A NAIRU ASSESSMENT: AN APPLIED CASE TO COLOMBIA'S LABOR MARKETS

Nicolás Molina Velásquez

20-May-2015

"Bendito el hombre que confía en Dios, y pone su confianza en Él."

Given the discrepancy over the optimum levels of employment for Colombia, this research targets both, the national and urban, Non-Accelerating Inflation Rate of Unemployment (NAIRU) for the Colombian markets. In doing so, there is a strong pertinence in estimating the constant NAIRU through raw and minimally altered data and providing the reader with a complete brief of the theory in which the model is founded. The introduction of supply shocks is considered to attain improved estimations and a more reliable assessment of the NAIRU to those that have previously been attempted. The backbone of the analysis is conducted through the relationship established by the Phillips curve from 2001 until 2015.

Contents

Briefing on the NAIRU

In 1968 Milton Friedman refines the perception of full-employment and the optimum rate of unemployment to meet macroeconomic policy targets through the maneuvering of prices. Convergence of actual unemployment levels towards the Non-Accelerating Inflation Rate of Unemployment (NAIRU) in time accounts for the inverse relationship of inflation with unemployment, (Phillips, 1958) and it also postulates an opportunity to determine the impact of the structural qualities within the labor market under analysis; such as wage flexibility, labor mobility, employment retention, education levels, labor market segmentation, and the effectiveness of labor policy decisions, among other vital tenacities. According to Friedman's (1968) definition of the NAIRU, it is the minimum benchmark level of unemployment at which prices remain unaffected in the economy, after accounting for any shocks. The NAIRU is [$8\% \leq NAIRU_{Colombia} \geq 15\%$] believed to be dynamic in nature but it is inclined to hover near 6% for developed economies. (Gordon, 1997) Moreover, if an economy's unemployment rate were to dip below the NAIRU, it would cause a rise in price indexes for as long as the economy persisted to "over-employ" its labor force. (Laubach, 2001) For lack of a better measure the NAIRU is considered synonymous to the optimum level of employment in a given economy.

The relationship between inflation and unemployment has been host to debate arguing their causality from a wide array of plinths. Robert Solow and Paul Samuelson were the first economists to publically illustrate the mutual relationship between both socioeconomic indicators represented as the "Phillips Curve." (Gordon, 1997) Furthermore, the convergence principle of economic variables is based on Robert Solow´s growth model, which demonstrates that economies should approach their steady states at a decreasing pace given the law of diminishing marginal returns. (Solow, 1957) Given the latter fundamental advancements in growth theory, Milton Friedman (1968) argued that unemployment should also converge to a steady state that is presumed to be the NAIRU, whose effect should be captured through the operation of the Phillips curve relationship. Since then, a wide array of econometric models that attempt to target the evasive NAIRU of differing economies through the use of reference variables ranging from transitional measures of inflation, wage segmentations, and different unemployment measures, among other variables. However, few have asserted with absolute certainty or through corroborated results that, in fact, economies are approaching their potential levels regardless of the time period or the method of analysis utilized.

Since Friedman (1968) published his results on the NAIRU, economists struggle to establish concrete data that genuinely supports the inert relationship between prices and unemployment and that in fact the NAIRU is the steady state of unemployment. The strongest opposition to the Phillips Curve relationship is known as Lucas' critique based on David Humes writing that monetary injections contradict the existence of a contingent relationship between prices and unemployment. He criticizes the reliability of forecasts implemented to determine current econometric policy evaluations and its future bearing on the economy based solely on past economic data. Furthermore, Lucas points out that agents behave rationally by systematically varying their expectations with respect to changes in policy since estimated coefficients vary in response to new policies.

However, the question still remains; is Colombia's unemployment path heading towards its NAIRU? There are few studies that target Colombia's NAIRU levels, but none of the studies available have measured the individual urban NAIRU levels. Analyzing Colombia's labor market is a difficult task with the quality of fuzzy data coupled with drastic structural variations in the labor structure as the ongoing development of government is still underway and mingled with the high levels of labor informality that are present in Colombia. (DANE, 2015)

The issue at hand lies within the actual precision in the measurement of the NAIRU assessed from the data published by the National Administrative Department of Statistics (DANE), or *Departamento Administrativo Nacional de Estadistica* in Spanish. The objective of this study is to obtain reliable insight into the relationship of inflation with the current national and average urban unemployment rates. The estimation of the constant NAIRU for the national and urban markets yields 11.79% and 13.62%, respectively. This research is divided into; the briefing on the NAIRU and its development; a review of previous studies that trace the unemployment rate across the Colombian economy; the specification of a constant NAIRU through ordinary least squares regression (OLS); a brief on the data considered; an illustration of yielded estimates and casual relationships, and lastly; a retrospective analysis of the process that is implemented and follow-up research recommendations.

Utilizing monthly reported unemployment levels and inflation rates for each available city from 2001 until 2015 yields robust measurements since it is the largest available frequency in Colombia. As seen in Staiger *et al* (1997) and Gordon (1982), a higher frequency of data improves recent inconsequential measurements of the Colombian NAIRU. Without the need to construct complicated "spline" polynomials[1] or apply filters to detrend the data, the estimation of the constant NAIRU takes place through OLS regressions relating inflation, unemployment, possible meaningful supply shocks, and their combined effect in time as outlined by Laubach (2001). It is imperative that the relationship between the reference variables is not regarded in a contemporaneous timeframe to avoid any simultaneity bias, but rather reference the permeating effects that continue to affect each other form past periods of time, or lags.

The analysis commences in 2001 after any effects from the inflation control measures that took place with the renewed emphasis on inflationary jurisdiction in 1991 and the country's shift in its macroeconomic policy (post-adoption of the Taylor rule

1 "Specifically, a cubic spline with two knot points is used. Between the knot points, the spline is a third degree polynomial. These polynomials are constrained to be equal, and to have equal first and second derivatives, at the knot points. The knot points used are equally spaced values along the time axis." (Gordon, 1997)

period that took place) in 1999 have both subsided. For the purpose of isolating the effect that unemployment has on inflation levels, lagged or differentiated supply shock variables are considered contingent on their level of significance throughout the research, such as, the exchange rate and reference commodity prices. The introduction of plausible supply shocks into the model means that the NAIRU is no longer considered a random walk with drift that is determined by unexpected changes in prices; but it is rather a rate that is determined by a stationary process. (Laubach, 2001)

Previous NAIRU Assessments

The majority of the studies that estimate a constant or a time-varying NAIRU, they do so with the implementation of the Phillips curve relationship and conclude the corresponding deviation of the "unemployment gap," as the difference between the rate of unemployment and the optimum level of employment. (Laubach, 2001) Optimum, or steady state of, unemployment is regarded as an undetermined parameter; hence, the NAIRU is traditionally thought of as the "next best measure" to take the place for the optimum employment benchmark.

There is convincing evidence that the NAIRU may vary over a period of time due to ample structural changes driven by increases in international trade and technological advancements. (Ball and Mankiw, 2002) Improvements in labor matching decrease unemployment turnover undertaken by temporary employment agencies to locate general and specific redundancies that are known for their lasting impact on the reduction of frictional unemployment in labor force. (Krueger, 1999; Staiger *et al*, 2001) Researchers detect a decrease in the estimation of the NAIRU with the introduction computers and technological advancements in processing methods, yielding an ocean of data that is visceral for the socioeconomic measurement of development. Braun (1984) and Grubb, Jackman and Layard (1982) propose that drastic increases in productivity are directly proportional to real wage increases that translate into decreases in the NAIRU.

Blanchard and Summers, (1986) and Layard *et al* (1991) introduce evidence that unemployment, and therefore the NAIRU, garnishes permanent momentum that precludes the variable from returning to its original state; this push is known as "hysteresis." The momentum permeates into shifts in aggregate demand that cause a permanent deviation of unemployment from the NAIRU. The NAIRU would in turn expose itself to a perpetual deviation from its previous state. Hysteresis occurs when job seekers become accustomed to a jobless life, hence, decreasing their probability of employment in the future. Layard *et al* (1999) find that the NAIRU permanently increases immediately after a significant disinflationary recessive stage where unemployment levels typically spike. However, the latter pretext of hysteresis conforms a conflicting debate among the real causes of a time-varying NAIRU.

Hodrick-Prescott (1997) engineers a filter that minimizes the sum of the squared deviations between trending and actual values of the time series while adding a penalty for any sudden changes in curvature resulting in selective smoothening of any formation of a trend. (Ball and Mankiw, 2002) In other words, it generalizes the linear time trend and allows for its slope to vary over a given period of time. The application of this filter to the calculation of the NAIRU serves to smoothen any sudden jumps due to breaks in the data, giving the regression a "looser" fit. Some economists prefer conceptualizing the time-varying NAIRU as a heavier curve that gradually moves in time as it is weighted down by lagged effects of unemployment.

Layard and Nickel (1986) acquire one of the first estimations of Britain's NAIRU based on trade and employment balance, wages, and prices. In which they base on a four-equation model on imperfect labor and goods markets. In their publication, they explain the shifts of the labor market from a theoretically perfectly competitive market towards a semi-monopolistic labor market, noting the growing presence of labor unions in the U.K. The authors note that collective bargaining within the labor force causes significant pressure that drives prices higher every time those employers are forced to increase wages when they face with an ultimatum from the labor syndicates. Hence, they provide an explanation for a relatively high level for Britain's NAIRU when comparing it to other developed nations.

Joyce and Wen-Lewis (1991) determine the effect that real exchange rates have on the estimation of the NAIRU through NIDEM modeling, and they identify evidence of a converging unemployment rate about the resulting NAIRU. They conclude that a positive demand shock prolongs the effect approximately five years in which the output capacity and unemployment levels remain superior to their steady states. They cite that the cause of the retreating effect of a demand shock sprouts a push in nominal inertia during the wage and price setting phases. The real exchange rate diminishes the impact of direct and indirect employment levies, which have significant influence on the NAIRU.

Gordon (1997) refines previous estimations with the empirical specifications of a "triangle model" that reduces inflation into three segments[2]. The inclusion of lagged changes of inflation into the model captures the dynamic inertia that could originate from price expectations or salary contract increments. Excess demands and supplies are regarded as positive gaps in employment and output gaps respectively.[3] He omits the inclusion of any explicit wages into the model due to their implicit presence in price levels; contractual wage fixation is not permanent. The triangle model predicts inflation measures while simultaneously omitting the presence of the money supply that is inherent in ordinary inflation rates. Gordon reports that changes in the NAIRU are due to "labor militancy, strong labor syndicates, high minimum wages, [and when there is a] marked increase in labor's share of national income." (Gordon, 1997) He concludes that other factors that might influence the NAIRU are, labor migration of unskilled labor, increases in global competition, a weaker demand, and increases of supplied output.

