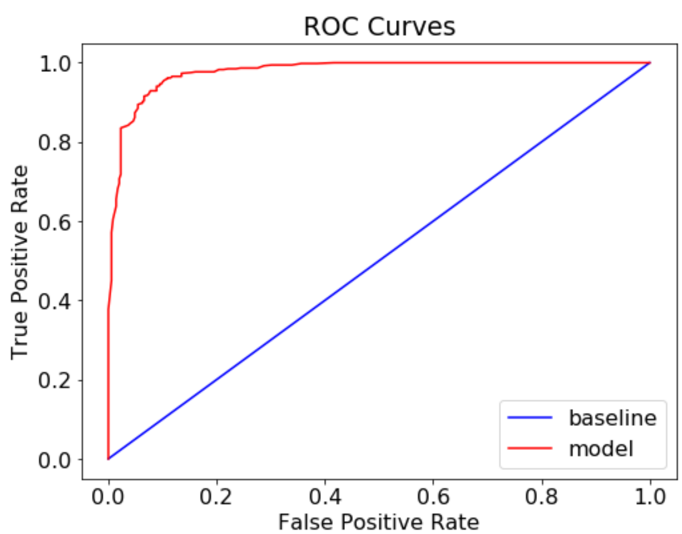
3.4 Random Forest

The random forest is a method that uses a specific number of individual trees, where each tree uses a random set of observations and only a subset of the futures are used for making a split.

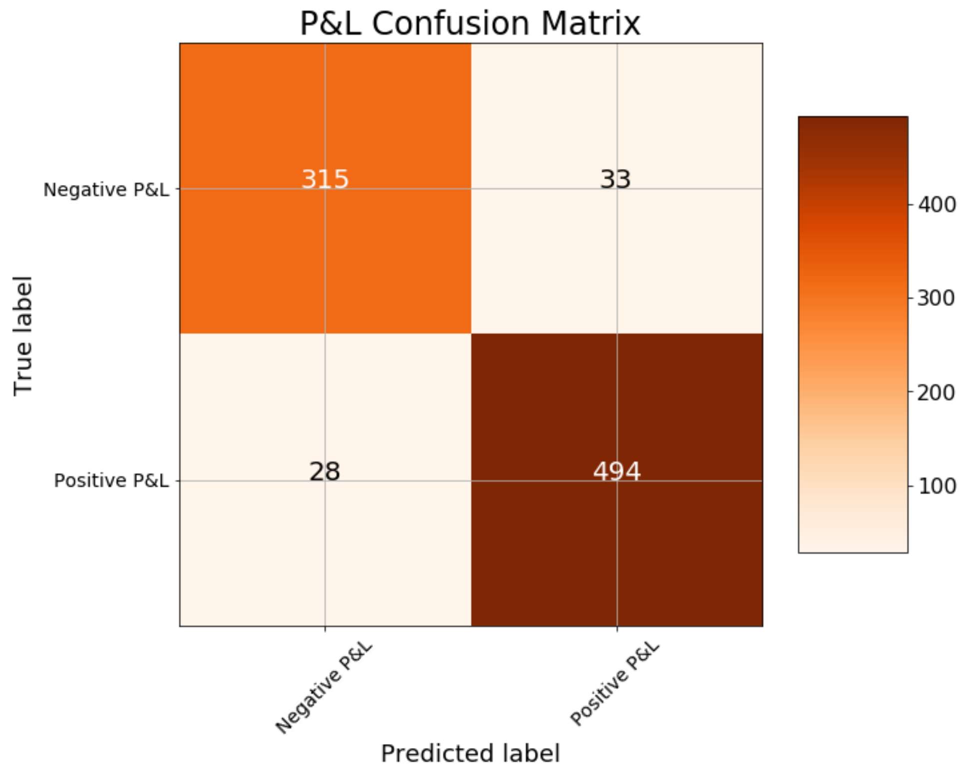
The random forest with 100 individual trees has an average number of nodes of 274, and an average maximum depth of 15.

Table 5

	Baseline	Test	Train
Recall	1	0.95	1
Presicion	0.6	0.94	1
ROC	0.5	0.98	1



The result of the random forest improves the results obtained before on the data test, without affecting the performance on the training set, now the precision, recall and ROC are 0.95, 0.94 and 0.98 respectively.



The improvement is evident too in the confusion matrix showing an error in classifying samples that were positive when in fact they were negative of 33 samples, and an error in classifying samples that were negatives when in fact they were positive in 28 samples.

The importance of the variables change in the random forest, the most important futures for the method are in table 6.

