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Alfredo Trespalacios, Lina M. Cortés, Javier Perote

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Centro de Investigaciones Económicas y Financieras
Universidad EAFIT

Modeling electricity price and quantity uncertainty: An application for hedging with forward contracts[†]

Alfredo Trespalacios[‡]
Lina M. Cortés[§]
Javier Perote[**]

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Abstract

Energy purchases/sales in liberalized markets are subject to price and quantity uncertainty, which should be jointly modeled by relaxing the unreliable normality assumption for capturing risk. In this paper, we consider the spot price and energy generation to follow a bivariate semi-nonparametric distribution defined in terms of the Gram-Charlier expansion. This distribution allows to jointly model not only mean, variance, and correlation, but also skewness, kurtosis, and higher-order moments. Based on this model, we propose a static hedging strategy for electricity generators that participate in a competitive market where hedging is carried out through forward contracts that include a risk premium in their valuation. For this purpose, we use Monte Carlo simulation and consider information from the Colombian electricity market as the case study. The results show that the volume of energy to be sold under long-term contracts depends on each electricity generator and the risk assessment made by the market in the Forward Risk Premium. The conditions of skewness, kurtosis, and correlation, as well as the type of risk indicator to be employed, affect the hedging strategy that each electricity generator should implement.

Keywords: Semi-nonparametric approach; multivariate distribution; electricity market; hedging; forward contracts.

JEL Classification: C14, C22, C53, L94, L98, Q2

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[‡] Department of Finance, Faculty of Economic and Administrative Sciences. Institucion Universitaria Instituto Tecnológico Metropolitano (ITM). Address: Calle 54A No 30-01 Medellin, Colombia. E-mail: alfredotrespalacios@itm.edu.co
[§] Department of Finance, School of Economics and Finance. Universidad EAFIT. Address: Carrera 49 No 7 Sur-50 Medellin, Colombia. E-mail: lcortesd@eafit.edu.co. Corresponding author. Phone: +5742619500 ext. 9756. Fax: +5743120649.
[**] Department of Economics and IME, University of Salamanca. Address: Campus Miguel de Unamuno 37007 Salamanca, Spain. E-mail: perote@usal.es

1. Introduction

Electricity is one of the most efficient ways to transform, transport, and use energy. It contributes to the economic growth of countries and enables them to achieve the Sustainable Development Goals (SDGs) by helping to end poverty, protect the planet, and ensure prosperity for all.

Electrical energy is usually traded in a short-term market (referred to as the spot market) and in a long-term market via contracts for future delivery (i.e., forward contracts). The electricity market is characterized by being highly volatile when compared to other commodity markets. This high volatility in terms of price and quantity is due to market conditions (e.g., expectations or strategies of each company and economic dynamics of a region) and physical conditions (e.g., climate, water availability, fuel production or transportation capacity, and even damage to the power transmission network) (Mosquera-López, Uribe, & Manotas-Duque, 2017). To face this situation, electricity generators − companies that make large investments with long capital recovery periods − must implement effective market risk management strategies to ensure compliance with their business objectives.

In order to achieve the best results in their energy transactions and reduce the inherent risks, electricity generators must define, in their energy portfolio, the volume of electricity to be sold, at each time in the future, under long-term contracts with a price previously established (i.e., forward contracts) and that to be traded at a spot price. To address this issue, it should be noted that electricity generators face price and quantity uncertainty, unlike in other types of financial products, as mentioned by Woo, Karimov, and Horowitz (2004); Nässäkkälä and Keppo (2005); Oum and Oren (2010); and Boroumand, Goutte, Porcher, and Porcher (2015), among other authors. Quantity risk (or volumetric risk) is driven by different conditions, such as the economic cycle, the availability of fuels, hydrologic inflows, or climate. These conditions also affect price; hence, generated quantity and price tend to be correlated. In this paper, we examine different conditions to hedge market risks associated with spot price and energy generation in a multivariate environment.

A key factor when structuring hedging portfolios is the price at which forwards are traded. Due to the limitations regarding electricity storage for long periods (i.e., months or years), the cost-of-carry model (commonly used for other commodities) is not applicable in this case. Therefore, market agents set the forward price based on their expectations and the risks they assume, which gives rise to the Forward Risk Premium (FRP).

This risk premium − defined as the discrepancy between the spot price and the forward price − has been studied and explained by Longstaff and Wang (2004); and Xiao, Colwell, and Ramaprasad (2014) for the Pennsylvania–New Jersey–Maryland (PJM) electricity market; Botterud, Kristiansen, and Ilic (2010) for the Nord Pool; Pantoja (2012) for the Colombian electricity market; Redl and Bunn (2013) for the European Electricity Exchange (EEX); and Bunn and Chen (2013) for the British electricity market (Ruddell, Downward, & Philpott, 2018). The incorporation of a FRP immediately leads to a difference between the forward price and the spot price expectations. Thus, the results reported by Nässäkkälä and Keppo

(2005) to define the optimal number of forward sales cannot be extended to markets where such premium is included.

Regarding the behavior of uncertainty sources, the studies in the literature typically address the hedging problem in electricity markets by assuming normality either on the variables or on their logarithm. Although this is a common assumption – used by Nässäkkälä and Keppo (2005), Oum and Oren (2010), and Trespalacios, Rendón and Pantoja (2012), among other authors – to properly select the number of forward contracts to hedge the risk associated with transactions in electricity markets, it presents limitations to deal with problems that involve cases of skewness and kurtosis.

In this regard, Trespalacios, Cortés, and Perote (2020) indicate that some variables in electricity markets exhibit conditions of skewness and kurtosis and higher-order moments that are not adequately represented only by means of normal distributions; they demonstrate that Semi-NonParametric (SNP) distributions allow a better fit to hydrologic inflows, spot price, and even demand for electricity data. Consequently, these authors recommend to manage risks in this type of market from a flexible SNP approach, where normality is a particular case, and to not only consider normal distributions to describe uncertainty.

Likewise, Brunner A. D. (1992) shows that SNP distributions serve to treat historical variables featuring skewness and heavy tails. Gallant, Rossi, and Tauchen (1992) use SNP modeling to describe the comovements of price and volume in the stock market of the United States (US), and Mauleon and Perote (2000) also employ the SNP distribution to model returns in the US and United Kingdom (UK) stock markets. Other works that adopt SNP approaches to expand series beyond the traditional normal or lognormal distributions are those by Cortés, Perote, and Mora (2016), who measure the productivity of researchers worldwide, and Cortés, Mora-Valencia, and Perote (2017), who estimate the size distribution of US firms. In this study, we go a step further by considering the uncertain components of price and energy generation of each electricity generator under study to follow a joint SNP distribution. Perote (2004) described this type of distribution and explained how it is estimated and, more recently, Ñíguez and Perote (2016) and Del Brio, Mora-Valencia and Perote. (2017) applied related denstities to forecast financial variables. However, as far as we know, this is the first attempt to model electricity markets in a multivariate SNP framework.

In this paper, we propose a static hedging strategy for electricity generators that participate in a competitive market where hedging is carried out through forward contracts that include a risk premium in their valuation. We consider the spot price and energy generation variables to follow a bivariate SNP distribution in terms of the Gram-Charlier expansion. This distribution allows us to not only model the mean, the variance, and their correlation but also the skewness, the kurtosis, and higher-order moments. Moreover, we employ Monte Carlo simulation to analyze the effect of three risk indicators (standard deviation, Value-at-Risk [VaR], and Conditional VaR [CVaR]) on net profit from energy sales, using data from the Colombian electricity market as the case study.

The main contribution of this study to the analysis of electricity markets is the structuring of an energy portfolio that does not impose the assumption of normality in both price and energy generation. The SNP approach allows a natural and flexible procedure to model departures from normality as well as the dependence structure. From a statistical point of view, however, its usefulness lies in the simulation of a SNP probability distribution by means of the Monte Carlo method.

The results show that the optimal quantity of energy to be sold under long-term contracts is dependent not only on the conditions of spot price and quantity uncertainty but also on the way market agents weigh the assumed risk levels. Therefore, to reduce the risk levels faced by generators, such optimal quantity will depend on the conditions of price and energy generation uncertainty explained by variance, skewness, kurtosis, and higher-order moments. Furthermore, the number of forward sales is determined by the correlation between price and energy generation, as well as by the FRP.

The rest of this paper is structured as follows. Section 2 introduces the mathematical model and the methodology implemented to solve the problem addressed here. Section 3 describes the data used in the case study. Section 4 discusses the results. Finally, Section 5 draws some conclusions.

2. Model and methodology

The cost of the energy purchases made by Local Distribution Companies (LDCs) and hedged using forward contracts can be calculated as $C = P(Q - q) + F \cdot q$, where P is the energy spot price; Q, the total amount of energy consumed by the LDC; and q, the amount of energy bought under the forward contract at price F – Woo et al. (2004). This suggests that when LDCs do not make such purchases under forward contracts, their value is simply defined as $C = P \cdot Q$, exposing themselves to variations in the spot price and volume of energy consumed. This matter is addressed by Oum and Oren (2010) to analyze the earnings obtained by LDCs and different types of hedging strategies. In this study, we combine the proposals by Woo et al. (2004) and Oum and Oren (2010) to estimate the profit obtained by electricity generators from their energy sales.

Unlike LDCs, electricity generators tend to hedge their sales of energy which they may supply through self-generation or spot purchases. Regardless of the activity they perform in the supply chain, market agents focus on defining strategies to achieve the best possible financial results that guarantee the continuity of the companies over time. Therefore, their decisions regarding energy sale or purchase must always aim to maximize their expected value, π^i, under adequate risk conditions. Achieving this is usually seen as an optimization problem applied to utility functions.

In this regard, Nässäkkälä and Keppo (2005) propose the hedging strategy of electricity generator i as an optimization problem with a mean–variance utility function over its net energy sales (I). This problem is represented by Equation (1) and depends on the level of risk aversion λ^i. The mean–variance utility function exhibits limitations when the forward price does not match the expected spot price, i.e., when there is a FRP. Trespalacios et al. (2012) state that when there is a risk premium in electricity markets, the optimization of the mean–variance utility function is subject to the decision makers' level of risk aversion and, thus, specific recommendations on the hedging strategy to be implemented cannot be made.

$$max \quad E[I^i] - \lambda^i \cdot E\left[\left(I^i - E[I^i]\right)^2\right] \tag{1}$$

Another decision criterion that does allow us to find concrete solutions to the problem addressed in this paper is worst-case maximization. In this paper, we propose using the VaR and the CVaR indicators, which do not require symmetry for the data for their interpretation and are not dependent on unobservable variables, such as the level of risk aversion.

Like Oum and Oren (2010), we consider two timings in our analysis: the time at which the hedging decision is made ($t = t_0$) and the time of maturity of the forward contract, i.e., the period for which the hedging strategy is being designed ($t = T$). In other words, electricity generator *i* faces uncertainty over its net sales at time T; therefore, it decides to sell long-term electricity contracts beforehand starting from time ($t = t_0$). If such generator participated in an electricity market whose spot price and energy generation derived from a multivariate SNP distribution, how many contracts should it trade to hedge its risk?

3.1. Spot price and energy generation

The uncertainty faced by electricity generators over their financial results is subject to the dynamics of the spot price and their energy generation. In this study, we propose modeling these two variables by means of multivariate SNP functions. Such functions account for the modeling of skewness, kurtosis, and higher-order moments of each random variable, as well as the correlation structure. Below we describe the portfolio multivariate SNP distribution, which generalizes the multivariate normal in terms of Gram-Charlier (Type A) series for every portfolio variable.