---

2 1) built-in inflation or "inertia"; 2) demand pull inflation; and 3) cost-push inflation.

3 Positive gaps in employment and output is justified by King and Watson (1994).

Gomez, Rebollo, and Usabiaga (2002) implement several direct methods of estimation to obtain an estimate of the NAIRU for the Spanish economy. They tend to emphasize their analysis in identifying any traces of employment through methods based on the Phillips relation. The authors divide their analysis into three separate approaches to estimate, (1) the direct estimation of the NAIRU by methods proposed by Bellod (1999) and Estrada et al (2000), (2) rigorous checks to verify the precision and robustness of the estimated results are preformed to determine their cause and dependability, and finally (3) the hysteresis phenomenon of unemployment; or garnished inertia. Their estimation scope is larger than that of this study[4] but it is important to note that their findings include evidence confirming the existence of hysteresis in the Spanish labor market. Indicating that there is a causal link between unemployment deviations and diverse estimations of the NAIRU through direct methods of estimation. They admit that the expanded implementation of differing methods for estimating the NAIRU proved to convolute their conclusions and hindered their selection of a significant measure.

Similar to other states, there is a considerable amount of discrepancy regarding the estimation of the Colombian NAIRU. Estimated NAIRU levels wildly range anywhere from 8% to 15% depending on the time period and reference variables chosen. Since there is a wide spectrum of possible filters and methods to dissect the path of unemployment with respect to its NAIRU, the methods included in this research attempt to represent precise measurements by assessing the constant national and urban NAIRU levels with organic and minimally altered data. Some authors cite the cause of variations in the assessment of the Colombian NAIRU due to its inconsistent labor market. Structural changes in the labor market such as, increases education levels, new wage legislation that regiment wage augmentation proportionate to the CPI increases; laws 789 and 779 established in 2002 and 2003 respectively, and labor informality levels also transform the representative NAIRU benchmark. (Arango et al, 2011) The following studies, illustrated in *Table 1*, present some of the most recent estimations that appraise the Colombian NAIRU:

4 Gomez et al (2002) estimate the NAIRU as, (1) a structurally constant relationship through the Phillips relation results of an approximate 15% and 3.3% when a supply shock is introduced, (2) a structural constant in specific sub-periods within their timeframe and, finally, (3) as a time-varying tendency through the use of the Hodrick-Prescott series filter.

Table 1: Previous NAIRU Assessments for Colombia's National Market

Investigators	Estimated NAIRU	Verified by Mayorga	Timeframe
Clavijo (1994)	8.2%	N/A	1975 – 1989
Farne Vivas and Yepes (1995)	6.2% - 7.2%	N/A	1974 – 1994
Cardenas and Gutierrez (1997)	8%	8.5%-9.0%	1993 – 1996
Henao and Rojas (1998)	10.50%	10%-11%	1990 – 1998
Nuñes and Bernal (1998)	10.8%	10%-11%	1991 – 1998
Gomez and Julio (2000)	8.5% – 10.5%	8.4% - 11.8%	1990 – 2000
Julio (2000)	13%	12.20%	1999 – 2000
Julio (2000)	11%	8% - 15%	1988 – 2000
Julio (2001)	10.7% (7.3 – 12.4%)	N/A	1978 – 2000
Arango and Posada (2006)	13.09%	N/A	1990-2005
Fuentes (2007)	11.19%	N/A	1990-2004
Arango *et al* (2011)	10.8%*	N/A	1991 – 2010

Values in parenthesis are confidence intervals; N/A means not applicable; * values indicate urban assessments

Source: Generated by Author

Clavijo's (1994) estimation of the Colombian NAIRU from 1974-1997, utilizes productivity of labor based on the Phillips Curve relationship with adaptive expectations as seen in Farné *et al* (1995). Clavijo includes efficiency ratings and the downward tendency of unemployment into his model as representative variables of the price difference. He cites some inherent issues with serial autocorrelation and regards the NAIRU as an undetermined parameter. However, his variables of reference are not representative with those that commonly undergo such analysis, as there is an innate biased between the aggregate unemployment rate and the industrial wages he uses to establish his conclusions.

Cardenas and Gutierrez (1998) apply a model that is founded on Layard *et al*'s (1991) work to estimate the determinants of optimum unemployment levels with the additions of demand shocks to isolate any redundant pricing effect. They find significant causality in the first difference in inflation by the USD to COP exchange rate for

Colombia. They conclude that an overvaluation of the exchange rate is caused when the unemployment levels rise above the NAIRU.

Henao and Rojas (1998) implement three different methods also applied by Posada and Gonzales (1997) to estimate the NAIRU; 1) a Hodrick and Prescott filtering; 2) a two stage least squares regression; and 3) through an augmented Phillips Curve. In their findings they cite an inherently conflicting and correlating relationship between unemployment and price levels. They took advantage of data describing the valuating salaries to further the Phillips relationship. They point out that an ARIMA model could have stochastic tendencies of unit roots within the unemployment rate. They recommend that active labor market policies such as, training programs, government incentives, and flexible unemployment programs, can prevent the formation of frictional and cyclical unemployment in the Colombian labor markets.

Lopez and Misas (1999), implement the Phillips curve estimations to demonstrate significant causal evidence linking the GDP gap[5] to the variation of Colombian inflation. They apply a Hodrick-Prescott filter and formulate their estimations basing their model on a VAR structure and a space-state representation estimated through a Kalman Filter. They encapsulate seasonal effects in their estimations by transferring the autocorrelating effect present in the residuals and placing it into dummy variables. They complete the estimation of the NAIRU by citing that the un-normal residuals remain unexplained by the difficulties of measurement until adjustments for atypical seasonal components, such as prices of imports, are introduced into their model.

Gomez and Julio (2000) present a non-linear revised estimation of the Phillips Curve, for the purpose of calculating the NAIRU. They also apply a Kalman filter. Following up the pair's research, Julio (2001) revised their estimates and applied a confidence interval through the use of spline polynomials. Splines allowed Julio to trace unemployment under a more flexible criterion and constrain the model from any sudden structural breaks providing a better measure for the dynamic NAIRU. Splines are also applied as a constraint method by Gordon (1982), Weiner (1993, 1994), and Eisner (1995a), among others.

In an attempt to expose the non-linear relationship of the Phillips Curve, Nigrinis (2003), utilizes supply shocks and the installed capacity gap to draw a relationship of price levels with the exclusion of food prices. He utilizes the same SVAR methods with an added Kalman Filter of estimation that Lopez and Misas (1999) implemented for their regression.

Cancelado (2011) conducts his research in the conventional form by estimating the Phillips curve, a constant NAIRU estimation, and a regression with a Hodrick Prescott filter. In addition, he estimates Elmeskov NAIRU that regards the relationship between the inflation rate and wages over time. The Elmeskov NAIRU operates under the assumption that the changes in wages are directly proportional to changes in the

5 the difference between expected production output and actual output.

unemployment gap. The data is arranged in a quarterly frequency and his estimates for all of the regressions are insignificant at the 90% level for three (3) differing periods of time ranging from 1984 until 2001. He notes significant effects of monetary policy and structural changes in the labor market through governmental reforms as part of his estimations.

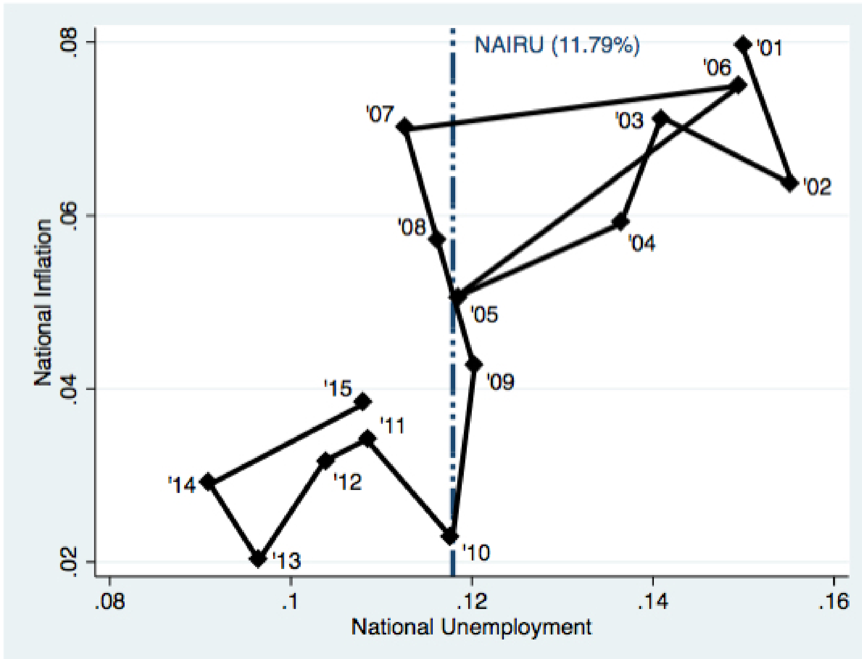
Mayorga and Escalante (2011) update previous estimates of the NAIRU, utilizing Greenslade et al. (2003) and Benes and D'Niaye's (2004) methods to discover the NAIRU's dynamic tendency to increase over time. They utilize Kalman's filter, founded on the Phillips theory, to include plausible supply shocks and expected inflation rates that might arise during OLS regression. They obtain significant evidence that unemployment congregates about the NAIRU's pathway in time. With two observed breaks, the first one being in 1992 when the unemployment levels were less than the estimated NAIRU and, the second one in 1994 when unemployment levels rose above the NAIRU.

Arango, Posada, and García (2011) estimate the NAIRU with the aggregate values of the thirteen mayor Colombian cities from 1991 until 2010, as well as for the seven principal metropolitan areas of Colombia with interesting contributions. They utilize married men's unemployment measure, since married men of 30 years or older serve as a more precise measure of unemployment given their "strong attachment to the labor force" as it is carried out by Staiger (2007). They narrow the scope of this market by analyzing if the job position and education level of these men affects their deterrence for unemployment. They found that married men, who are compromised to their current employment position, have approximately -1% NAIRU, which translates into a lower tolerance for unemployment from those who are single and not as attached to their current position. A brief comparison between expected and actual inflation rates results in a differentiated optimum rate, higher than the NAIRU estimated. They denominate this new measure as the Equal Inflation to the Expected Inflation Compatible with the Unemployment Rate (TADECIM) of 11.8%.