Table 6

Feature	Importance
Node 180	0.116424
Node 360	0.088857
Node 90	0.077711
Node 720	0.074946
Node 3240	0.066314
Node 2880	0.056533
Node 2160	0.052062
10 year Inflation	0.051956
Node 1800	0.051123
5 year Inflation	0.045169

For this method is very important the nodes of the zero-coupon curve, contrary to what happened when we used the method with a simple tree. In conclusion the random forest is capable to generalize to new data much better than the previous method.

3.5 Random Forest Optimization through Random Search

When we adjust the model, we have the option to select among many other parameters the following.

- **bootstrap:** Whether bootstrap samples are used when building trees. If False, the whole dataset is used to build each tree.
- **n estimators:** The number of trees in the forest.
- **min sample split:** The minimum number of samples required to split an internal node.
- **max leaf nodes:** limits the number of leaf nodes as long as there is no improvement of impurity.
- **min samples split:** The minimum number of samples required to split an internal node.

To maximize the performance of the random forest, we can perform a random search for better hyperparameters. This will randomly select combinations of hyperparameters, evaluate them using cross validation of training data, and return the best performing values.

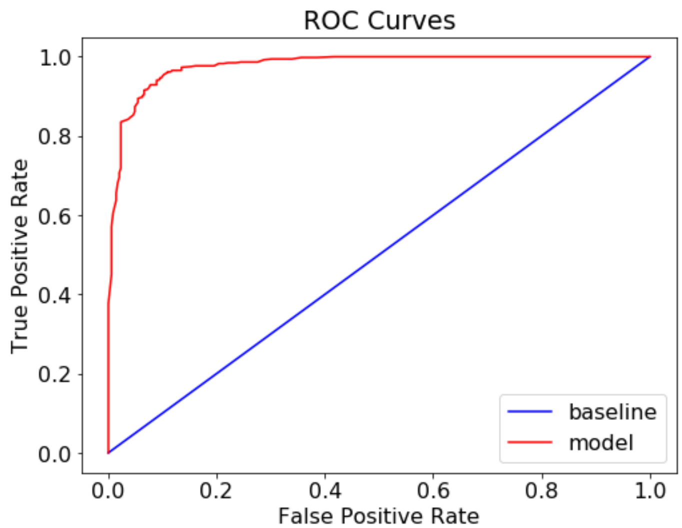
Table 7

	Best Parameters
bootstrap	16
n estimators	auto
min sample split	43
max leaf nodes	5
min sample splits	29

In table 7 it can be seen the result of the exercise to find the best hyperparameters, using this, now we have a random forest with average number of nodes of 274 and average maximum depth of 15. In table 8 are the results of the best model.

Table 8

	Baseline	**Test**	**Train**
Recall	1	0.95	0.98
Presicion	0.81	0.94	0.87
ROC	0.5	0.98	0.88



The model has the same performance in the test data that the previous model, it has the same number of nodes and depth but train data no longer has perfect results and the baseline is better than the previous.

3.6 Using Random Forest Optimization

Now that we understand the essence of the decision tree and random forest, we will use the best model found, so we must first optimize the parameters for each of the strategies, then train the random forest and finally collect the results for Recall, Precision and ROC to understand how good it is to classify the result of the strategy.

Table 9

	Baseline	**Test**	**Train**	**Baseline**	**Test**	**Train**
		90, 90			**180, 180**	
Recall	1	0.95	0.98	1	0.96	0.99
Presicion	0.81	0.94	0.87	0.72	0.72	0.99
ROC	0.5	0.98	0.88	0.5	0.96	1
		360, 360			**720, 360**	
Recall	1	1	1	1	0.99	1
Presicion	0.86	0.99	1	0.85	0.98	0.99
ROC	0.5	1	1	0.5	1	1
		1080, 360			**1440, 360**	
Recall	1	1	1	1	0.99	1
Presicion	0.93	0.99	1	0.98	1	1
ROC	0.5	1	1	0.5	1	1
		1800, 360			**2160, 360**	
Recall	1	0.94	1	1	0.96	1
Presicion	0.66	0.95	0.99	0.25	0.94	1
ROC	0.5	0.98	1	0.5	1	1

As it can be seen in the Table 9 the model has an excellent capacity to classify the negative or positive performance of the strategies in the test data, only three data are lower than 0.95 in this one, the precision in the strategy for the periods (90,90), (180,180) and (2160,360), where it highlights the data for the strategy (180, 180) of 0.72 equaling the result of the baseline.

So, in conclusion we can say that the result of the random forest for the classification between positive and negative of the profit and losses generated by the strategies is a very good. Now, it is necessary to add one more element to our analysis.