Let us assume that Z_t is a vector that contains *J* variables distributed with zero mean and multivariate SNP distribution, then its joint probability density function (pdf) F_Z is written, as proposed by Perote (2004), as follows:

$$F_Z(Z_t) = G_Z(Z_t) + \left[\prod_{j=1}^{J} g_{z_j}(z_{jt})\right]\left[\sum_{j=1}^{J} k_{z_j}(v_{jt})\right] \; ; \; -\infty < z_{jt} < \infty \tag{2}$$

where $G_Z(Z_t)$ is a multivariate normal pdf with zero mean, covariance matrix Σ – with general element $\{\sigma_{ij}\}$ – and marginal pdfs represented by $g_{z_j}(z_{jt})$ – i.e. $z_{jt} \sim N(0, \sigma_j^2)$, $\sigma_j^2 = \sigma_{jj}$; and $k_{z_j}(v_{jt})$ is a linear combination of the first M_j terms of the Gram-Charlier series Hermite polynomial in variable $v_{jt} = z_{jt}/\sigma_j$, as shown in Equation (3). The terms of these expansions, $H_m(v_{jt})$, is the so-called Hermite Polynomials (HP) or order m – see equation (5), which is weighted by parameter d_{jm} capturing the raw moment of order m for the marginal variable j.

$$k_{z_j}(z_{jt}) = \sum_{m=2}^{M_j} d_{jm} H_m(z_{jt}) \quad (3)$$

Hence, the marginal pdf of z_{jt} is:

$$f_{z_j}(z_{jt}) = \frac{1}{\sigma_j\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{z_{jt}}{\sigma_j}\right)^2\right\}\left\{1 + \sum_{m=2}^{M_j} d_{jm} H_m\left(\frac{z_{jt}}{\sigma_j}\right)\right\} \quad (4)$$

It can be easily proved that the even (odd) moment of order *r* of variable z_{jt} depends linearly on d_{jm} for all $j \leq r$ and j even (odd). Particularly, mean and variance are $E[z_{jt}] = 0$ and $E[z_{jt}^2] = (1 + 2d_{j2})\sigma_j^2$, respectively, and the covariance between the variables z_{jt} and z_{it} is defined by the corresponding entry in matrix Σ, i.e. $E[z_{jt}z_{it}] = \sigma_{ji}$. Furthermore, if $d_{j3} > 0$ ($d_{j3} < 0$) then the *j*-th marginal pdf features positive (negative) skewness, when $d_{j4} > 0$ the marginal pdf of z_{jt} exhibits leptokurtosis and the higher-order even parameters account for extreme values. A further discussion on the interpretation of such parameters can be found in the studies by Mauleon and Perote (2000); Cortés et al. (2017); and Trespalacios et al. (2020) for the case of electricity markets.

The HP of order *m*, $H_m(v)$, is defined in terms of the *m*-th derivative of the standard normal pdf and thus can be calculated by solving Equation (5).

$$H_m(v) = \frac{(-1)^m}{g(v)} \cdot \frac{d^m g(v)}{dv^m} \quad (5)$$

$$g(v) = \frac{1}{2\pi} e^{-\frac{v^2}{2}}$$

Hence, the first ten HPs used to represent the standardized random variable v are:

$$H_0(v) = 1 \quad (6)$$

$$H_1(v) = v$$

$$H_3(v) = v^2 - 1$$

$$H_3(v) = v^3 - 3v$$

$$H_4(v) = v^4 - 6v^2 + 3$$

$$H_5(y) = v^5 - 10v^3 + 15v$$

$$H_6(v) = v^6 - 15v^4 + 45v^2 - 15$$

$$H_7(v) = v^7 - 21v^5 + 105v^3 - 105v$$

$$H_8(v) = v^8 - 28v^6 + 210v^4 - 420v^2 + 105$$

$$H_9(v) = v^9 - 36v^7 + 378v^5 - 1260v^3 + 945v$$

$$H_{10}(v) = v^{10} - 45v^8 + 630v^6 - 3150v^4 + 4725v^2 - 945$$

It is noteworthy that the HPs form an orthonormal basis, since

$$\int H_s(v) H_m(v) g(v)\, dz_t = 0 \;\; \forall s \neq m. \tag{7}$$

According to the theory above, we proceed to define the joint SNP pdf for spot price and energy generation variables. Let p_T be the natural logarithm of the spot price P_T; and q_T^i, that of the energy generation by generator i, Q_T^i. Since p_T and q_T^i are represented by a bivariate SNP distribution written as $\left(p_T, q_T^i\right) \sim SNP$, then their joint pdf is given by:

$$F_Z\left(p_T, q_T^i\right) = G_Z\left(p_T, q_T^i\right) + g_p(p_T) \cdot g_{q^i}\left(q_T^i\right) \cdot \left\{k_p(p_T) + k_{q^i}(q_T^i)\right\} \tag{8}$$

Thus, marginal distributions can be written as follows – see proof in Appendix 2:

$$f_P(p_T) = g_p(p_T) + g_p(p_T) \cdot \left\{k_p(p_T)\right\} \tag{9}$$

$$f_{q^i}\left(q_T^i\right) = g_{q^i}\left(q_T^i\right) + g_{q^i}\left(q_T^i\right) \cdot \left\{k_{q^i}\left(q_T^i\right)\right\}$$

3.2. Forward price

A long-term electricity contract (or forward contract) is a financial derivative whose underlying asset is the spot price of electricity. In this contract, a seller (short position) and a buyer (long position) agree to trade electricity at a specific future date. Hull (2009) describes this type of contract as a spot contract, which is an agreement to buy or sell a commodity for immediate delivery. The purpose of forward contracts is to stabilize the price of the energy to be traded in the future by eliminating the effects of the short-term variations in the variables involved in the market. This, in turn, increases agents' certainty levels over future cash flow, uncover future expectations regarding the behavior of market fundamentals, and reach financial closure for investment projects that are capital intensive.

When it comes to defining a project finance model, for instance, financiers (either banks or shareholders) demand that the desired Internal Rate of Return -IRR and Net Present Value -NPV be hedged using forward contracts with a supply period equal to that required for capital recovery. In addition, shareholders who have invested in operating power generation companies request adequate levels of sustainability, liquidity, and value generation from these companies' management; in this case, long-term contracts also become useful tools to meet said requirements. Therefore, according to such examples, the purpose for which these derivatives are used depends on the strategy defined by decision-makers in terms of supply period, contracting time, requested counterparty, and minimum or maximum transaction prices.

If an agent purchased a forward contract at time t_o to receive an amount of electricity at maturity time T at price $F_T^{t_o}$, it would receive such agreed amount at maturity at the agreed price $F_T^{t_o}$. Since such electricity received at time T is valued at the spot price P_T, its net income will be given by the difference between the spot price and the agreed price stated in the contract, as follows:

$$Long\ position's\ profit = P_T - F_T^{t_o} \tag{10}$$

As this is a zero-sum game, the seller's profit will be equal to that of the buyer but with opposite sign. Thus, the seller will be paid at the agreed price in exchange of delivering the agreed amount of electricity, which will be valued at the spot price at maturity. Due to limitations regarding electricity storage, it is assumed that the only way for the seller to deliver the agreed amount of electricity at time T is to procure it from the spot market at that very moment.

$$Short\ position's\ profit = F_T^{t_o} - P_T \tag{11}$$

When defining the fair forward price, market agents face limitations because the tools required for the valuation of electricity considerably differ from those used for common financial assets. In this regard, Bessebender and Lemmon (2002) indicate that traditional cost-based valuation models are not applicable in electricity markets due to technical limitations regarding large-scale long-term electricity storage. For this reason, the models for pricing electricity forward contracts depend on expectations about the future performance of the spot price and the risk levels assumed by market agents due to variations in demand, the occurrence of climate phenomena, risks inherent to primary fuels, reservoir levels mainly for hydraulic systems, or even market power. The assessment of these assumed risk levels is reflected in the price of the traded contracts as demonstrated by Longstaff and Wang (2004); Botterud, Kristiansen, and Ilic (2010); Pantoja (2012); Redl and Bunn (2013); Bunn and Chen (2013); Xiao, Colwell, and Ramaprasad (2014); Ruddell, Downward, and Philpott (2018), who studied the FRP in electricity markets.

The spot price P_T and the forward price $F_T^{t_o}$ denote the valuation of the same commodity: electricity supplied. However, defining the forward price in advance adds risk conditions that should be considered by agents when making the transactions. In other words, the price of an electricity forward contract bought at time t_0 and with a maturity T will be different from the expected spot price. This difference is known as the Forward Risk Premium (FRP), which is represented by Equation (11).

$$FRP_T^{t_0} = E_{t_0}(P_T) - F_T^{t_o} \tag{12}$$

The sign of the FRP indicates who the agent hedging the risk and paying for it is: when the sign of the FRP is positive, the seller is the one paying for the hedging, while when it is negative, the buyer is the one paying for it. Moreover, according to Pantoja (2012), a positive FRP value denotes a normal backwardation (i.e., the forward price is below the expected spot price) whereas a negative one indicates a contango (i.e., the forward price is above the expected spot price). According to Bessebender and Lemmon (2002), this risk premium incorporated in forward prices may be associated with the variance and skewness of the spot price of electricity.

This FRP may describe the expected spot price or the forward price based on the following relationships:

$$E_{t_0}(P_T) = F_T^{t_o} + FRP_T^{t_0} \quad (13)$$

$$F_T^{t_o} = E_{t_0}(P_T) - FRP_T^{t_0}$$

where $FRP_T^{t_0}$ is the FRP for T measured at time t_0. Due to the nature of forward contracts, convergence must occur at maturity, i.e., a forward price agreed in t_0 which matures at T should be equal to the spot price, as stated by Hull (2009) when explaining the concept of basis risk. Trespalacios et al. (2012) show that, as maturity approaches, the FRP becomes null in electricity markets, as follows:

$$\lim_{t_0 \to T} FRP_T^{t_0} = 0 \quad (14)$$

$$\Rightarrow F_T^T = P_T$$

3.3. Pay-off function

Assuming that an electricity generator that has produced an amount of electricity Q_T^i at time T sells all this electricity at the market spot price P_T, then its net sales I_T^i are given by the product of the spot price and its energy generation, as follows:

$$I_T^i = P_T \cdot Q_T^i \quad (15)$$

Due to their nature, the spot price and the energy generation are considered to be random variables, and, thus, their values can only be known ex post. At time $t = t_0$, their values are unknown, which means that the generator is taking a risk due to variations in price and quantity. In this paper, we consider the relationship $P_T \cdot Q_T^i$ to be the initial risk condition.

When it comes to energy sales, function I_T^i does not represent the generator's profit because production costs are ignored. We do not take into account such costs in this analysis because of three reasons: (i) The problem to be solved involves choosing the hedging instruments to be used and its values do not depend on production costs. (ii) If power generation costs were incurred, these should be reflected in the generation decisions, so variable Q_T^i would include the effects of the production costs and their relationships with the spot price, P_T. (iii) Some power generation technologies (e.g., a hydroelectric power plant with a dam) have an opportunity cost that can be subject to conditions that are beyond the scope of this work. In any case, variable Q_T^i must capture the opportunity cost.

If, to hedge the assumed risk, the generator decides to implement certain strategy (or transaction) j at time t_0 whose pay-off function is $\phi_T^{ij}(Q_T^i, P_T|\theta_{t_0}^{ij})$, its net energy sales will be given by:

$$I_T^i = P_T \cdot Q_T^i + \phi_T^{ij}(Q_T^i, P_T|\theta_{t_0}^{ij}) \tag{16}$$

where $\theta_{t_0}^{ij}$ is the vector of the parameters necessary to specify strategy j implemented by electricity generator i at the initial time t_0. According to Boroumandet al. (2015), if the hedging strategy corresponds to the sale of fixed-price (forward) contracts, its pay-off function ϕ_T^{ij} will be equal to $C_T^{t_o} \cdot \left(F_T^{t_o} - P_T\right)$. This function depends on the amount of electricity sold under the forward contract, $C_T^{t_0}$, and the difference between the fixed price of the contract, $F_T^{t_o}$, and the spot price, P_T. Hence, the net energy sales of a generator that hedges its risk through forward sales is given by:

$$I_T^i = P_T \cdot Q_T^i + C_T^{t_0} \cdot (F_T^{t_o} - P_T) \tag{17}$$

The variables $C_T^{t_0}$ and $F_T^{t_o}$ denote the energy generation and the forward price, respectively, that were agreed at time t_0 in the contract with a maturity T. If we also assume that, at time t_o, the conditional expected value of the energy generation is denoted by $E_{t_0}(Q_T)$, the previous expression can be written relatively to the expected generation unit, as follows:

$$\frac{I_T^i}{E_{t_0}(Q_T)} = P_T \cdot \frac{Q_T^i}{E_{t_0}(Q_T)} + \frac{C_T^{t_0}}{E_{t_0}(Q_T)} \cdot (F_T^{t_o} - P_T) \tag{18}$$

$$\Rightarrow \pi_T^i = P_T \cdot {Q^*}_T^i + \eta_T^{t_0} \cdot (F_T^{t_o} - P_T)$$

where ${Q^*}_T^i$ will be the energy generation with respect to the expected value. The previous equation can also be rewritten by grouping the random variables the first term $\left({Q^*}_T^i - \eta_T^{t_0}\right) \cdot P_T$; and the deterministic ones in the second term $\eta_T^{t_0} \cdot F_T^{t_0}$, as shown below:

$$\pi_T^i = \left(q_T^i - \eta_T^{t_0}\right) \cdot P_T + \eta_T^{t_0} \cdot F_T^{t_0} \tag{19}$$

3.4. Risk indicators

The mean-variance utility function creates a dependency between the optimal contracting levels and the FRP, as shown by Trespalacios et al. (2012), not allowing decision makers to have a specific proposal for action. Other decision criteria (e.g., VaR or CVaR maximization), conversely, allow us to find contracting levels that do not depend on subjectivity and may be assigned confidence levels. In this study, the VaR and CVaR were estimated at a 95% confidence level and measured in monetary units, as well as the standard deviation (another risk indicator traditionally used to describe the behavior of net income from energy sales).