Specification for a Constant NAIRU Estimation Model

The two following graphs, *Graphs 1 and 2*, illustrate the Phillips curve relationship within the time frame for which the NAIRU of each market that is assessed. The vertical axis represents the inflation rates while the horizontal axis represents the unemployment rates. The time period commences with high inflation and unemployment rates and, through time, both variables stagger across the graph and eventually reach much lower measures for both rates. Both markets have similar pathways in time but the average urban markets feature higher rates than those of the national market.

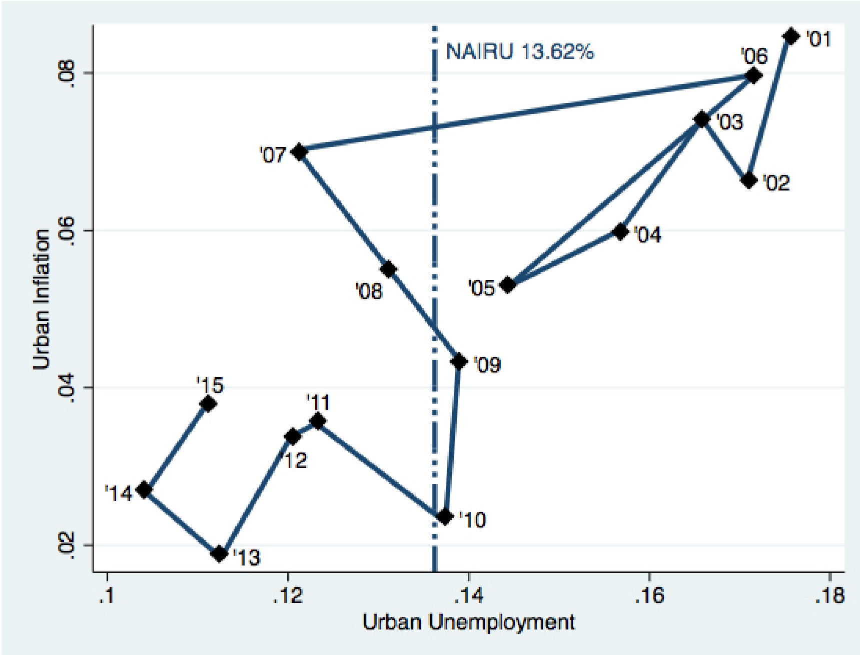
Graph 1: Phillips Curve Relationship in Time,
Yearly Data for Colombia's National Market (2001-2015)



Source: Generated by Author

Graph 1, illustrates the National Phillips relationship where there is an erratic decrease in inflation and unemployment at the beginning of the period, from 2001 until 2005, only to increase back to similar levels by 2006 for both, inflation and unemployment rates observed at the commencement of the period (2001). From 2005 until 2006 there is a sharp increase of both measures. Prior to the American credit crisis, from 2006 until 2008, there is a gradual decrease in unemployment coupled with a sharp decrease in prices from 2006 to 2007. Post-2008 inflation leads by dropping significantly until 2010, then unemployment follows by decreasing from 2010 until 2014. From 2013 until 2015 inflation increases and from 2014 until 2015 both rates increase.

Graph 2: Phillips Curve Relationship in Time,
Data for Colombia's Average Urban Markets (2001-2015)



Source: Generated by Author

Both of the previous relationships show similar tendencies but differ in their reaction to economic events and their actual values. From 2005 until 2006 there is a sharp increase of both measures. Prior to the American credit crisis of 2008, the urban Phillips curve rapidly declines until it drops well below the estimated NAIRU, mainly due to a decrease in unemployment, until the 2008 crisis where unemployment begins to rise but inflation continues to decrease. From 2009 until 2010 urban unemployment remains almost the same albeit a steep drop in inflation. The latter decrease in inflation seems to have a lagged effect on unemployment as it decreases from 2010 until 2014, where it increases slightly until present time. From 2013 until 2015 there is an increase in inflation to similar levels of 2009.

In hopes of attaining statistically significant estimations, the assessment of a constant NAIRU is conducted as seen in Douglas Staiger *et al* (1997), Gordon (1982), the Congressional Budget Office (1994), Eisner (1995a), Tootell (1994), Weiner (1993, 1994), and Fuhrer (1995). The principles exposed in the previously mentioned investigative works, are founded on the same Phillips curve principle but it interprets the optimum level of employment as an unknown parameter. The scope of this research lies in obtaining a robust measure with minimally altered data that could possibly be utilized for policymakers or forecasting purposes between both Colombian markets.

Through the model implemented, and as seen in *Graphs 1 and 2*, it is clear when the average of the major Colombian cities are approaching or when they have surpassed their constant full employment rates, or long-term unemployment levels. When the model is adjusted for seasonal components through the SARIMA method, the regression yields a higher degree of robust and significant estimates for Colombia's NAIRU. This method provides valuable insight into the current labor market structure and the impact that variations in inflation have on the economy given the stature of the unemployment level with respect to the NAIRU. It is possible to indicate through OLS regression if there is a causal relationship of hysteresis, or inertia, in the monthly unemployment levels, but it falls outside the scope of this research.

The model is based on the relationship in the form of difference in inflation, $\Delta\pi_{i,t}$, as the dependent variable for market i. The latter reference variable serves the purpose of encapsulating the difference between the actual inflation value and that of the expected inflation (represented as π_{t-1}). The basic relationship is based on the principle that the inflation gap, $\Delta\pi_{i,t}$ is explained by the unemployment gap $(u_{i,t-1} - \bar{u})$; which is the difference of actual unemployment and the optimum level of employment ($\bar{u}$). The period of analysis pertains to the past 14 years, from 2001 to 2015, for reasons previously mentioned regarding macroeconomic policy changes that have taken place in Colombia. The representation relating the unemployment gap with the first difference of inflation is demonstrated in the following equation:

$$\Delta\pi_{i,t} = ß_1\left(u_{i,t-1} - \bar{u}\right) + ß_2\left(u_{i,t-2} - \bar{u}\right) + \omega_t \tag{1}$$

Where $\Delta\pi_{i,t} = \pi_{i,t} - \pi_{i,t-1}$ is the first difference of inflation for each market *i* in time *t*; $u_{i,t}$ is the unemployment rate, $\bar{u}$ represents optimum level of employment that, according to Staiger *et al* (1997b) and others, it is entered as an unknown parameter. The model is extended for as many significant lags as it is necessary for either of the variables. After separating out the previous collecting terms involving the NAIRU the equation is rewritten, as seen in Staiger *et al* (1997b), King *et al* (1995), Gordon (1997), and Laubach (2001) in the following form:

$$\Delta\pi_{i,t} = \alpha + ß_1 u_{i,t-1} + ß_2 u_{i,t-2} + \omega_t \tag{2}$$

Where ω_t is assumed i.i.d. normal N(0,σ_u^2). The hypothesis of a unit root present in the inflation data for the examined markets is not rejected at the 99% significance level, given by augmented Dickey-Fuller or Phillips-Perron tests[6]. Considering a controlled variable to account for the difference of inflation lags; it is represented in the following equation by λX:

$$\Delta\pi_{i,t} = \alpha + ß_1 u_{i,t-1} + ß_2 u_{i,t-2} + \lambda X + \omega_t \tag{3}$$

6 as seen in *table A4 and A5* in the appendix.

$$\lambda X = \lambda_1 \Delta\pi_{i,t-1} \quad (4)$$

The difference in autoregressive lags is contained by the vector X. There is a seasonal correction about the first and twelve month of each series due to the surge in the aggregate demand during Colombian holidays and a general increase in the aggregate yearly inflation rate that takes place at the beginning of each year.

Take Colombia's national market for example, these effects represented in the forecasting of $\Delta\pi_{i,t}$~SARIMA (1,0,1,12) are identified in the seasonal component coupled with a presence of ARMA (1,1). For Colombia's national market there is a presence of an AR(3) in the regular portion of the $\Delta\pi_{i,t}$. These corrections are included in the model in the following way:

$$\Delta\pi_{i,t} = \alpha + ß_p u_{i,t-1} + ß_p u_{i,t-2} + \lambda X + \omega_t - \Phi_j \Delta\pi_{i,t-s} - \theta_k \omega_{t-s} \quad (5)$$

Where ω_t represents the stochastic perturbation (assumed to be a white noise); $\Phi_j \Delta\pi_{i,t-s}$ is the seasonal autoregressive component (AR); and $\theta_1 \omega_{t-s}$ is the seasonal moving average component (MA). Note that *s* is equivalent to twelve (12) since data values under analysis are arranged in a monthly frequency. The model implements single equation methods that reflect a strong caution to avoid any simultaneity bias. In order to dodge any issues of simultaneity the variables loaded on the right side of the equation are not entered as contemporaneous values but rather as lagged variables. However, some researchers such as Staiger *et al* (1997b) and Gordon (1997) enter some variables, like supply shocks or unemployment, in their contemporaneous state.

To extract any distortive spikes caused by supply factors and obtain an accurate measure of the optimum level of unemployment regarded as the NAIRU, it is essential that the lagged or second difference of the price of the supply shock are introduced in the following form;

$$\Delta\pi_{i,t} = \alpha + ß_1 u_{i,t-1} + ß_2 u_{i,t-2} + \lambda X + \rho Z - \Phi_j \Delta\pi_{i,t-s} - \theta_k \omega_{t-s} + \omega_t \quad (6)$$

where Z_t represents the vector of variables capturing the effect of a given supply shock. In this research we utilize the uncontemporaneous nominal prices of crude oil, coffee, and the COP to USD exchange rate as brute supply shock data.

The NAIRU is considered an unobserved and stochastic process, which is derived from the ratio of the constant term and the resulting coefficients of unemployment in the following way:

$$\alpha = -(ß_1 + ß_2)\bar{u} \quad (7)$$

The NAIRU, U^N, is interpreted as a synonymous measure of the optimum level of unemployment. Under this assumption, $\bar{u}_i = \mathrm{U}_i^{\mathrm{N}}$, we obtain the following estimation method for the NAIRU_i.

$$\mathrm{U}_i^{\mathrm{N}} = -\frac{\alpha}{(ß_1 + ß_2)} \tag{8}$$

Where U^{N} is the NAIRU and it is assumed that it follows a random walk without drift prior to the introduction of unexpected changes in prices:

$$\mathrm{U}_{i,t}^{\mathrm{N}} = \mathrm{U}_{i,t-1}^{\mathrm{N}} + v_t \tag{9}$$

Where v_t is assumed i.i.d. normal $\mathrm{N}(0,\sigma_u^2)$ and uncorrelated with ω_t.