Table 10

	90 - 90	180 - 180	360 - 360	720 - 360	1080 - 360	1440 - 360	1800 - 360	2160 - 360	2520 - 360	2880 - 360	3240 - 360
<0	1,166	748	373	375	147	30	568	1,058	-	1	1
>0	1,732	2,085	2,329	2,066	2,033	1,888	1,090	340	1,137	874	613
N	2,898	2,833	2,702	2,441	2,180	1,918	1,658	1,398	1,137	875	614

As we can see, for the periods in which we do not adjust the classification model there are not or there are very few negative results for this reason the problem is no longer a classification problem the problem now is determine the value of the profit of the strategy, for this reason is necessary to apply a different method.

3.7 Regression Tree

Similar to the the previous model we are going to train the regression tree, this time to identify how good is our method, we are going to use the mean squared error as a metric of evaluation, this will give a gross idea of how wrong all predictions are (0 is perfect), so the closer it gets to zero the better our model will be.

In theory the regression tree divide the space using high dimensional rectangles, the approach is top-down, as it starts at the top of the tree (where all observations fall into a single region) and successively divides the prediction space into two new branches to finally determine what is the value of the variable. The goal is minimized the RSS equation.

The process described is too optimistic in terms of training data, i.e. it may overfit the data and may underperform on data test. The overfit model determines a high variance, but a smaller tree with fewer divisions could result in lower variance and a little higher bias. One strategy would be to split the nodes only if the decrease in the RSS of the split exceeds a certain threshold. But this strategy does not guarantee better results. Tree size is tuning parameter that governs the complexity of the model and the optimal tree size should be chosen from the data.

We are going to use the bagging method of random forest regressor and extra tree regressor to improve the performance of the method.

It is evident in table 11 that the performance of the method improve when is used bagging as it happens in the classification methods when we optimized to find the best hyperparameters. the performance in all nodes is very good, nearly to zero that will be the perfect result, so

Table 11

	Baseline	RF	ET		Baseline	RF	ET
90, 90	0.0101%	0.0004%	0.0003%	**1800, 360**	0.0038%	0.0003%	0.0003%
180, 180	0.0160%	0.0008%	0.0004%	**2160, 360**	0.0046%	0.0014%	0.0012%
360, 360	0.0154%	0.0009%	0.0005%	**2520, 360**	0.0057%	0.0009%	0.0007%
720, 360	0.0060%	0.0003%	0.0002%	**2880, 360**	0.0009%	0.0002%	0.0002%
1080, 360	0.0046%	0.0003%	0.0003%	**3240, 360**	0.0016%	0.0004%	0.0003%
1440, 360	0.0046%	0.0004%	0.0002%				

it could be say that the method are good to predict the benefit to the strategy in all the periods, highlighting that the best performance occur in (2880,360).

4 Conclusions

- It is demonstrated that for all the periods in which the strategy is carried out, which depends on the realization of the theory of liquidity, more than 50% of the observations show a positive result except for the period (2160, 360), so the result of the study shows that in most cases for all the nodes of the curve, the theory of liquidity is complied with during the period analyzed in the Colombian public debt market.

- In the periods (1080, 360), (1440, 360), (2520, 360) and (3240, 360) it is evident that the strategy is positive in more than 90% of the periods analyzed, so the strategy may have a higher probability of producing a positive result when executed in the middle and long part of the curve. In addition, for the period (2520, 360) the strategy always showed positive results, having the highest average and second highest performance value in all the sample.

- It is demonstrated that for the classification of the benefit of the strategy between positive and negative, the random forest method gives good metrics in the test data, and that these results are improved by optimizing the parameters of the model, after adding the analysis of the balance of the data, that is, how much data shows that the strategy is positive or negative, it can be concluded for the period (180, 180) the method fails in classified a negative performance of the strategy in the 28% of the samples, being the worst performance of all, follow by the classification of the performance of the strategy for the period (90, 90) failing in a 6% of the samples, it is therefore concluded that the method gives results with an accuracy above 94% for all the periods analyzed except for the period (180, 180).

- It is evident that the result of the ROC indicator is better 100% of the time when using the random forest optimized method in relation to the baseline when we are classifying the benefit of the strategies between positive and negative, for all the periods analyzed.

- It was not possible to use classification methods for the benefit obtained in the periods (2520, 360), (2880, 360) and (3240, 360) since these showed only one negative data or less for the whole sample, so it is important not only to classify the benefit but also to be able to predict it.

- Finally, it is evident that for all the periods in which the strategy is executed, it is possible to use any of the methods proposed for the prediction of its benefit, since they all give good results for the periods analyzed, according to what was expected, by using the ensemble methods, Random Forests (RF) and Extra Trees (ET) a better result is found for the MSE indicator. So we conclude that is possible with the limits of this study, to carry out the strategy and together with the monitoring of the variables used to predict the profit, produced a positive rate of benefits in the majority of the times.

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