The standard deviation measures the dispersion of the data from the mean. Thus, if the net income from the energy sales of agent i at time T is given by I_T^i, its standard deviation is denoted by:

$$Std[I_T^i] = \sqrt{E\left[{I_T^i}^2\right] - \left(E\left[I_T^i\right]\right)^2} \tag{20}$$

The VaR captures the lowest net income that would be expected at a desired confidence level. In short, it can be calculated as the 5th percentile of the I_T^i value, which can be written as:

$$5\% = \int_{-\infty}^{VaR_{5\%}} f\left(I_T^i\right) \cdot dI_T^i \tag{21}$$

where $f\left(I_T^i\right)$ is the probability density function of the net income from energy sales. When Monte Carlo simulation is used to calculate the risk indicators, the VaR requires more simulations to be accurately estimated than the standard deviation with the CVaR. This is further illustrated in the results section when the VaR and the CVaR are compared at different forward contracting levels.

The CVaR also determines the lowest profit that would be expected, but given that the VaR level has been exceeded. Therefore, it does not consider the highest profit level of the 5% of the lowest profits, but it averages all the net income levels that are below the VaR. One way to understand this risk indicator is to associate it with the average of the lowest possible profits and thus compute it as in equation (22).

$$CVaR_{5\%} = \int_{-\infty}^{VaR_{5\%}} I_T^i \cdot f\left(I_T^i\right) \cdot dI_T^i \tag{22}$$

Regarding decision criteria, the structuring of optimal hedge portfolio for risk averse decision-makers who face price and quantity uncertainty should include the search for contracting levels that maximize the VaR and the CVaR or minimize the standard deviation of their net income from energy sales.

3.5. Methodology

To solve the problem addressed in this paper, we propose a stepwise procedure in three stages: (i) estimation of the deterministic component parameters, (ii) estimation of the random (bivariate) component parameters, and (iii) sensitivity analysis and simulation of electricity generator's portfolios.

Estimating the deterministic component parameters (stage 1) allow a decomposition of the time series under study in order to determine the parameters of the random component for each pair of spot price and energy generation of each electricity generator (stage 2). Finally, we conducted Monte Carlo simulations and performed sensitivity analyses of the generator's portfolio in order to assess the risk levels using different indicators and optimize the hedging strategy to be implemented.

(i) Parameter estimation of spot price and energy generation

Regarding the spot price P_t, we consider an exponential model represented by a stochastic process with a deterministic long-term mean and mean reversion, as described by equation (23). This structure was developed based on the models proposed by Lucia and Schwartz (2002) and Geman and Roncoroni (2003), which have been applied to the case of Colombia by Trespalacios et al. (2012) and Uribe and Trespalacios (2014).

$$p_t = \ln(P_t) \tag{23}$$

$$p_t = \mu_p(t) + x_t^p$$

$$x_t^p = \phi_p \cdot x_{t-1}^p + \epsilon_t^p$$

where $\mu_p(t) = \beta_{0p} + \beta_{1p} \cdot t$ is a deterministic trend and x_t^p a stochastic autoregressive of order 1 component, AR(1), which is assumed to be stationary, i.e. $|\phi_p| < 1$, and ϵ_t^p being a random noise, thus the long-term variance of x_t^p is given by $1/(1-\phi_p^2)$ times the variance of ϵ_t^p.

Similarly, as for the energy generation of agent *i,* we have:

$$q_t^i = \ln(q_t) \tag{24}$$

$$q_t^i = \mu_{q^i}(t) + x_t^{q^i}$$

$$x_t^{q^i} = \phi_{q^i} \cdot x_{t-1}^{q^i} + \epsilon_t^{q^i}$$

where $\mu_{q^i}(t) = \beta_{0q} + \beta_{1q} \cdot t$ is the deterministic trend and $x_t^{q^i}$ the AR(1) random component of the energy generation of agent *i*, with $|\phi_{q^i}| < 1$ and $\epsilon_t^{q^i}$ being a random noise.

As the stochastic variables $\epsilon_t^{q^i}$ and ϵ_t^p are clearly correlated, the random component of the model, i.e. the vector $\boldsymbol{\epsilon_t^i} = \left(\epsilon_t^p, \epsilon_t^{q^i}\right)$, must be jointly modeled to properly account for the uncertainty of the spot price and the energy generation of agent *i*.

The set of equations used to estimate the uncertain components of the spot price can also be written in matrix form, with $\boldsymbol{z_t^i} = \left(p_T, q_T^i\right)$, as follows:

$$\boldsymbol{z_t^i} = \boldsymbol{\mu^i}(t) + \boldsymbol{x_t^i} \tag{25}$$

$$\boldsymbol{x_t^i} = \boldsymbol{\phi^i x_{t-1}^i} + \boldsymbol{\epsilon_t^i}$$

where

$$\boldsymbol{\mu^i}(t) = \begin{pmatrix} \mu_p(t) \\ \mu_{q^i}(t) \end{pmatrix},\ \boldsymbol{x_t^i} = \begin{pmatrix} x_t^p \\ x_t^{q^i} \end{pmatrix},\ \boldsymbol{\epsilon_t^i} = \begin{pmatrix} \epsilon_t^p \\ \epsilon_t^{q^i} \end{pmatrix} \text{ and } \boldsymbol{\phi^i} = \begin{pmatrix} \phi_p & 0 \\ 0 & \phi_{q^i} \end{pmatrix}.$$

(ii) Bivariate distribution estimation for price and energy generation

The vector $\boldsymbol{\epsilon_t^i} = \left(\epsilon_t^p, \epsilon_t^{q^i}\right)'$, is assumed to follow a the bivariate SNP distribution defined by the pdf in equation (8). This procedure is very flexible to generalize the normal distribution by considering the impact of high order moments in a natural way, but at the cost of a more complex parametric structure. However, Del Brio, Ñíguez and Perote (2011) proved that the model can be consistently estimated in two steps: Firstly, (Quasi) Maximum Likelihood (QML) estimation of every mean-variance process independently – equations (23) and (24) – and under normality; Secondly, joint Maximum Likelihood (ML) of the rest of the parameters of the bivariate pdf, $\left\{d_{im}^j\right\}_{m=2}^{M_i}$ for $j = p, q^i$, which we denote by the vector $\boldsymbol{\theta}^i = (\boldsymbol{\theta}_p \quad \boldsymbol{\theta}_{q^i})'$, as well as the correlation between both variables, denoted by ρ_{pq^i}. This second step considers the series standardized by using the mean-variance estimates in the previous step and its loglikelihood function to be maximized for generator *i* and given a sample of size *T* is shown in Equation (26).

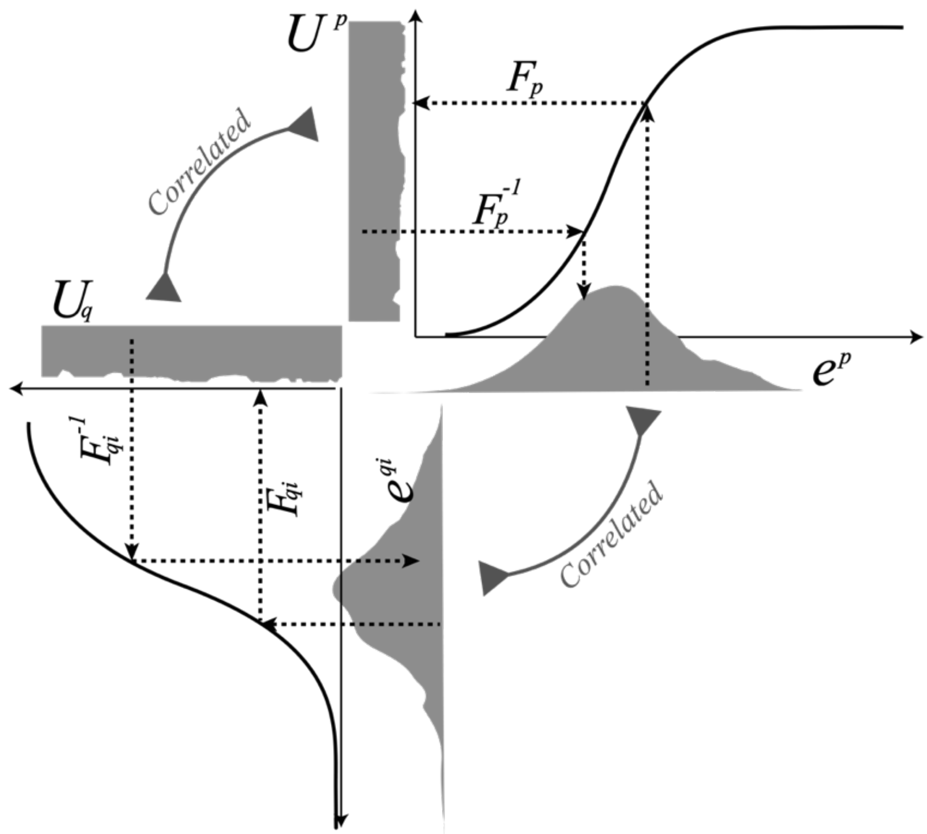
$$l(\boldsymbol{\theta}^i) = \log\left[L(\boldsymbol{\theta}^i)\right] = \sum_{t=1}^{T} log\left[F_\epsilon(\boldsymbol{\epsilon_t^i}|\boldsymbol{\theta}^i)\right] \tag{26}$$

(iii) Monte Carlo simulation of bivariate SNP distribution

Once the model is estimated, we analyze the sensitivity of the results to the parametric uncertainty and its effects on the electricity markets hedging under the SNP distributional assumption for the random component. For this purpose, we perform Monte Carlo simulations, which require the extraction of (correlated) random numbers from the bivariate SNP distribution of spot price and energy generation ($\boldsymbol{\epsilon_t^i}$). According to Fusai and Roncoroni (2008) No truly random number can be generated by a computer code, as long as it can only perform sequences of deterministic operations. From uniform pseudo-random samples, it can be obtained numbers from any other kind of density distributions applying a specific transformation. This transformation can be performed by taking the inverse cumulative distribution function to a sample of uniformed pseudo-random number. They propose a two-step algorithm to generate a sample from a number which cumulative function distribution is given by F: (i) Simulate $U \sim uniform[0,1]$ and then (ii) Return $F^{-1}(U)$. In this work, we need to simulate not only one sample but two correlated samples in order to recreate the bivariate SNP.

A straightforward method to simulate the SNP distribution series can be obtained by implementing the methodology proposed by Meucci (2007), valid for any type of joint distribution. Intuitively, this methodology involves filtering out the joint distribution through its marginal density functions to obtain uniformly distributed functions with a dependence structure.

Figure 1 Monte Carlo simulation of bivariate distribution



This figure illustrates the method used to simulate random numbers based on the proposal of Meucci (2007).