According to Laubach (2001) three lags of the difference in inflation is sufficient precedent to remove any significant autocorrelation when it is regressed onto itself; however, this research covers the optimum lags for each economy even if they differ in temporal significance. In order to determine the adequate number of optimum lags, the Box-Pierce test is implemented. Both, Colombia's aggregate market and the average of the twelve cities, require two significant lags for the unemployment rate and vary in the number of lags for autoregressive inflation. Three (3) autoregressive lags are included for the national market and two (2) for the urban market.

Raw Data

Even though the previous NAIRU assessments include different variations in the measurement of the countrywide NAIRU, this research focuses primarily on the comparison of the NAIRU between Colombia's urban and national markets. The following twelve (12) cities compose the basis of our analysis: Bogotá, Barranquilla, Bucaramanga, Cali, Cartagena, Cucuta, Manizales, Medellín, Monteria, Pasto, Pereira, Villavicencio, and the national values for Colombia as a whole. All of the indicators were applied in a monthly frequency to increase the likelihood of obtaining robust estimations. The unemployment surveys were last revised on March of 2015.

A brief analysis of the data[7] reveals that while the inflation data is stationary in nature, the unemployment data for Colombia and the majority of its cities have a downward tendency, which resulted insignificant during the estimation phase. In general terms, inflation values seem to behave with similar tendencies across the individual urban centers with the slight exception that unemployment for Bogotá and Medellín seems to be less erratic for certain segments in time, which differs from Cali[8].

The unemployment data is obtained from a governmental release detailing large-scale surveys conducted by the National Administrative Department of Statistics

7 See appendix *Table A1*

8 as seen in *Graphs A1-A3* in the appendix.

(DANE), or *Departamento Administrativo Nacional de Estadistica* in Spanish. The purpose of the survey is to publish nationwide urban labor market statistics. The data is presented in a versatile and decomposed modus. Within the survey data obtained, there is wide variety of descriptive statistics in the report, from occupational data, gender employment distribution, to education levels within the labor market. The unemployment values reported as a three-month moving average to adjust for seasonality, or any inertia, that might be present in the data as it is demonstrated in Gordon (1997) and Laubach (2001). Mankiw (2002) cited Ball and Mofitt's (2001) advancements through the use of data and its moving averages in the calculation of the unemployment gap. The use of moving-averages provides forth a smoothed out relationship somewhat similar to that obtained by the application of a Hodrick-Prescott filter to extract a trend.

The consumer pricing source data, inflation, for each city is obtained form Colombia's National Bank (B.R.), or *Banco de la Republica de Colombia* in Spanish. The monthly increments in pricing data are in percentage points for each major Colombian city from 1979 until present time. The data is constrained from January 2001 until January of 2015. The bank publishes pricing data for additional Colombian cities after January of 2009, which do not meet the temporal constraints established in the scope of this work. Thus, the exclusion of cities outside the defined scope results in the selection to investigate the twelve (12) previously mentioned cities.

The supply shocks that are considered in this work include monthly average values for oil and monthly USD to COP exchange rate prices obtained from Bloomberg market exchange data from January of 2001 until January of 2015. There are close to two million people employed in the Colombian coffee industry, hence we considered the trade market price impact in our model as a supply shock. The coffee price data implemented in the model is obtained by the Colombian Federation of Coffee Growers (FNC); or *La Federación Nacional de Cafeteros de Colombia* in Spanish.

Preliminary Analysis of the Data

Staiger (1997) notes that empirical examinations of inflation and unemployment, for any given economy, lack precision in the measurement of the raw data and the ample configuration of the variables utilized. The latter implies that analysts have ample flexibility for their choice of variables implemented to analyze the Phillips curve relationship. As previously mentioned, some researchers' dependent reference variables range from the GDP deflator index to a varied array of exclusions in the price index implemented. For this case, there is not an abundance of raw data available to determine the measurement of the urban NAIRU. Thus, the main findings for Colombia's national market and its average urban market are derived from the second difference in the monthly consumer price index and the monthly moving average of the unemployment rates.

Surges in consumer behavior due to holidays or cyclical components have a direct impact on prices as increasing costs[9] translate into higher prices year by year. Seasonal wage adjustments also have a direct impact in prices, which in turn translate into a seasonal component in the data analyzed. Seasonal effects are encapsulated, in the national and urban models through a SARIMA (1,0,1,12) annual cycle (monthly).

Since the timeframe analyzed covers the American financial crisis of 2008 that quivered markets at a global scale, its effects must be accounted for in the model. Hence, the introduction of a dummy variable, represented in the models as *crisis*, to capture the prolonged effect[10] of the crisis and therefore obtain econometrically sound estimates for which to base the calculation of the NAIRU.

On October of 2012, the Colombian Minister of Interior and Public Credits or *Ministerio de Hacienda y Credito Publico* in Spanish, Mauricio Cardenas, enacts a massive tax reform, represented in the models as *tax*, known as *La Reforma Tributaria 2012* in Spanish; to promote employment; increase tax discounts for poultry, livestock, fish, corn, and milk producers; and the overall reduction of export centered levies. This tax reform has a direct effect on employer's costs of operation and, thus, their ability to hire more workers or increase their wages. The Minister of Interior reports an increase of 1 million jobs. (Cardenas, 2012) Therefore, the use of a dummy variable, represented in the models as *tax*, is introduced in the estimation to account for the effects that such policy changes have on the labor market.

In response to the current supply shock to the petrol market by the OPEC cartel intends to make inexpensive fracking techniques financially unsustainable for recently established U.S. companies. (Egan, 2015) Another dummy variable is accounted for in order to capture a reduction in the market price of crude of approximately 50% since 2014, represented in the models as *crudesh*. According to the DANE, Colombian petrol exports constituted for 53% of total export transactions in 2014.

Resulting Estimates

Most studies that target the Colombian NAIRU do so utilizing a similar methodology to the one outlined in this study, but with added caveats of insignificant optimum lags within the series, several filters, and complex spline polynomials. Some researchers[11] publish feeble estimates, which transcends into insignificant outcomes. Imprecise estimations could form due to the endogenous relationship between prices and wages that cause circular argumentation in hopes of justifying the outcomes; or due to excessive manipulation (smoothing, detrending, or breaking) of the data as it is explained by Staiger *et al* (1997). Consequently, investigators seek other methods in order to

9 production costs, increased demand, higher wages, increased competition for the labor force, the formation of syndicates, and levies.

10 crisis dummy variable includes de effects of the crisis from august of 2008 until 2009.

11 see Cancelado (2011) and Gomez and Julio (1997).

improve the clarity of evidence that describes the causality between unemployment and inflation. For the case of this research, there is minimal manipulation of the data that boosted the robustness of the values inherent in the estimation of the Colombian NAIRU with comparison to previous measures.

After correcting each econometric model for the seasonal component among other influencing factors, such as crises, international market shifts, national policy changes, and possible supply shocks, the following model is the best predictor[12] of the variation in inflation for the aggregate Colombian economy:

$$\begin{aligned}\Delta\pi_{col,t} = {} & 0.0003876 + 0.0672404\, u_{col,t-1} - 0.0705285\, u_{col,t-2} - 0.2609681\, \Delta\pi_{col,t-1} \\ & - 0.2766989\, \Delta\pi_{col,t-2} - 0.2591371\, \Delta\pi_{col,t-3} + 0\,.0000348\, \Delta^2 dpcup \\ & - 0.7229801\, \omega_{col,t-12} - 0.9592468\, \Delta\pi_{col,t-12} + \omega_t\end{aligned}$$

For the average urban sector, the following model is the best predictor of the variation inflation:

$$\begin{aligned}\Delta\pi_{urb,t} = {} & 0.0006035 + 0.117156\, u_{urb,t-1} - 0.1215869\, u_{urb,t-2} - 0.1538489\, \Delta\pi_{urb,t-1} \\ & + 0.0000335\, \Delta^2 dpcup + 0.0000702\, rate - 0.738473\, \omega_{t-12} \\ & + 0.9444069\, \Delta\pi_{urb,t-12} + \omega_t\end{aligned}$$

12 Based on equation (6): $\Delta\pi_{i,t} = \alpha + ß_1 u_{i,t-1} + ß_2 u_{i,t-2} + \lambda X + \rho Z - \Phi_j \Delta\pi_{i,t-s} - \theta_k \omega_{t-s} + \omega_t$

Table 2: Estimates of Constant Measures for the NAIRU[13]

Estimate	*National [2001-2015]*	*Urban [2001-2015]*
NAIRU	11.79%	13.62%
α	0.0003876 (0.0009844)	0.0006035 (0.0011836)
$ß_1$	0.0672404 *** (0.0251157)	0.117156 ** (0.0585382)
$ß_2$	- 0.0705285 *** (0.0259136)	- 0.1215869 ** (0.0578884)
λ_1	- 0.2609681 *** (0.075274)	- 0.1538489 ** (0.0790985)
λ_2	- 0.2766989 *** (0.0768947)	.
λ_3	- 0.2591371 *** (0.0818625)	.
ρ_1	0.0000348 *** (0.0000161)	0.0000335 *** (0.0000131)
ρ_2	.	0.0000702 * (0 .0000383)
θ_1	-0.7229801 *** (0.1065252)	- 0.738473 *** (0.1155256)
Φ_1	0.9592468 *** (0.0305356)	0.9444069 *** (0 .0417972)

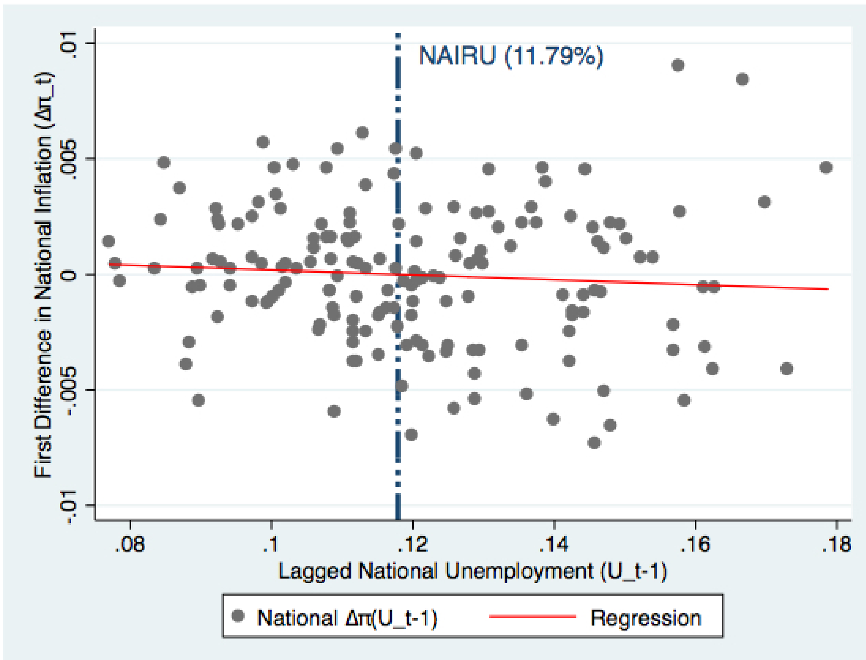
* 90% significance; ** 95% significance; ***: 99% significance; values in parenthesis are the standard errors.
Source: Generated by Author