Based on Meucci's methodology, each component of vector $\boldsymbol{\epsilon_t}$ can be standardized, using the cumulative functions $F_p(\epsilon_t^p)$ and $F_{q^i}(\epsilon_t^{q^i})$, towards a space where each variable contains a uniform probability distribution, as follows:

$$\boldsymbol{U} \equiv \begin{pmatrix} U_p \\ U_g^i \end{pmatrix} \equiv \begin{pmatrix} F_p(\epsilon_t^p) \\ F_{q^i}(\epsilon_t^{q^i}) \end{pmatrix} \quad (27)$$

Therefore, to generate a random number from a joint SNP distribution whose cumulative marginal functions are F_p and F_{q^i}, it will suffice, for the purposes of this study, to evaluate the quantile function Q(.) of the random numbers that maintain the distribution of $\boldsymbol{U} \sim \mathrm{U}[(0,1)]$ and its correlations. Figure 1 illustrates the set of proposed transformations.

Once the parameters of the random components were calibrated, we implemented this simulation methodology to perform the sensitivity analyses (presented in the following sections) of the portfolio hedging problem.

3. Data description

We use information for the electricity spot price from the Colombian electricity market and the main electricity generators, from January 2000 to December 2018. The Spot Price series corresponds to the average price of the monthly energy traded in the Colombian energy market, measured in COP (Colombian pesos) per kWh (kilowatt-hour) (COP/kWh); meanwhile the generation correspond to the total energy produced monthly for a generator, and it is measured in GWh (10^6 kWh).

Table 1 shows the descriptive statistics of the series employed in this study. The generators considered are EPM (EPMG), ISAGEN (ISGG), AES Chivor (CHVG), and Enel (ENDG) – which predominantly manage hydraulic resources.

The Spot Price series exhibits the highest value of skewness, while all the Energy Generation series, except that of EPMG, show a positive skewness. Regarding kurtosis, Spot Price again exhibits the highest value; the kurtosis of CHVG and ENDG is above that of a normal distribution, while that of EPMG and ISGG is below 3. For the sake of comparisons, the series in logarithms and relatively to the mean are also displayed.

Table 1 Descriptive statistics of Spot Price and Energy Generation of each generator

Type	Generator	Unit	Mean	SD	Skewness	Kurtosis	Percentile				
							5th	25th	50th	75th	95th
Series without transformations											
Spot	Spot	COP/kWh	124.1	125.8	4.38	27.3	40.9	64.6	85.5	145.8	249.2
Energy Generation	EPMG	GWh	1,038	209	-0.25	2.26	686.2	866	1,068	1,198	1,359
	ISGG	GWh	798	258	0.34	2.92	391.7	620	794	949	1,267
	CHVG	GWh	337	130	0.80	3.22	160.3	246	319	409	616
	ENDG (1)	GWh	1,110	169	0.38	3.57	882.4	1,002	1,093	1,190	1,468

Natural logarithm of the series (2)

Spot	Spot	log	4.58	0.62	1.01	4.51	3.71	4.17	4.45	4.98	5.52
Energy Generation	EPMG	log	6.92	0.22	-0.68	2.91	6.53	6.76	6.97	7.09	7.21
	ISGG	log	6.63	0.35	-0.55	3.06	5.97	6.43	6.68	6.86	7.14
	CHVG	log	5.75	0.38	-0.09	2.60	5.08	5.51	5.76	6.01	6.42
	ENDG	log	7.00	0.15	-0.22	3.86	6.78	6.91	7.00	7.08	7.29

Relative to the mean. Series with transformation: x / E(x) (3)

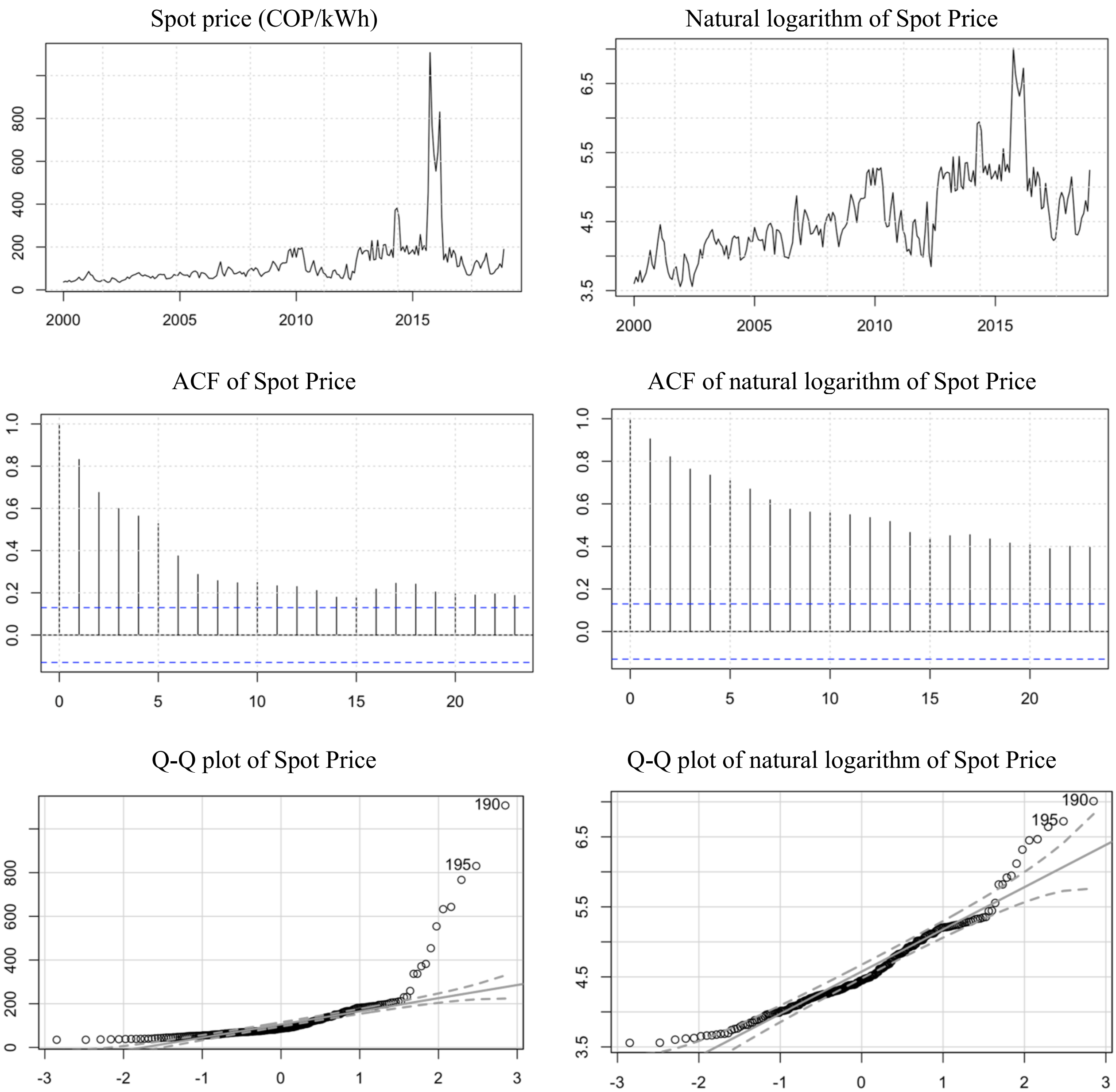
Spot	Spot	pu (4)	1.00	1.01	4.38	27.25	0.33	0.52	0.69	1.17	2.01
Energy Generation	EPMG	pu	1.00	0.20	-0.25	2.26	0.66	0.83	1.03	1.15	1.31
	ISGG	pu	1.00	0.32	0.34	2.92	0.49	0.78	0.99	1.19	1.59
	CHVG	pu	1.00	0.38	0.80	3.22	0.48	0.73	0.94	1.21	1.83
	ENDG	pu	1.00	0.15	0.38	3.57	0.79	0.90	0.98	1.07	1.32

This table contains the descriptive statistics of the time series which were observed at a monthly frequency from January 2000 to December 2018. The series used in this study include the spot price in the Colombian electricity market and the energy generation of the four major electricity generators in the country: EPM (EPMG), ISAGEN (ISGG), AES Chivor (CHVG), and Enel (ENDG). (1) This series contains information since September 2007 to December 2018. (2) The natural logarithm of each measure in the series is calculated. (3) Each measure in the series is divided by the series mean. (4) pu stands for per unit; the values are relative to the mean of each series; hence, the mean of all variables here is equal to one.

Figure 2 illustrates the spot price series, as well as its autocorrelogram, Q-Q plot, and natural logarithm. According to this figure, Spot Price exhibits a trend and jumps; its highest jump occurs after 2015 due to the occurrence of the El Niño-Southern Oscillation (ENSO) together with the shortage of natural gas for power generation. This situation led the country to supply its power demand through power plants that operate with liquid fuel and whose production cost is relatively high.

In Figure 2 , the autocorrelograms of Spot Price and of its natural logarithm show the memory condition of this time series, which is similar to the structures analyzed by Lucia and Schwartz (2002), Geman and Roncoroni (2003), and Uribe and Trespalacios (2014), among other authors. According to the Q-Q plots, which assess the percentiles of the samples, the data move away from the normal distribution in both the left and right tails, a condition represented through SNP distributions by Trespalacios et al. (2020).

Figure 2 Spot Price time series



This figure shows the evolution of the Spot Price series in Colombia since 2000, its natural logarithm, and its Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) – which serve to identify its aucorrelation structure, as well as the Q-Q plots of the Spot Price and of its natural logarithm, which are compared to the normal distribution at a 95% confidence interval.

ISGG, followed by EPMG, and Spot Price exhibit the highest level of first-order autocorrelation, which means that short-term distortions remain for a longer time in these series than in the other ones. Since the Colombian electricity market is mainly composed by hydraulic sources, climate phenomena affect the behavior of the availability of resources for power generation and the spot price. Furthermore, since the electricity generators under analysis manage dams, their decisions in terms of power generation are subject to expectations regarding future spot prices. This interaction between supply and price leads to correlations between energy generation and spot price, which are measured later in this paper.

Another aspect to highlight is that when the natural logarithm of the series was calculated the direction of the skewness changed. Regarding dispersion of the series, estimating the percentiles relative to the mean allowed us to observe, for instance, that spot price is between 0.33 times and 2.01 times its mean at a 90% confidence level. After spot price, CHVG is the series with a wider 90% confidence interval, followed by ISGG, EMPG, and ENDG.

The autocorrelation levels of the time series (shown in Table 2) suggest that the data must be transformed using first-order autoregressive processes, as evidenced by the decline in the correlation levels when the first difference of the series is estimated (as explained in the methodology section).

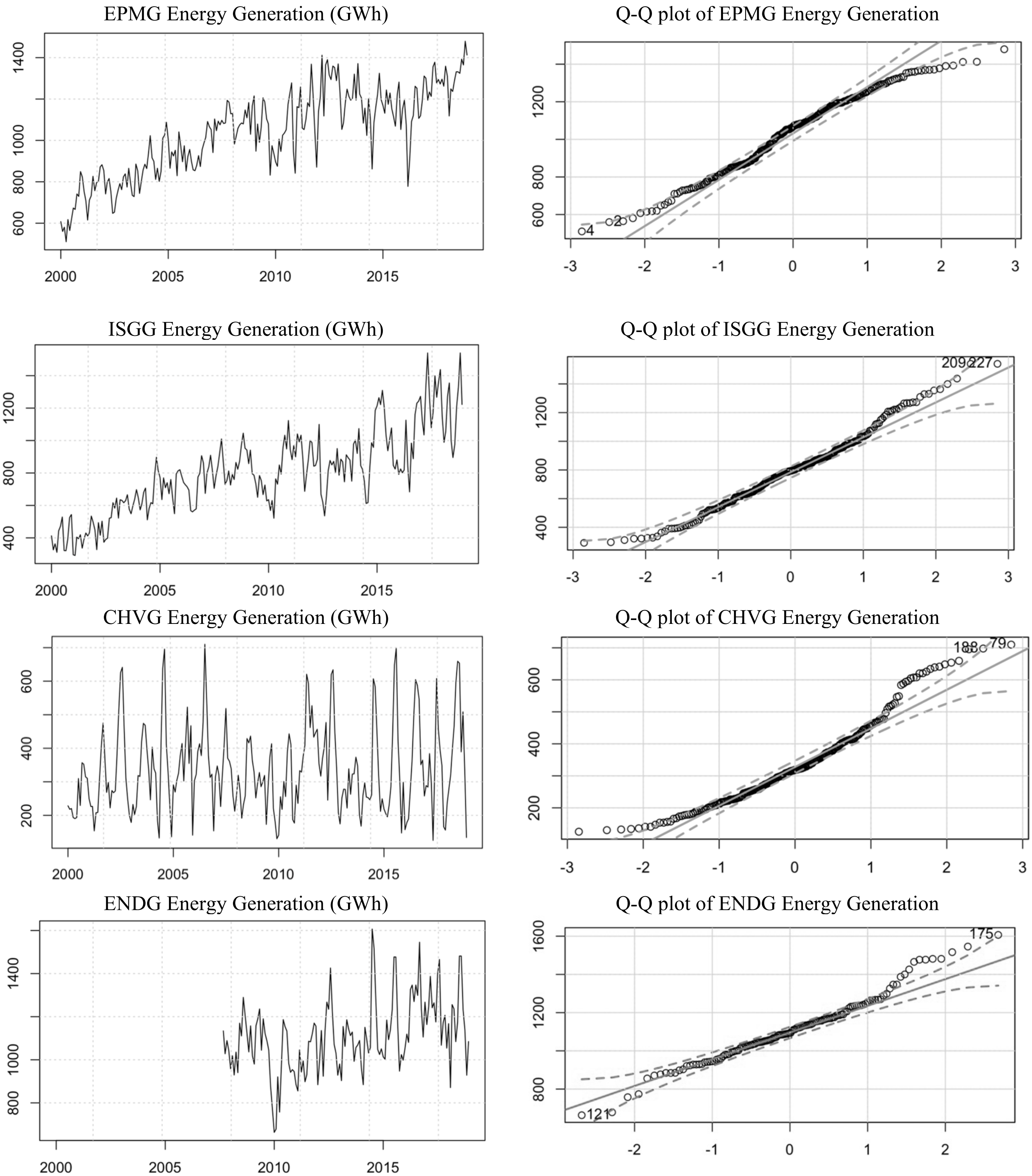
Table 2 Autocorrelation of the series

Lag	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
							Series								
Spot	1.0	0.83	0.68	0.6	0.56	0.53	0.37	0.29	0.26	0.25	0.25	0.23	0.23	0.21	0.18
EPMG	1.0	0.86	0.78	0.73	0.71	0.70	0.68	0.65	0.61	0.59	0.60	0.62	0.62	0.59	0.58
ISGG	1.0	0.88	0.77	0.71	0.69	0.69	0.66	0.62	0.57	0.54	0.55	0.57	0.58	0.52	0.47
CHVG	1.0	0.65	0.29	0.00	-0.16	-0.24	-0.26	-0.26	-0.20	-0.03	0.22	0.39	0.44	0.31	0.12
ENDG	1.0	0.61	0.37	0.23	0.12	0.04	-0.08	0.01	0.15	0.21	0.28	0.35	0.40	0.33	0.19
							First differences (Delta of x)								
Spot	1.0	-0.03	-0.24	-0.12	0.00	0.35	-0.20	-0.17	-0.06	-0.04	0.05	-0.03	0.05	0.04	-0.09
EPMG	1.0	-0.25	-0.11	-0.14	0.00	0.02	0.05	0.02	-0.05	-0.11	-0.01	0.03	0.13	-0.07	0.08
ISGG	1.0	-0.14	-0.17	-0.16	-0.04	0.12	0.03	-0.01	-0.05	-0.17	-0.05	0.05	0.27	-0.05	-0.11
CHVG	1.0	0.04	-0.09	-0.20	-0.11	-0.07	-0.04	-0.09	-0.18	-0.12	0.11	0.18	0.24	0.10	0.02
ENDG	1.0	-0.19	-0.13	-0.02	-0.06	0.06	-0.27	-0.07	0.11	-0.01	-0.01	0.04	0.15	0.08	-0.07

This table presents the autocorrelation of the series and that of their first differences.

The Energy Generation series of each electricity generator (shown in Figure 3 along with their Q-Q plot) corresponds to the sum of its hourly energy generation for each month, measured in GWh (i.e., 10^6 kWh). According to Figure 3, EPMG and ISGG show a trend, which is explained by the expansion processes of these electricity generators which have constructed new power plants in recent years. CHVG and ENDG exhibit higher stationarity because they have not had a substantial change in their assets.

Figure 3 Energy Generation time series of different electricity generators in Colombia



This figure shows the evolution of the energy generation (observed at a monthly frequency) of four electricity generators in Colombia, as well as the corresponding Q-Q plots, which are compared to the normal distribution.

4. Results and discussion

Our analysis is focused on three main tasks: (i) estimating the parameters of the model, both the deterministic and the random components, (ii) analyzing the sensitivity of the risk indicators to the simulation of the parameters, and (iii) determining the optimal hedging level for each electricity generator under study.

The results in this study show that the optimal level of electricity forward sales is subject to different factors: the conditions inherent to the uncertainty over each generator's energy generation and their correlation with the spot price, the risk assessment performed by the market – reflected in the FRP value; and the type of risk indicator that is expected to be managed.

Regarding the type of indicator, our findings indicate that when the VaR is maximized, it is more advisable to increase forward sales than when the CVaR is maximized. Likewise, when the FRP value is negative, it is better to maintain a higher forward contracting level. Only in the case of the standard deviation minimization, the FRP value does not affect the forward contracting level.

In the following subsections, we use the term Eta, (η), to refer to the forward contracting level. If Eta is 0.8, an electricity generator sells (short position), under forward contracts, a volume of electricity equal to 80% of its expected energy generation. As for parameter calibration, we use the notation set forth in the methodology section.

4.1 Parameter estimation of the Spot Price and Energy Generation series

The model for Spot Price and Energy Generation is estimated in two stages as explained in Section 3.5. Table 3 presents the parameter estimates of the model in equations (23) and (24) for the Spot Price and Energy Generation series. The model explains the natural logarithm of the series considering a deterministic trend and an AR(1) component, both of them are highly significant. Descriptive statistics for the residuals are also displayed to identify the initial parameters to be considered when estimating the bivariate SNP functions of the series. The Jarque-Bera test, whose null hypothesis is normality to the residuals, is also displayed revealing the non-normality of the series, except for ENDG.

Table 3 Estimates for the deterministic trend and AR(1) component of the (log)series

Parameter		Spot Price	Energy Generation			
			EPMG	ISGG	CHVG	ENDG (1)
Beta 0	Coeff	3.84	6.61	6.14	5.68	6.75
	t-value	65.32	416	235	113	134
	p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Beta 1 (2)	Coeff	0.0065	0.0027	0.0043	0.0006	0.0016
	t-value	14.58	22.40	21.73	1.49	5.16
	p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Phi 1 (3)	Coeff	0.829	0.652	0.673	0.608	0.544
	t-value	22.79	12.96	13.87	11.40	7.57

	p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Residuals (4)						
Statistics	Mean	0.00	0.00	0.00	0.00	0.00
	SD	0.25	0.09	0.15	0.30	0.12
	Skewness	0.51	-0.60	-0.65	-0.43	0.04
	Kurtosis	5.64	4.27	3.76	3.64	3.35
JB test (5)	Statistic	72.1	28.2	21.2	10.6	0.5
	p-value	< 0.0001	< 0.0001	< 0.0001	0.00495	0.77195
	Ho	rejected	rejected	rejected	rejected	accepted

This table contains the parameter estimates of the deterministic trend and AR(1) components for the natural logarithm of the spot price and the energy generation of various electricity generators in Colombia, with information since January 2000. (1) This series contains information since January 2007. (2) Coefficient of the deterministic trend. (3) Coefficient for the stochastic AR(1) component. (4) Residuals are calculated after a two-stage process in which the trend is fitted first and then the autoregressive component. (5) Jarque–Bera test, whose null hypothesis is normality of residuals.

Table 4 reports the fitted parameters of the bivariate SNP distributions of the vector $\boldsymbol{\epsilon_t^i} = \left(\epsilon_t^p, \epsilon_t^{q^i}\right)'$, where the series were filtered from the estimates of the model in the previous stage. As can be inferred from the descriptive statistics, the Spot Price requires a forth-order SNP expansion and parameter d_4 is positive and significant, reflecting the excess kurtosis of this series. However, skewness parameter (d_3) seems not to be relevant and is excluded from the model. On the other hand, the Energy Generation series exhibit negative and significant skewness but not a salient kutosis and thus a third-order SNP is enough to account for non-nomality. ENDG series, however, seems to be an exception since in this case the coefficient d_3 is not statistically significant and thus a normal distribution fits data accurate, as indicated by the Jarque Bera statistic. The marginal distributions of each bivariate SNP pair are depicted in the Appendix 1.

As far as correlation is concerned, only the distribution of Spot Price and EPMG has a positive value, although insignificant at a 95% confidence level, whilst for the other couples is negative and significant. Note that the level of correlation between ISSG and Spot Price is higher than that of CHVG–Spot Price and ENDG–Spot Price (which are quite similar). These similar levels of correlation may be explained by the close geographical location of two dams with similar generation capacity between these two electricity generators.

Table 4 Bivariate SNP fitted distributions of the Spot Price and Energy Generation

Epsilon	Descriptive statistics				Multivariate SNP estimation (2)				
					Standardized series				
Bivariate	Mean	SD	Skewness	Kurtosis		d3	d4		Rho
Spot Price	0.00	0.25	0.51	5.64	Coeff.		0.166		
					p-value		0.001	Coeff.	0.278
EPMG	0.00	0.09	-0.60	4.27	Coeff.	-0.224		p-value	0.106
					p-value	0.002			

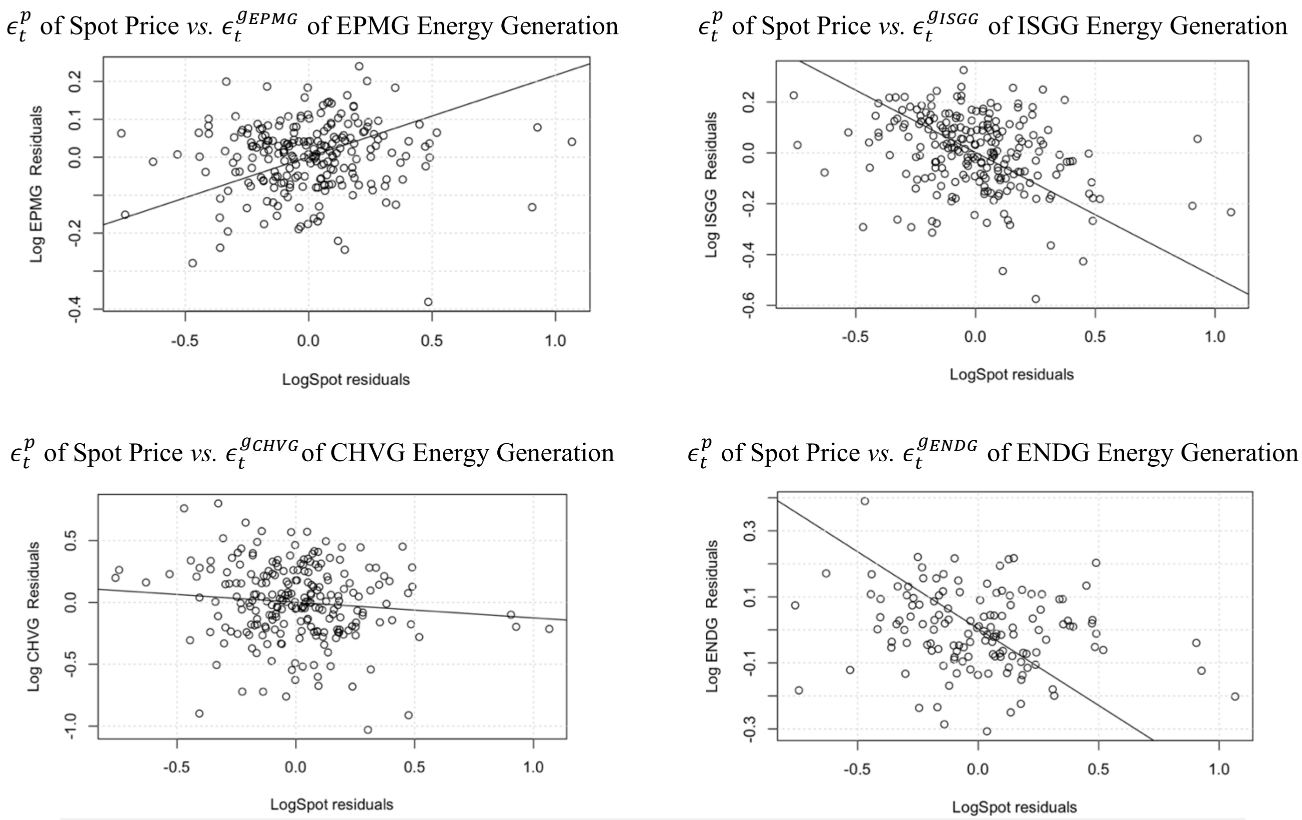
Spot Price	0.00	0.25	0.51	5.64	Coeff.		0.178		
					p-value		0.000	Coeff	-0.676
ISGG	0.00	0.15	-0.65	3.76	Coeff.	-0.410		p-value	0.000
					p-value	0.000			
Spot Price	0.00	0.25	0.51	5.64	Coeff.		0.160		
					p-value		0.001	Coeff	-0.427
CHVG	0.00	0.30	-0.43	3.64	Coeff.	-0.251		p-value	0.002
					p-value	0.003			
Spot Price	0.00	0.25	0.51	5.64	Coeff.		0.146		
					p-value		0.023	Coeff	-0.420
ENDG (1)	0.00	0.12	0.04	3.35	Coeff.	-0.041		p-value	0.029
					p-value	0.700			