As seen in *Table 2*, there is a higher measure for the urban NAIRU with respect to the national NAIRU. Which is concurrent with to the higher unemployment levels Colombia's urban labor market in comparison to national employment levels. (Dane, 2015) From all of the previously mentioned shocks, only the second difference of coffee is significant at the 99% level for the national market and 95% at an urban level; the lagged exchange rate is significant at 90% only for the urban market. All of the other previously mentioned supply shocks and dummy variables introduced into the models to capture perturbations that caused spikes within the intended measures were discarded due

13 Based on equation (9): $U_i^N = -\frac{\alpha}{(ß_1 + ß_2)}$

to their insignificant relationship. By extracting any significant jumps in the NAIRU that were caused by supply shocks, the practical measure of the NAIRU further resembles the theoretical principle of optimum employment level. (Gordon, 1997)

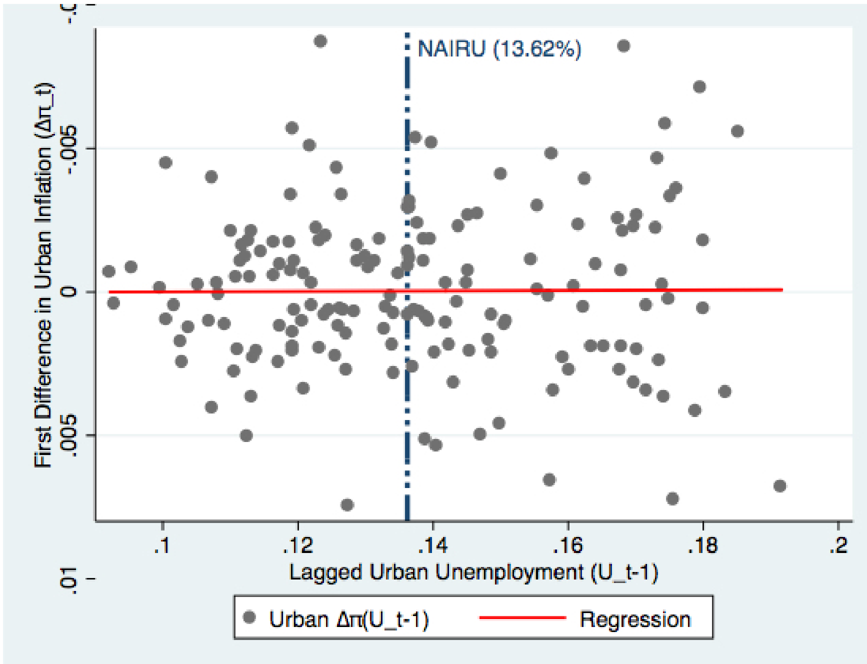
Graph 3: Change in inflation vs. Total Unemployment of Previous month and the Regression Line, Monthly Data for Colombia's National Market (2001-2015)



Source: Generated by Author

Graph 3: Shows the horizontal axis shows the unemployment rate of the previous month; and the vertical axis shows the monthly change in inflation. There is an evident negative relationship. In addition of the relationship, the red line represents the OLS regression line that is estimated over the national Colombian sample. The intersection point of the regression line is equal to, the NAIRU, 11.79% that represents the minimum benchmark of unemployment for which inflation is predicted to remain at a constant level. The measure for the national labor markets differ -1.83% from the urban assessment.

Graph 4: Change in inflation vs. Total Unemployment of Previous month and the Regression Line, Monthly Data for Colombia's National Market (2001-2015)

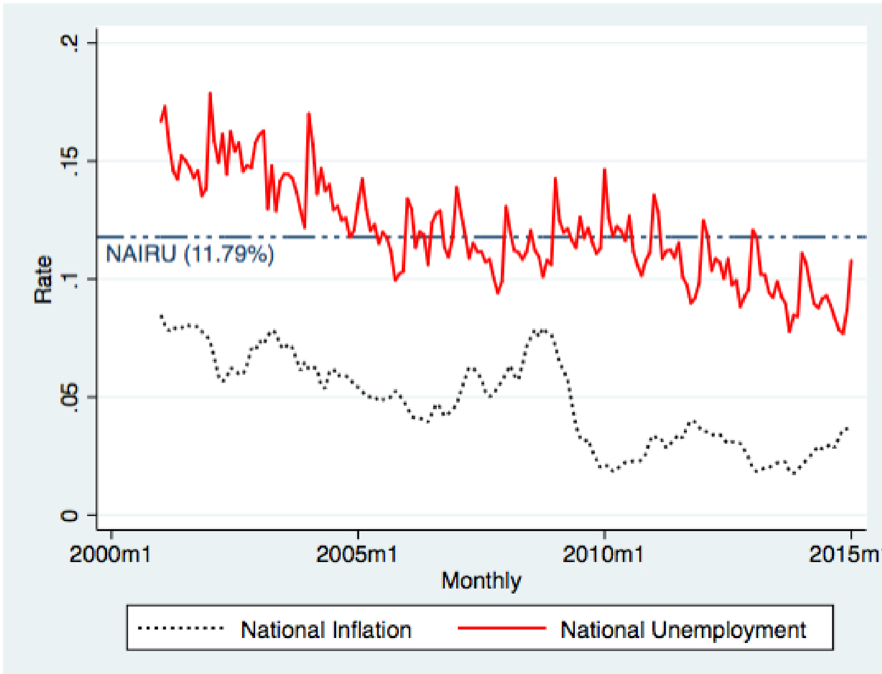


Source: Generated by Author

Graph 4: Shows the horizontal axis shows the unemployment rate of the previous month; and the vertical axis shows the monthly change in inflation. There is an evident negative relationship. In addition of the relationship, the red line represents the OLS regression line that is estimated over the urban Colombian sample. The intersection point of the regression line is equal to, the NAIRU, 13.62% that represents the minimum benchmark of unemployment for which inflation is predicted to remain constant. The measure for the urban labor markets is +1.83% higher from the national NAIRU measure.

As illustrated in *Graph 5,* When national unemployment moves above the national NAIRU, inflation has a downward tendency, however, when unemployment dips below the NAIRU, national inflation rates move with an upward trend.

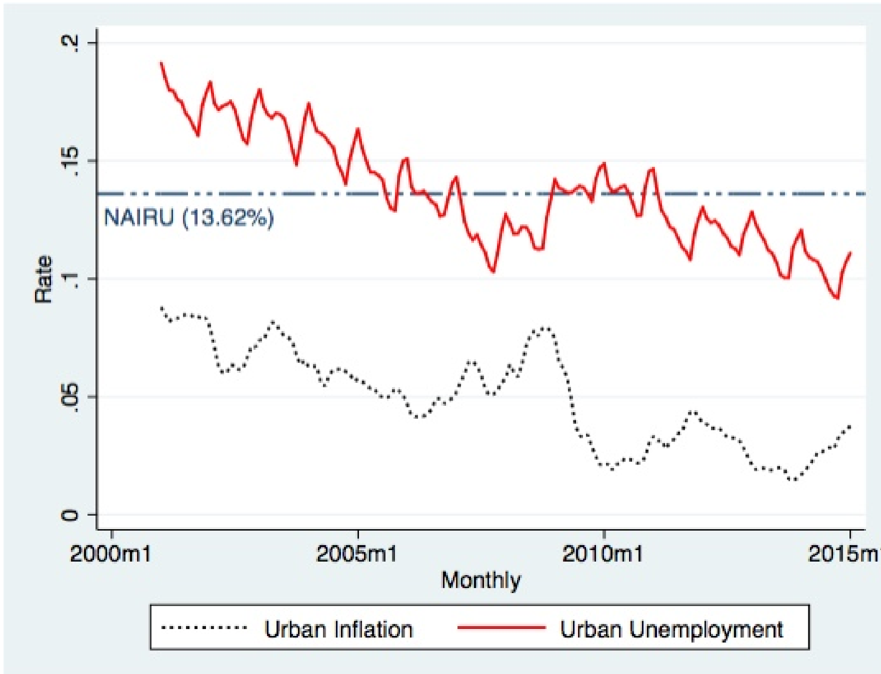
Graph 5: Path of National Unemployment in Time With Respect to the Constant NAIRU, Monthly Data for Colombia's National Market (2001-2015)



Source: Generated by Author

Graph 5 and Graph 6 illustrates the path in time of unemployment for both, urban and national markets, with respect to the respective NAIRU estimated. Similar surges of unemployment resonate within the urban path and that of the aggregate level. The vertical axis demonstrates the unemployment level and the horizontal axis represents the time in which unemployment varies from January of 2001 until January of 2015. Both markets dip below the NAIRU in the last quarter of 2005 and rise above it in 2009. In about mid 2011 both markets dip below the NAIRU, once again, and do not rise above it for 6 years. In periods when the unemployment rate is above the NAIRU inflation seems to remain constant or decrease, from 2001 until 2006 and from 2008 until 2011.

Graph 6: Path of Urban Unemployment in Time With Respect to the Constant NAIRU, Monthly Data for Colombia's Urban Market (2001-2015)



Source: Generated by Author

Estimation Issues

In order to adjust for heteroskedasticity or serial correlation, the NAIRU is estimated as the sample mean of unemployment. The assumption that the average of the unemployment gaps averages zero over time or the NAIRU can, also, be regarded as a parameter in the Phillips curve. Hence, the filtering through the assumption of parametric uncertainty translates, in part, into the yields of the estimation process. The introduction of supply shocks or dummy variables reduces, but fails to abolish, the uncertainty inherent within the model and yields a considerably more meaningful estimate of the NAIRU.