This table contains the estimated parameters of the multivariate SNP for the zero-mean random component of Spot Price and Electricity Generation of the different Colombian generators. Descriptive statistics for the series are also displayed. (1) This series was fitted with information since January 2007. (2) SNP expansions include the relevant terms to account for non-normality, which are a fourth-order expansion for Spot Price and a third-order expansion for Energy Generation. The fitted SNP model outperforms the normal density, except for the ENDG series, for which neither the skewness (d3) nor the kurtosis (d4) seem to be significant.

The scatterplots in Figure 4 compare the residuals of Spot Price, ϵ_t^p, with those of each Energy Generation, $\epsilon_t^{q_i}$, to illustrate the comovements among these variables. Furthermore, figures for both bivariate pdf and cdf of the Spot Price and Energy Generation series can be found in Appendix 3.

This subsection discusses the behavior of the risk indicators at different contracting levels (Eta), the value that the contract market is paying to risk holders (measured using the FRP), and the correlation between the logarithm of the spot price and that of the energy generation (Rho). The sensitivity analyses performed in this study correspond to simulations based on a hypothetical generator's portfolio. This portfolio is characterized by having a mean and standard deviation of $\epsilon_t^{q_i}$ of 1 and 0.09133, respectively. In addition, it follows a third-order SNP distribution, with coefficient d_3 equal to -0.07415 and a correlation between the transformed components of spot price and energy generation of -17%. For the spot price, we consider the parameters estimated for the Colombian electricity market (see Tables 3 and 4), which are the conditions that a hypothetical generator would be facing. A thousand simulations were conducted according to the Monte Carlo method.

Figure 4 Scatterplots of the residuals of Spot Price versus those of each Energy Generation



This figure presents scatterplots of the residuals of Spot Price, ϵ_t^p , versus those of each Energy Generation, $\epsilon_t^{q_i}$. The lines correspond to the the best linear regression whose slopes capture the correlation among the variables.

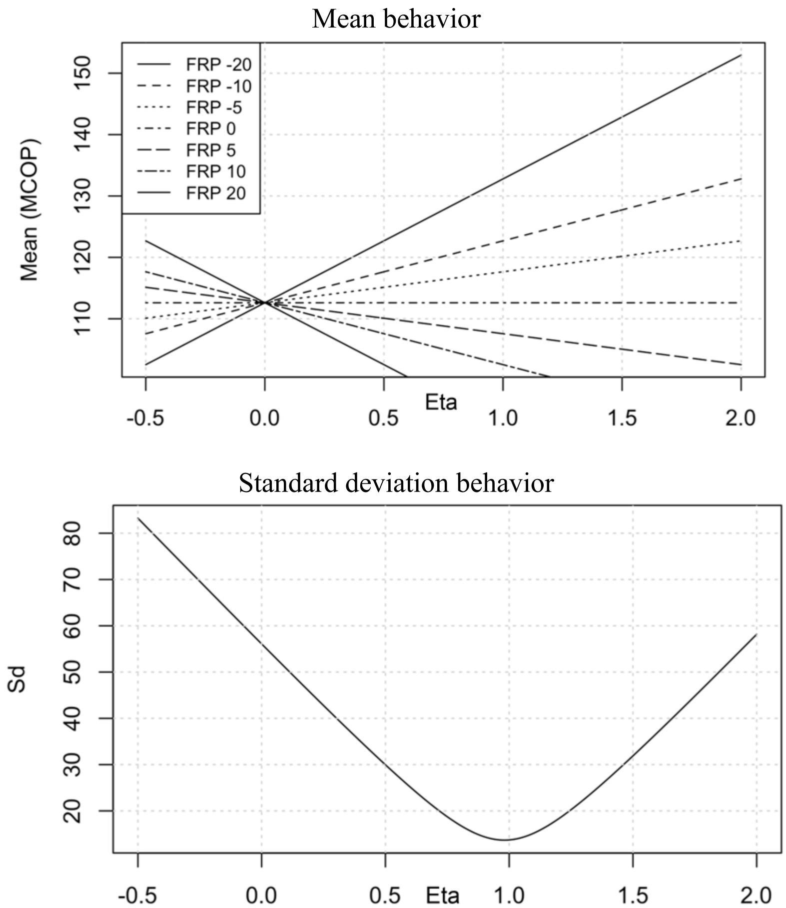
4.3 Sensitivity of the risk indicators to the contracting level, forward risk premium and correlation

The contracting level, Eta (η), captures the percentage of the expected energy generation that is sold under long-term contracts. For instance, if an electricity generator is expected to produce an average of 100 GWh of electricity in a specific time and its Eta is 0.9, it will be selling 90 GWh through a long-term contract. If this generator produced 110 GWh at maturity, it would have sold 90 GWh through the contract at a known price; and 20 GWh, at the spot price. In the event that it had sold 90 GWh in the contract but only generated 80 GWh at maturity, it would have to procure the remaining 10 GWh from the spot market to meet its obligations.

Figure 6 illustrates the behavior of the mean and the standard deviation of the hypothetical generator's portfolio at different Eta (η) and FRP values. From this figure, we observe that when the FRP value is zero (i.e., the forward price is equal to the expected spot price) the mean of the net income from energy sales will remain constant and does not depend on the Eta. Its standard deviation, on the contrary, does depend on the Eta because, as illustrated by this figure, the curve of the standard deviation is concave up decreasing (increasing) at low (high) Eta values. Unlike the mean of the portfolio, the curve of its standard deviation does not change at different FRP values.

With respect to the expected value of the portfolio, its relationship with the Eta depends on the sign and value of the FRP. A negative FRP is translated into a higher profit for the electricity generator. Moreover, increasing the contracting level when the FRP value is negative will result in a higher expected net income, as shown in Figure 6 when the FRP value is -5, -10, and -20. Based on this, it is thus clear that the more negative the FRP value, the higher the expected value for the electricity generator when it increases its forward contracting level. Conversely, the more positive the FRP value, the lower the expected value of the portfolio.

Figure 6 Sensitivity of the mean and the standard deviation of the net income from the sale of energy

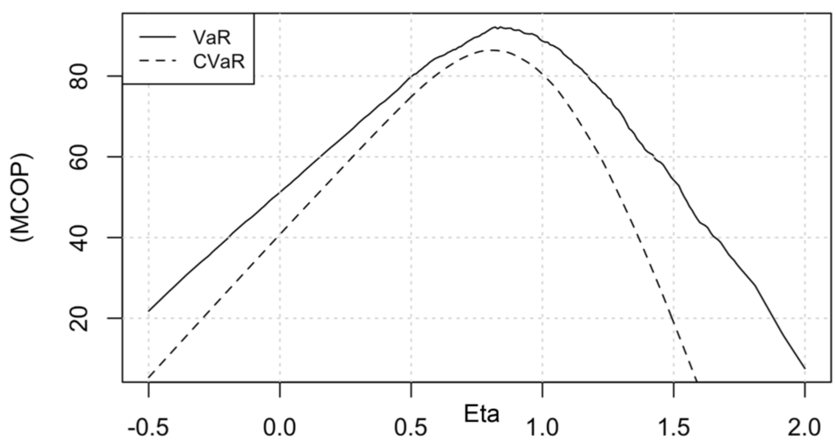


This figure presents two scatterplots to illustrate the sensitivity of the mean and the standard deviation of the net income from the sale of energy at different contracting levels, η, and Forward Risk Premium (FRP) values.

In addition to the mean and the standard deviation, we also present indicators of left-tail risk on the hypothetical generator's net income from energy sales. Figure 7 shows the typical behavior and concavity of the VaR and CVaR curves at different Eta values, which differ from those of the standard deviation. For the standard deviation, it is more likely to find an Eta value that minimizes it, while, for the VaR and CVaR, it is more likely to find an Eta value that maximizes both.

Due to the type of estimators and their calculation method under Monte Carlo simulation, we observe, from Figure 7, that the CVaR is consistently lower than the VaR and maintains a smoother behavior compared to the distortions that this latter seems to have. Additionally, note that the CVaR curve increases faster than that of the VaR once the Eta value exceeds the optimum (when indicators reach the maximum value). This situation occurs due to the thickening of the left tail of the generator's net income when it must procure the volume of energy that it did not generate in the spot market.

Figure 7 Sensitivity of the VaR and the CVaR to the contracting level (Eta)



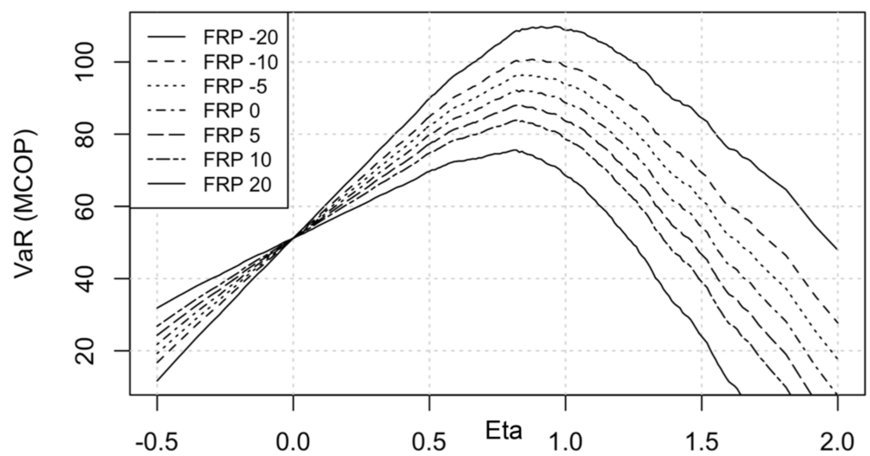
This figure plots the behavior of the Value-at-Risk (VaR) and Conditional VaR (CVaR) at different contracting levels (Eta). The VaR is computed as the 5th percentile of the generator's net income from energy sales and the CVaR as the average of all its net income from energy sales that are below the VaR.

Similar to the behavior of the expected value, the VaR and the CVaR change at different FRP values. When the Eta values are positive, a negative FRP involves a better risk condition for the electricity generator. In other words, in order to improve the mean and the left-tail risk indicators, the generator will always prefer to sell at a higher forward price; hence, it should be aware of the high-risk opportunities identified by market agents. This situation is illustrated in Figure 8.

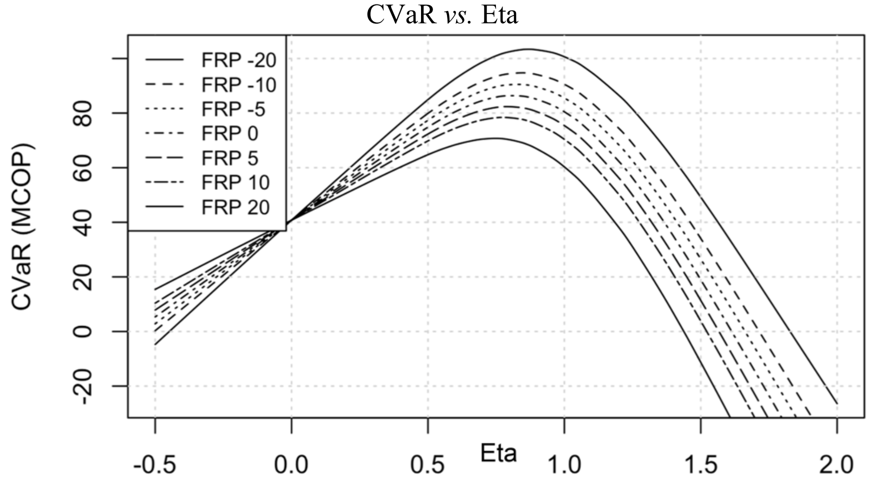
Furthermore, variations in the FRP also seem to move the maximum point of the indicators, as shown in Figure 8. For instance, if an electricity generator considers that it it is optimal to sell a number of contracts equal to 75% of its expected energy generation under a FRP value of zero, the optimal Eta value will change if the FRP value drops to -10. Therefore, the optimal forward contracting level depends not only on the uncertainty conditions of the spot price and the energy generation but also on market conditions and the way the market values the assumed risk levels.

Figure 8 Sensitivity of the VaR and the CVaR to the Eta and FRP

VaR *vs.* Eta



CVaR *vs.* Eta

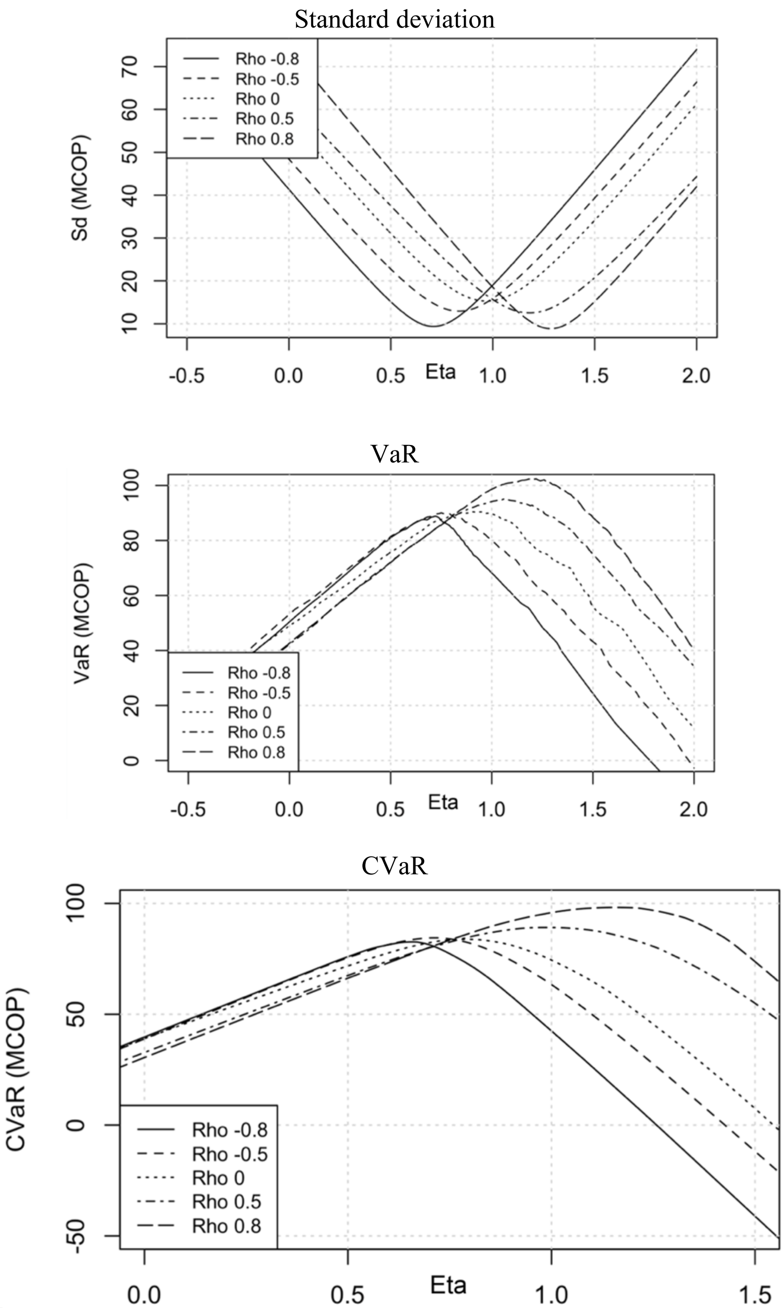


This figure illustrates the sensitivity of the Value-at-Risk (VaR) and Conditional VaR (CVaR) to different contracting levels (Eta) and Forward Risk Premium (FRP) values. It is worth recalling that a negative forward risk premium represents an additional profit for the electricity generator (seller in the forward contract) regarding the expected spot price; and a positive forward risk premium, an additional profit for the client (buyer in the forward contract).

Moreover, the correlation between Spot Price and Energy Generation affects the optimal decision. Figure 9 – constructed assuming an FRP equal to zero – presents the behavior of the risk indicators under analysis at different correlation values. A positive correlation tends to increase the optimal contracting level, while a negative one tends to reduce it, which is consistent with the belief that a negative correlation represents a natural hedge. Another perceived effect is that the VaR and CVaR leves rise as the correlation values increase: a positive correlation shows a higher CVaR value compared to a negative one, at the same uncertainty and contracting levels.

A negative Eta represents an opportunity for the electricity generator to buy, instead of selling, forward contracts. When this generator purchases forward contracts, the positive FRP values, compared to the negative ones, improve their position. Although this scenario of contract purchase is considered in the sensitivity analyses performed, it is not relevant for the type of generator under analysis. This situation is confirmed by the fact that the Eta values that maximize the VaR and the CVaR are considerably far to the right of zero.

Figure 9 Sensitivity of the standard deviation, the VaR, and the CVaR to the Eta and Rho

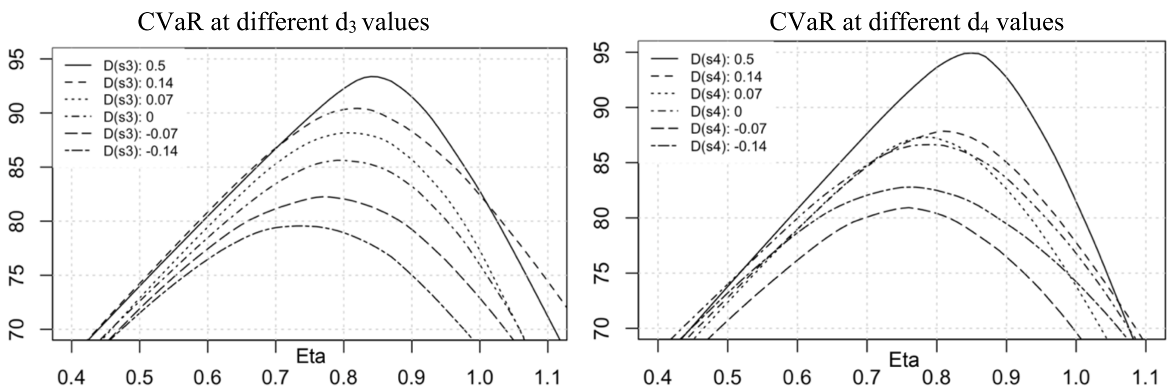


This figure shows the sensitivity of the standard deviation, the Value-at-Risk (VaR), and the Conditional VaR (CVaR) of the net income from the sale of energy to different contracting levels (Eta) and correlation values (Rho).

4.4 Effect of SNP parameters

The d_m parameters of the SNP function serve to describe the conditions of skewness, kurtosis, and higher-order moments. In this section, we present the sensitivity of the CVaR and the contracting level (Eta) at different d_3 and d_4 values for the hypothetical generator's energy generation. The first graph of Figure 10 plots the behavior of the CVaR of the portfolio when the marginal distribution of the energy generation only contains the thrid-order HP and parameter d_3 takes values of 0.5, 0.14, 0.07, 0 , -0.07, and -0.14. The second graph shows a similar sensitivity analysis, but for the fourth-order HP. It is worth noting that that $d_3 = 0$ and $d_4 = 0$ indicates a normal distribution.

Figure 10 Sensitivity of the CVaR to different SNP parameters



This figure presents the behavior of the Conditional Value-at-risk (CVaR) of the net income from energy sales at different values of the parameters d_3 and d_4 of the SNP marginal function for the natural logarithm of the EPMG energy generation. The y axis in both graphs is measured in million Colombian pesos (MCOP).

The d_3 (skewness) and d_4 (excess kurtosis) coefficients affect the curve of the CVaR at different Eta values. If we take the case of a normal distribution as a reference, negative coefficients tend to decrease the CVaR levels, while positive ones tend to increase them and move the maximum point to the right. Normal conditions do not allow us to properly represent the sensitivity of risk indicators to different hedging levels. Therefore, professionals in this area should adequately represent levels of skweness or kurtosis in power generation in order to find appropriate levels of hedging.

Our results suggest that the risk levels, measured by the CVaR, actually depend on the effect that the SNP function captures. Figure 10 indicates that even the optimal contracting level is affected by the SNP expanison terms and their coefficients. Thus, measurements of moments, such as skewness and kurtosis, should be included in the assessement and management of risk levels in electricity markets.

4.5 Optimal forward contracting level

Table 6 reports the optimal contracting levels suggested for each electricity generator under study at different FRP values. For the mean, the VaR, and the CVaR, we considered the contracting level that maximizes each of them; and for the standard deviation, the contracting level that minimizes it.
Based on the sensitivity analysis conducted, the net profit expected value maximization problem has a corner solution: when the FRP values are negative, the best average is obtained by increasing the forward contracting level as much as possible, while if they are positive, forward sales should be preferably reduced to a minimum.

According to Table 6, the contracting level that minimizes the standard deviation of the portfolio depends on the particular conditions of each electricity generator and not on the market conditions (FRP). In general, the suggested contracting levels are below one, except for EPMG, which is required to increase its forward sales, maybe, because this generator has positive correlation levels higher than those of other three generators under analysis.

Moreover, for all generators, we observe that the negative FRP values tend to increase the optimal contracting level for the VaR and the CVaR indicators. In particular, based on the sensitivity analyses performed, the contracting levels can even reach variations up to 15%. In other words, an electricity generator that expects to produce 100 GWh of electricity may either have to sell 72 GWh under forward contracts if the FRP value is 20 or sell 87 GWh if such value is -20.

Regarding the type of indicator, portfolios structured based on the VaR, compared to those structured based on the CVaR, tend to increase the amount of energy traded under forward contracts. According to Table 6, the contracting level obtained for EPMG using the VaR is on average 10% higher than that obtained with the CVaR. In the case of ISGG, we observe that the suggested contracting levels are clearly lower than those of the other electricity generators. This situation may be explained by the fact that the unexplained component of the ISGG series exhibits a negative correlation higher than that of the other generators. A positive correlation tends to increase the contracting levels, while a negative one tends to reduce them.