However, out of the six variables introduced with for the purpose of capturing a portion of the effect exposed in the Phillips curve relationship, only the lagged exchange rate and the second difference of the price of coffee has a significant impact. Colombia's labor market and economy depends, to a meaningful degree, on the export of commodities such as, the international price of Colombian coffee, as well as the price of crude which its effect is surprisingly insignificant. This issue of estimation could be due, in part, to the parametric uncertainty and the hollow representation of the true model. Such parametric uncertainty could be due to the unrepresentative unemployment sample gathered. The unemployment values only embody a meager 56% of the entire labor market.[14]

Under the hood of the model, there is a strong seasonal pull on the data, given the cyclical consumer trends and recurring unemployment. The seasonal component is included in the model, as previously specified, to obtain econometrically sound estimates.

The focus of this research emphasized a reduction of filtering uncertainty that is corroborated when filters were applied to the data yielded insignificant and null estimates. Filters such as Hodrick-Prescott's were deemed ineffective and greatly distorted the significance of the estimated components in the NAIRU. The introduction of dummy variables to capture the effect of the crisis, the employer tax reform, and the recent petrol price drop, proved to be ineffective since they were particularly insignificant, as well.

Remaining within the objective to obtain trustworthy estimates, as previously stated, estimations were regarded with the utmost detail to obtain econometrically sound estimations. Albeit arduous corrections to the proposed theoretical model to extract any autocorrelating noise that might be present in the residuals and the amalgamation of supply shocks for better estimations; precise estimates for the Colombian NAIRU were obtained with a 99% regiment of significance and 95% significance for the average urban centers.

Conclusions

The current immobility of skilled labor within the more developed urban labor markets might come off as a stabilizing force of the unemployment levels of these individual urban centers. However, the contrary is expected for unskilled labor, as it is able to move with more ease across the country and find employment. It is imperative to reiterate the high levels of employment informality, almost 46%, (DANE, 2015) which are present throughout the country as a whole. Informality presents a significant setback in the search of a NAIRU that is representative for Colombian markets. High levels of labor informality impede a proper estimation of the NAIRU with respect to the aggregate labor market and it could be the source for discrepancies between studies.

The NAIRU's estimates in this research concur within the range of previous estimates of NAIRU for Colombia's national and urban labor market that range anywhere between 8% and 15%. A higher NAIRU for the average urban employment market is concurrent with existing employment measures that regard higher urban unemployment quantities due to a more complex and regimented urban labor market. The difference in the NAIRU between both markets, could be explained in the following way: since in an urban setting, the polarizing real vs. nominal wage forces give rise to an arm wrestling duel between laborers and employers. These opposing forces create an optimal assignation of labor intensity when output is tied to labor performance metrics. While in

14 According to the DANE (2015) there is measured labor informality rate of 46%.

the national setting the labor procedure is not as conflicting as it is in the urban markets and labor assignation occurs with ease. Urban markets have one less significant autoregressive lag of inflation, but are more dependent on the relative price of foreign goods.

Suggested Research

For further analysis, the individual urban and dynamic measurements of the NAIRU could be estimated, but only if strict regiments for precision and the preservation of organic data remain the central focus of the research. Individual urban NAIRU assessments could provide further insight, at a micro level, into the determinants of the optimum urban employment levels.

The structural differences of the individual urban employment markets could be cross analyzed and tangible conclusions regarding the catalysts of employment, the identification of possible bottlenecks, and areas of sluggish employment levels could be recognized. A dynamic, or time varying, NAIRU could serve to interpret future changes in prices or unemployment regarding the Phillips curve relationship. The extraction of money supply levels from consumer price measures could prove to be a useful tool in the estimation of a more representative NAIRU.

Moreover, the determination of conditional convergence measures of unemployment towards its NAIRU could prove to be strategically advantageous for urban and national policymakers. The cross-reference of urban employment policies with the urban employment curve, and to its inherent NAIRU, would be a prevailing indicator of the effect of policies and their consequential lag on prices. Convergence of actual unemployment levels towards the Non-Accelerating Inflation Rate of Unemployment (NAIRU) in time not only takes into account the inverse relationship of inflation with unemployment, (Phillips, 1958) but also provides forth an opportunity to determine the impact of the structural qualities within the labor market under analysis; such as wage flexibility, labor mobility, employment retention, education levels, labor market segmentation, and policy decisions, among other vital tenacities. This is true for economic agents whose operations are intensely sensitive to price changes and wage differences, such as national labor syndicates and citywide planners. It is plausible that some grouping of cities with similar market structures could move as one group, but end up diverging from other groups; as it is proposed by Daniel Quah (1993, 1996[a], 1996b, 1997).

Bibliography

Arango, L., y C. Posada (2006); “La tasa de desempleo de largo plazo en Colombia” Borradores de Economía, No. 388. Banco de la República.

Arango, L.; A. García, y C. Posada (2006); “La metodología de la Encuesta Continua de Hogares y el empalme de las series del mercado laboral urbano de Colombia”, Borradores de Economía, No. 41. Banco de la República.

Ball, L. (2001); “Productivity growth and the Phillips curve”, NBER Working Paper Series, No 8421. National Bureau of Economic Research.

Barro, Robert (1992). “Convergence”. En: Journal of Political Economy No100 (2). Pp. 223-251.

Barro, Robert (1997).“Determinants of Economic Growth” MIT Press.

Barro, Robert y Sala-i-Martin, Xavier (1995). “Economic Growth”. New York: McGraw-Hill.

Barro, Robert y Sala-i-Martin, Xavier (1997). “Technological Discussion, Convergence, and Growth”. En: Technological Discussion, Convergence, and Growth Journal of Economic Growth No 2, 1.26.

Bellod (1999): “Prueba de causalidad y determinación de la NAIRU”, El Trimestre Económico 66(264), 663-692.

Benes, J and D’Niaye, P. (2004). A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic.IMF, Working Paper No. 04-45

Blanchard, Olivier, and Lawrence Katz, (1997) "What We Know and Do Not Know About the Natural Rate of Unemployment, "Journal of Economic Perspectives 11 (1997), 51-72.

Blanchard, Olivier, and Lawrence Summers, (1986) "Hysteresis and the European Unemployment Problem,"(pp. 15-78), in Stanley Fischer (Ed.), NBER Macroeconomics Annual (Cambridge, MA: MIT Press, 1986)

Cancelado Dario. (2012). “Convergencia de la NAIRU: El caso Colombiano”. Revista Civilizar .Universidad Sergio Arboleda de Colombia.

Cárdenas, M. y Gutiérrez, C. (1998). Determinantes del Desempleo en Colombia. Debates de Coyuntura Social, No. 9. Colombia.

Cardenas, Mauricio, (2012) "Reforma Tributaria 2012." El Ministro De Hacienda, 8 Oct. 2012. Web. 19 May 2015.

Clavijo, S. (1994); “Inflación o desempleo: ¿Acaso hay escogencia en Colombia?”, Archivos de Macroeconomía No. 31. DNP.

Congressional Budget Office, (1994) "Re-estimating the NAIRU " In The Economy; and Budget Outlook. Washington, D.C.: U.S. Government Printing Office, August 1994, pp. 59-63.

Congressional Budget Office, (1996) The Economic and Budget Out/oak: Fiscal Years 1997-2006 Washington, D.C. U.S. Government Printing Office, May 1996.

Eisner, Robert, (1995b) "Opening Up the Growth Debate," Wall Stree Jtournal, September 25, 1995b, A18:3.

Eisner, Robert, (1997) "A New View of the NAIRU," manuscript, Northwestern University, July 1995a; in Davidson, P., and J. Kregel, eds., Improving the Global Economy: Keynesianism and the Growing Output and Employment. Cheltinham, UK, and Brookfield: Edward Elgar, forthcoming, October 1997; Cahiers de l'Espace Europe, forthcoming, March 1997

Estrada, Á.; Hernando, I.; López-Salido, J.D. (2000): Measuring the NAIRU in the Spanish Economy. Banco de Espaňa / Documento de Trabajo no 0009: 1.

Farné, S.; A. Vivas, y T. Yepes (1995); "Estimación de la tasa natural de desempleo en Colombia", Cuadernos de Empleo, No. 1, Ministerio de Trabajo y Seguridad Social. González, A.; L.

Friedman, Milton (1968) "The Role of Monetary Policy," American Economic Revieew March1968, 58, 1-17.

Fuhrer, Jeffrey C., (1995) "The Phillips Curve is Alive and Well," New England Economic Review of the Federal Reserve Bank of Boston, March/April 1995, 41-56.

Galton, Francis (1886). "Regression toward mediocrity in hereditary stature". En: Journal of the Anthropological Institute of Great Britain and Ireland No 15. pp. 246-263.

Galton, Francis (1989). "National Inheritance". London: Macmillan.

Gómez García, F. & Rebollo Sanz, Y. & Usabiaga Ibáñez, C., 2002. "Nuevas estimaciones de la NAIRU de la economía española: métodos directos," Estudios de Economía Aplicada, Estudios de Economía Aplicada, vol. 20, pages 509-530, Diciembre.

Gómez, J. y Julio, J.M. (2000). An Estimation of the Nonlinear Phillips Curve in Colombia. Banco de la República. Borradores de Economía No. 160.

González, A.; L. Melo, y C. Posada (2009); "Inflation and money in Colombia: Another P-Star model", Applied Economics, vol. 41, no. 10, 2009.

Gordon, R. (1997). The time-varying NAIRU and its implications for economic policy. Journal of Economic Perspectives, Vol. 11(1).

Gordon, Robert J., (1982) "Price Inertia and Ineffectiveness in the United States," Journal of Political Economy December 1982, 90, 1087-117.

Greenslade, JV, Pierse, RG and Saleheen, J (2003), ‘A Kalman filter approach to estimating the UK NAIRU’, Bank of England Working Paper no. 179.

Henao, M. y Rojas, N. (1998). La Tasa Natural de Desempleo en Colombia. DNP, Archivos de Economía. No. 89.

Hodrick, Robert J. and Edward C. Prescott. (1997) “Postwar U.S. Business Cycles: An Empirical Research.” Journal of Money, Credit and Banking. February, 29:1, pp. 1–16.

Julio, J. (2001); “How uncertain are NAIRU estimates in Colombia?”, Borradores de Economía, No. 184. Banco de la República.

King, Robert, and Mark Watson, (1994) "The Post-War U.S. Phillips Curve: A Revisionist Econometric Conference History, "Carnegie-Rochester Series on Public Policy 41 (1994), 157-219.

King, Robert, James Stock, and Mark Watson, (1995) "Temporal Instability of the Unemployment-Inflation Relationship, "Economic Perspectives of the Federal Reserve Bank of Chicago 19 (1995), 2-12.

Laubach, Thomas, (1997) "Measuring the NAIRU: Evidence from Seven Economies, "Federal Reserve Bank of Kansas City research working paper 97-13, (1997).

Layard, R., Nickell, S., & Jackman, R. (1991). Unemployment: Macroeconomic performance and the labour market. Nueva York: Oxford University Press.

López, E. y Misas, M. (1999). Un examen empírico de la curva de Phillips en Colombia. Banco de la República. Borradores de Economía No. 117.

Lucas JR, Robert (2002). “The Industrial Revolution: Past and Future”. En: Lectures on Economic Growth: Cambridge, Massachusetts: Harvard University.

Mankiw, Gregory, Romer, David y Weil, N (1992). “A Contribution to the Empirics of Economic Growth”. En: Quarterly Journal of Economics No 107. pp. 407-438.

Mayorga Mogollón, Wilson, Escalante Cortina, Rafael D. (2011). Actualización de la estimación de la NAIRU en ColombiaRevista de Ciencias Sociales (Ve) [online] 2011, XVII (Abril-Junio) : [Date of reference: 10 / mayo / 2015]

Meisel, Adolfo, and Jaime B. Moron. (1999) "LA CONVERGENCIA REGIONAL EN COLOMBIA: UNA VISION DE LARGO PLAZO, 1926 – 1995." Banco De La Republica. N.p., Feb. 1999. Web.

Melo, y C. Posada (2009); “Inflation and money in Colombia: Another P-Star model”, Applied Economics, vol. 41, no. 10, 2009.

Mincer, J. (1974). Schooling, experience, and earnings. Nueva York: Columbia University Press.

Modigliani, F., y L. Papademos (1975) “Targets for Monetary Policy in the Coming Year”, Bookings Papers on Economic Activity, 1, 141-165.

Mora, John J. (2003) "CRECIMIENTO Y CONVERGENCIA: A PROPÓSITO DE QUAH." SIELO. N.p., 30 May 2003. Web.

Moulton, B. R. (1986). Random group effects and the precision of regression estimates. Journal of Econometrics, 32, 385-397.

Nigrinis, M. (2003). ¿Es Lineal la Curva de Phillips en Colombia?. Banco de la República. Borradores de Economía No. 282.

Núñez, J. y Bernal, R. (1998). El desempleo en Colombia: tasa natural, desempleo cíclico y estructural y la duración del desempleo 1976-1998. DNP. Archivos de Economía, No. 97.

Phelps, E. (1967), “Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time”, Economica, 34 (3), 678-711.

Phillips, A. W., (1958) "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861 - 1957," Economica. November, 1958, 25, 283-99.

Quah, Danny (1993). “Galton.s Fallacy and Tests of the Convergence Hypothesis”. En: Scandinavian Journal of Economics No 95. pp. 427-443.

Sala-i-Martin, Xavier (1997). “I just ran two million regressions”. En: American Economic Review No 87. pp. 178-183.

Sala-i-Martin, Xavier (2006). "The World Distribution of Income: Falling Poverty and. Convergence Period". En: Quarterly Journal of Economics No 2. pp. 351-397.

Sánchez, F., & Núñez, J. (1998). La curva de salarios para Colombia. Estimaciones de las relaciones entre desempleo, la inflación y los ingresos laborales 1984-1996. Archivos de Macroeconomía, documento 80, DNP.

Shapiro, C., & Stiglitz, J. E. (1984). Equilibrium unemployment as a worker discipline device. American Economic Review, 74(3), 433-444.

Shin, D., & Solon, G. (2006). New evidence on real wage cyclicality within employer employer matches (Working Papers Series No 12262). NBER.

Solon, B., & Parker, J. (1992). Measuring the cyclicality of real wages: How important is composition bias? (Working Papers Series No 4202). NBER.

Solow, Robert M., (1969) Price Expectation and the Phillips Curve. Manchester, UK: Manchester University Press, 1969.

Staiger, D.; J. Stock y M. Watson (1997) “The NAIRU, Unemployment and Monetary Policy”, Journal of Economic Perspectives, 11, 1, winter.

Staiger, Douglas, James Stock, and Mark Watson, (1997a, 1997b) "The NAIRU, Unemployment and Monetary Policy, "Journal of Economic Perspectives 11 (1997a), 33-49. , "How Precise Are Estimates of the Natural Rate of Unemployment?" (pp. 195-242), in C. Romer and D. Romer (Eds.), Reducing Inflation: Motivation and Strategy (Chicago: Chicago University Press and NBER, 1997b)

Sun, Y. (2001). “Catching up, Forging ahead, and Falling Behind: A Panel Structure Analysis of Convergence Clubs”. UCSD mimeographed.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. Journal of Political Economy, 88(1), 1-23.

Tootel, Geoffrey (1994) M. B., "Restructuring, the NAIRU, and the Phillips Curve," New England Economic Review of the Federa Rleserve Bank of Boston, September/October 1994, 31-44.

Uribe, J.D. (2004). Situación actual y perspectivas de la economía Colombiana. Presentación, mayo de 2004.

Urrutia, M. (2002). El subempleo en Colombia. Revista del Banco de la República, 75(898), 5-23.

Weiner, Stuart E., (1993) "New Estimates of the Natural Rate of Unemployment," Economic Review of the Federal Reserve Bank of Kansas City, Fall 1993, 78, 53-69.

Weiner, Stuart E., (1994) "The Natural Rate and Inflationary Pressures," Economic Review of the Federal Reserve Bank of Kansas City, Summer 1994, 79, 5-9.

Whittaker, E. T. (1923). “On a new method of graduation.” Proceedings of the Edinburgh Mathematical Association. No 78. pp. 81-89.

Wren-Lewis, S. (I990). 'Nominal inertia and Keynesian effects.' National Institute Discussion Paper No. I74.

Zhu, F. (2004). “Three Essays in Macroeconomic Empirics and Monetary Theory”.Ph.D Dissertation, Yale University.

Appendix

Table A1: Summary of All Statistics Used

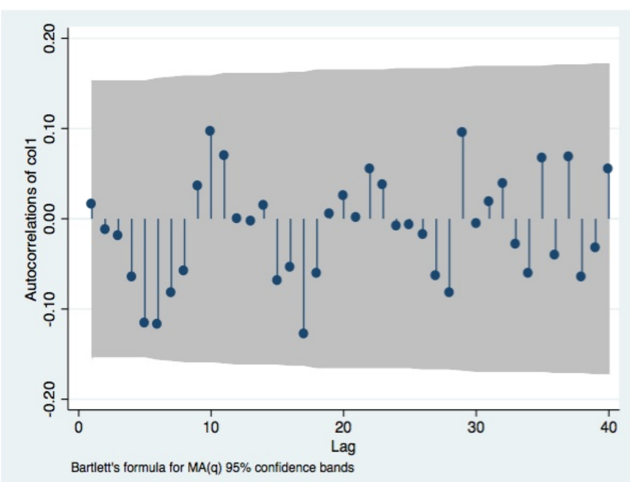
Variable	*Obs.*	*Mean*	*Std. Dev.*	*Min*	*Max*
National Inflation (N.I).	169	0.0038675	0.0038437	-0.0026	0.0189
Urban Inflation (U.I.)	169	0.0492856	0.0206367	0.01435	0.0881417
Difference N.I.	168	0.0000244	0.0030128	-0.0073	0.009
Difference U.I.	168	0.0000348	0.0028339	-0.00868	0.00748
National Unemployment					
Contemporaneous	169	0.120171	0.0215219	0.0771	0.1787
Lag 1	168	0.120244	0.0215652	0.0771	0.1787
Lag 2	167	0.1204419	0.0214766	0.0771	0.1787
Urban Unemployment					
Contemporaneous	169	0.1373189	0.0232609	0.092161	0.1916129
Lag 1	168	0.137476	0.0232403	0.092161	0.1916129
Lag 2	167	0.1376569	0.0231913	0.092161	0.1916129
Supply Shocks					
Price of Coffee (*cup*)	169	1.095935	0.4790629	0.412	2.312
Difference of cup	166	0.0461412	12.72889	-41.25713	34.73772
Price *crude*	169	67.04101	28.56752	19.39	133.88
Difference of *crude*	166	0.0569337	16.46183	-47.99461	44.49675
Exchange *rate*	168	0.1139733	3.598065	-10.1501	13.08362
Lag *rate*	167	0.1017446	3.605383	-10.1501	13.08362
Dummy Variables					
2008 Crisis	169	0.1420118	0.3500996	0	1
Tax	169	0.147929	0.3560847	0	1
Crude shock	169	0.0295858	0.1699452	0	1

Source: Generated by Author

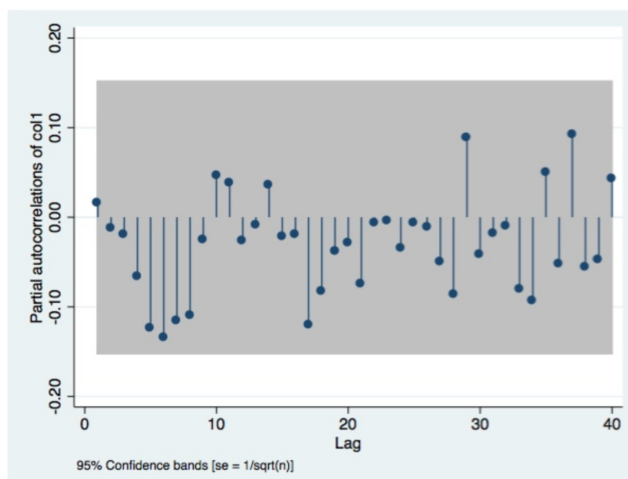
Table A2: Colombia's National Market ARIMA Regression

Sample: 2001m4 - 2015m1				Number of obs =		166
Log likelihood = 774.7411				Wald chi2(8) =		3936.87
				Prob > chi2 =		0.000
Diff. National Inflation	*Coefficient*	OPG *Std. Err*	*Z*	*P>|z|*	*[95% Conf. Interval]*	
National Unemployment						
Lag 1	0.0672404	0.0251157	2.68	0.007	0.0180146	0.1164662
Lag 2	-0.0705285	0.0259136	-2.72	0.006	-0.1213182	-0.0197388
Constant	0.0003876	.0009844	0.39	0.694	-0.0015419	0.002317
Supply Shocks						
Diff. Coffee Price	0.0000348	0.0000161	2.160	0.031	0.00000321	0.0000665
Autoregressive and Seasonal Adjustment						
ARMA (1,0)	-0.2609681	0.075274	-3.47	0.001	-0.4085025	-0.1134338
ARMA (2,0)	-0.2766989	0.0768947	-3.60	0.000	-0.4274097	-0.1259881
ARMA (3,0)	-0.2591371	0.0818625	-3.17	0.002	-0.4195846	-0.0986896
Seasonal AR (1)	0.9592468	0.0305356	31.41	0.000	0.8993981	1.019095
Seasonal MA (1)	-0.7229801	0.1065252	-6.79	0.000	-0.9317657	-0.5141945
Sigma	0.0021992	0.0001355	16.23	0.000	0.0019336	0.0024648

Source: Generated by Author



Source: Generated by Author



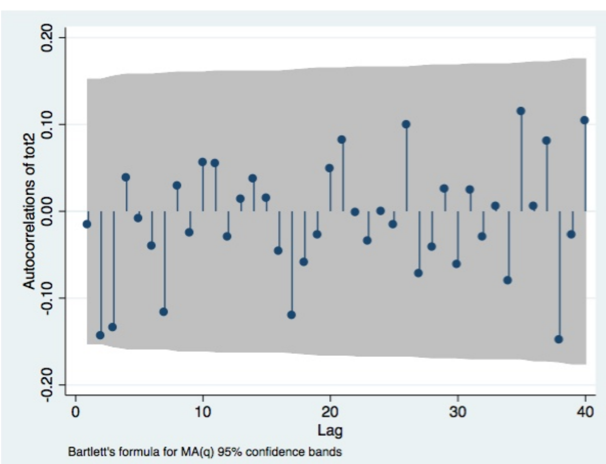
Source: Generated by Author

Table A3: Colombia's Urban Market ARIMA Regression

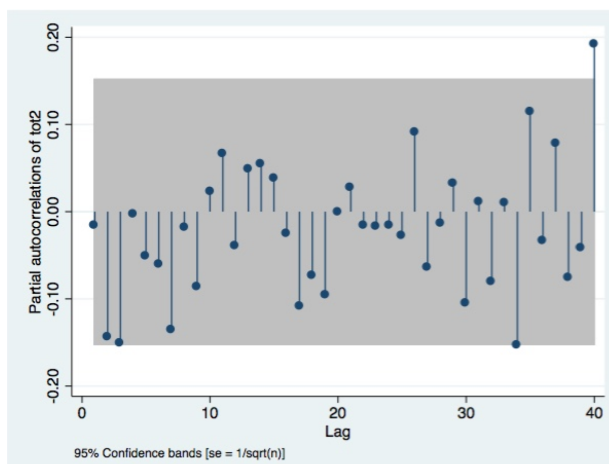
Sample: 2001m4 - 2015m1					Number of obs =	166
Log likelihood = 781.7146					Wald chi2(7) =	2012.09
					Prob > chi2 =	0.000

Diff. Urban Inflation	*Coef.*	*OPG Std. Err.*	*Z*	*P>|z|*	*[95% Conf.*	*Interval]*
Urban Unemployment						
Lag 1	0.117156	0.0585382	2.00	0.045	0.0024233	0.2318887
Lag 2	-0.1215869	0.0578884	-2.10	0.036	-0.2350461	-0.0081278
Constant (α)	0.0006035	0.0011836	0.51	0.610	-0.0017163	0.0029232
Supply Shocks						
Diff. Coffee Price	0.0000335	0.0000131	2.55	0.011	0.00000773	0.0000592
Lag Rate	0.0000702	0.0000383	1.83	0.067	-0.00000481	0.0001452
Autoregressive and Seasonal Adjustment						
ARMA(1,0)	-0.1538489	0.0790985	-1.95	0.052	-0.3088791	0.0011812
Seasonal AR(1)	0.9444069	0.0417972	22.59	0.000	0.8624859	1.026328
Seasonal MA(1)	-0.738473	0.1155256	-6.39	0.000	-0.9648989	-0.512047
Sigma	0.0021322	0.0001274	16.74	0.000	0.0018825	0.0023819

Source: Generated by Author



Source: Generated by Author



Source: Generated by Author

Table A4: Phillips-Perron for First Difference in National Monthly Inflation Rates

			Number of obs =	167
			Newey-West lags =	4
		Interpolated Dickey-Fuller		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(rho)	-146.795	-20.023	-13.834	-11.089
Z(t)	-12.647	-3.488	-2.886	-2.576
MacKinnon approximate p-value for Z(t) = 0.0000				

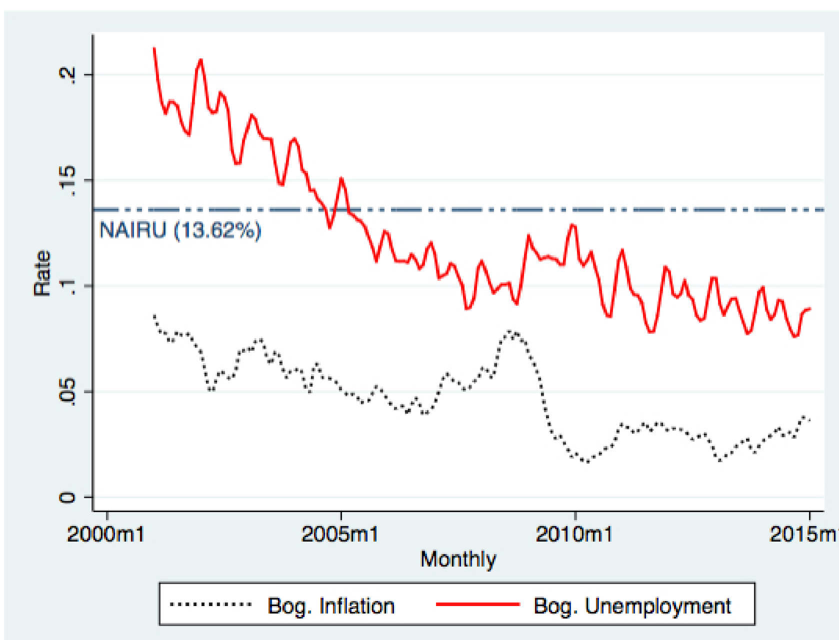
Source: Generated by Author

Table A5: Phillips-Perron for First Difference in Urban Monthly Inflation Rates

			Number of obs =	167
			Newey-West lags =	4
		Interpolated Dickey-Fuller		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(rho)	-150.09	-20.023	-13.834	-11.089
Z(t)	-12.394	-3.488	-2.886	-2.576
MacKinnon approximate p-value for Z(t) = 0.0000				

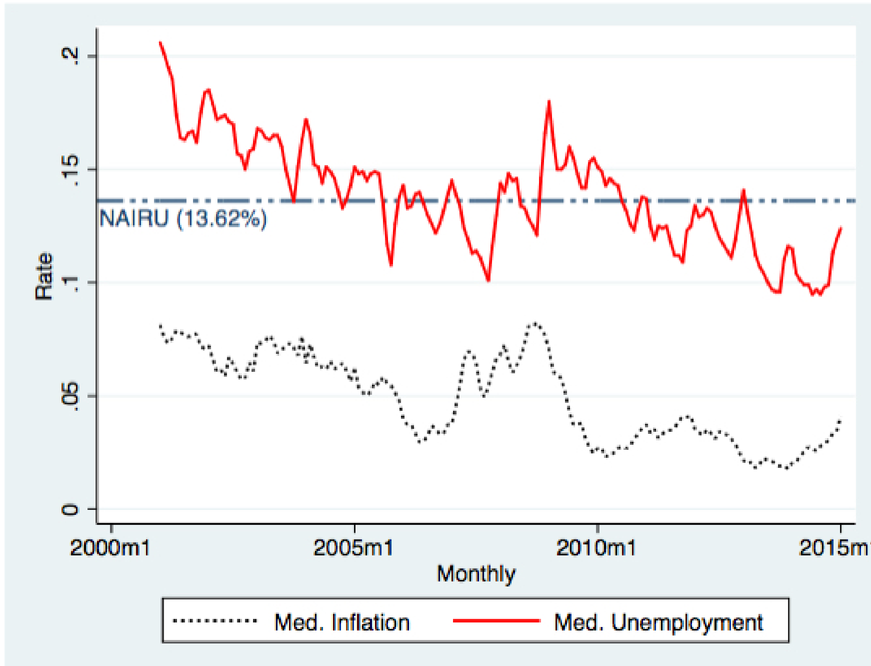
Source: Generated by Author

Graph A1: Bogotá's Monthly Unemployment and Inflation Path (2001-2015)



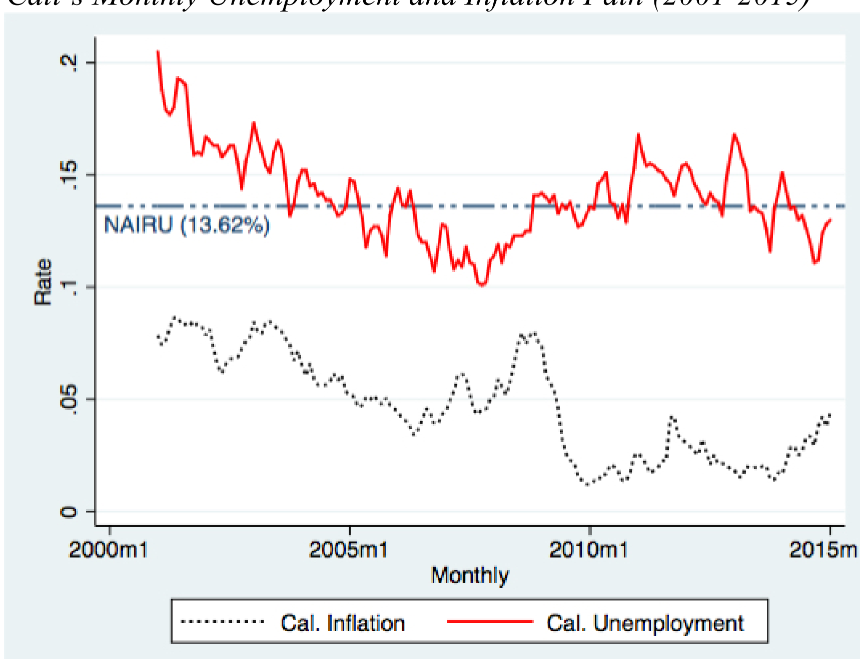
Source: Generated by Author

Graph A2: Medellín's Monthly Unemployment and Inflation Path (2001-2015)



Source: Generated by Author

Graph A3: Cali's Monthly Unemployment and Inflation Path (2001-2015)



Source: Generated by Author