Table 6 Optimal contracting level (Eta) suggested for each generator

FRP	EPMG				ISGG			
	Mean (1)	SD	VaR	CVaR	Mean	SD	VaR	CVaR
-20	2	1.13	1.20	0.96	2	0.61	0.62	0.49
-15	2	1.13	1.16	0.95	2	0.61	0.62	0.48
-10	2	1.13	1.16	0.95	2	0.61	0.61	0.48
-5	2	1.13	1.16	0.95	2	0.61	0.56	0.46
-2	2	1.13	1.16	0.95	2	0.61	0.56	0.45
0	0	1.13	1.06	0.93	0	0.61	0.56	0.43
2	0	1.13	1.04	0.90	0	0.61	0.44	0.43
5	0	1.13	1.04	0.85	0	0.61	0.44	0.41
10	0	1.13	1.00	0.83	0	0.61	0.26	0.40

15	0	1.13	0.95	0.76	0	0.61	0.26	0.40
20	0	1.13	0.95	0.76	0	0.61	0.24	0.40

FRP	CHVG				ENDG			
	Mean	SD	VaR	CVaR	Mean	SD	VaR	CVaR
-20	2	0.88	0.81	0.67	2	0.87	0.84	0.74
-15	2	0.88	0.81	0.66	2	0.87	0.84	0.73
-10	2	0.88	0.81	0.66	2	0.87	0.84	0.72
-5	2	0.88	0.79	0.65	2	0.87	0.77	0.72
-2	2	0.88	0.69	0.64	2	0.87	0.76	0.71
0	0	0.88	0.69	0.64	0	0.87	0.76	0.71
2	0	0.88	0.69	0.64	0	0.87	0.76	0.71
5	0	0.88	0.69	0.63	0	0.87	0.76	0.7
10	0	0.88	0.69	0.63	0	0.87	0.76	0.69
15	0	0.88	0.69	0.62	0	0.87	0.76	0.67
20	0	0.88	0.68	0.61	0	0.87	0.68	0.66

This table shows the optimal contracting level suggested for each electricity generator under study. In each case, we considered the optimal values for the four indicators (mean, standard deviation, Value-at-Risk [VaR], and Conditional Value-at-risk [CVaR]) of their net profit from energy sales. The contracting level (Eta) corresponds to the percentage of the expected energy generation that must be sold under long-term contracts (electricity forward contracts), depending on the risk indicator and the value of the Forward Risk Premium (FRP) in the market. (1) The simulation was performed using Eta values between 0 and 2. In the case of the mean, the Eta optima occur at a corner solution, i.e., if the FRP values are negative on average, it is preferable to increase forward sales, while if they are positive, it is better to reduce forward sales.

5. Conclusions

In this paper, we proposed a static hedging strategy for electricity generators that participate in a competitive market where hedging is carried out through forward contracts that include a risk premium in their valuation. We considered the spot price and energy generation variables to follow a bivariate SNP distribution defined in terms of the Gram–Charlier (Type A) expansion. This distribution allowed us to not only model the mean, the variance, and their correlation but also the skewness, the kurtosis, and higher-order moments. Moreover, we used Monte Carlo simulation to analyze the effect of three risk indicators (standard deviation, VaR, and CVaR) on the net profit from energy sales, using information from the Colombian electricity market as the case study.

The main contribution of this work to the analysis of electricity markets is the structuring of a hedging portfolio that does not impose the assumption of normality on price and energy generation. From a statistical point of view, its usefulness lies in the simulation of a SNP probability distribution by means of the Monte Carlo method.

In general, a negative FRP increases an electricity generator's net profit from its energy sales in the contract market, thus favoring electricity forward sales. Moreover, the FRP affects the behavior of the mean, the VaR, and the CVaR regarding the amount of electricity to be sold under forward contracts. This situation does not occur for the standard deviation, whose behavior, instead, is affected by the contracting level, regardless of the FRP value in the market.

The results show that the optimal quantity of energy to be sold under long-term contracts is dependent not only on the conditions of spot price and quantity uncertainty but also on the way market agents weigh the assumed risk levels. Therefore, to reduce the risk levels faced by generators, such optimal quantity will depend on the conditions of price and energy generation uncertainty explained by variance, skewness, kurtosis, and higher-order moments. Furthermore, the number of forward sales is determined by the correlation between price and energy generation, as well as by the FRP.

As a final remark, we recommend experts in electricity markets to structure company-specific portfolios based on the conditions of the market on which the analysis is performed. In addition, they should consider a flexible modeling that captures a greater number of moments than those allowed by a normal distribution for the variables involved, as well as the correlation between spot price and energy generation. The multivarite SNP distribution can be an appropriate tool for this purpose.

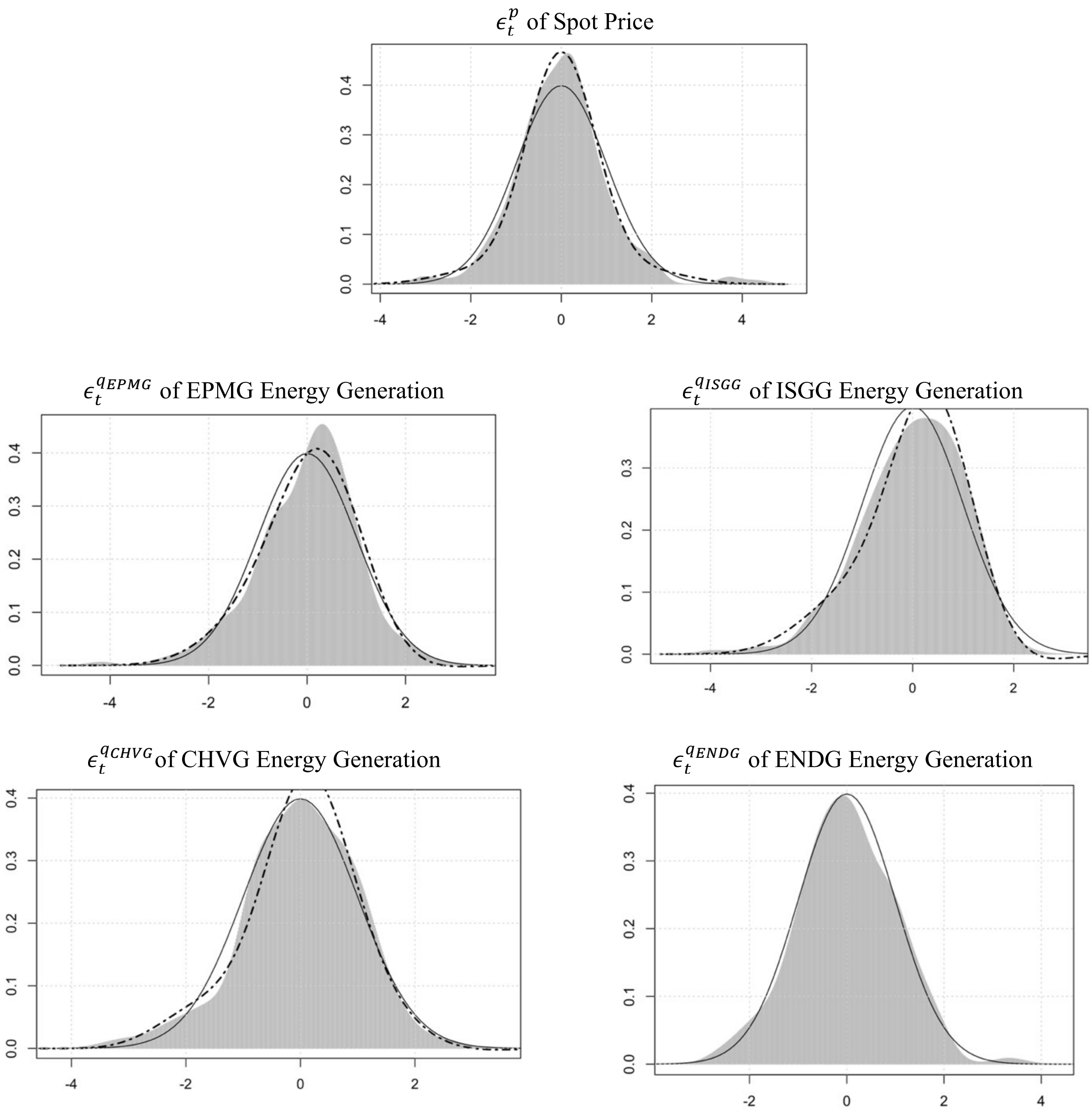
Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix 1. Marginal distributions fitting

This figure shows how the marginal distributions of the Spot Price and Energy Generation for each agent series. It is noteworthy that the estimates for marginals are used as initial seeds for the maximum likelihood estimation of the bivariate SNP for every pairwise series.

Figure Marginal density functions of residuals ϵ_t



This figure shows the sample density functions (shaded area), normal distribution (solid line), and SNP distribution (dashed line) of each residual, ϵ_t, of the Spot Price and Energy Generation series. They correspond to the marginal density functions of the bivariate pairs of Spot Price and Energy Generation.

Appendix 2. Proof for the marginal pdfs of the Spot Price and Energy Generation

Let the joint SNP density function of the random variables p_T and q_T^i be denoted by:

$$F_Z\left(p_T, q_T^i\right) = G_Z\left(p_T, q_T^i\right) + g_p(p_T) \cdot g_{q^i}\left(q_T^i\right) \cdot \left\{k_p(p_T) + k_{q^i}(q_T^i)\right\}$$

The marginal density function of p_T can be estimated as follows:

$$f_p(p_T) = \int_{q^i} F_Z\left(p_T, q_T^i\right) dq_T^i$$

$$\Rightarrow f_p(p_T) = \int_{q^i} G_Z dq_T^i + \int_{q^i} g_p\, g_{q^i}\, k_p dq_T^i + \int_{q^i} g_p\, g_{q^i}\, k_{q^i} dq_T^i$$

$$\Rightarrow f_p(p_T) = g_p(p_T) + g_p k_p \int_{q^i} g_{q^i}\, dq_T^i + g_p\, g_p \int_{q^i} g_{q^i}\, k_{q^i} dq_T^i$$

$$\Rightarrow f_p(p_T) = g_p(p_T) + g_p k_p(1) + g_p\, g_p(0)$$

Given the orthogonality property in equation (7).

$$\Rightarrow f_p(p_T) = g_p(p_T) + g_p(p_T) \cdot \left\{k_p(p_T)\right\}$$

Likewise, the marginal density function of q_T^i can be calculated as follows:

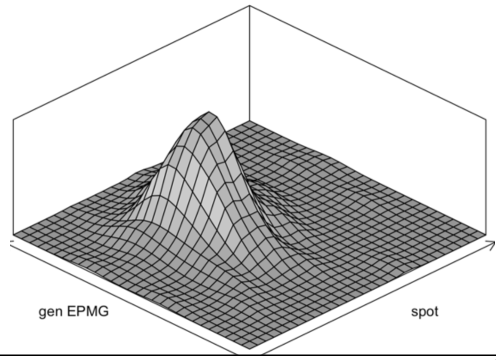
$$f_{q^i}\left(q_T^i\right) = \int_{p} F_Z\left(p_T, q_T^i\right) dp_T$$

$$\Rightarrow f_{q^i}\left(q_T^i\right) = g_{q^i}\left(q_T^i\right) + g_{q^i}\left(q_T^i\right) \cdot \left\{k_{q^i}(q_T^i)\right\}$$

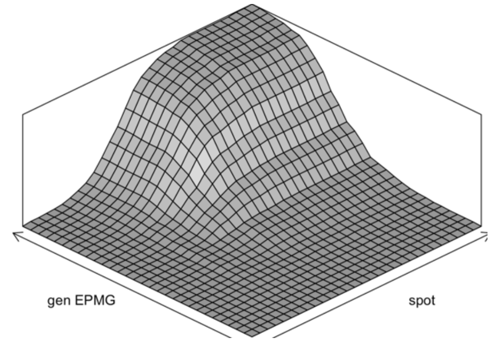
Appendix 3. Joint probability density (PDF) and cumulative distribution (CDF) functions for the stochastic component of the Spot Price and Energy Generation series

ϵ_t^p of Spot Price *vs.* $\epsilon_t^{g_{EPMG}}$ of EPMG Energy Generation

Joint PDF

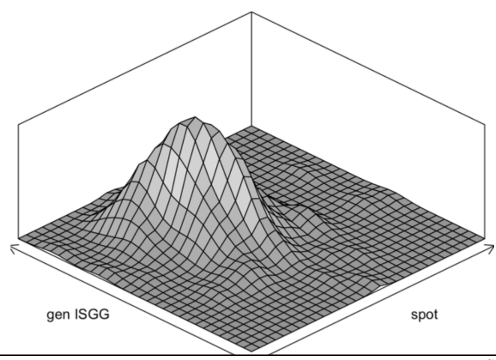


Joint CDF



ϵ_t^p of Spot Price *vs.* $\epsilon_t^{g_{ISGG}}$ of ISGG Energy Generation

Joint PDF



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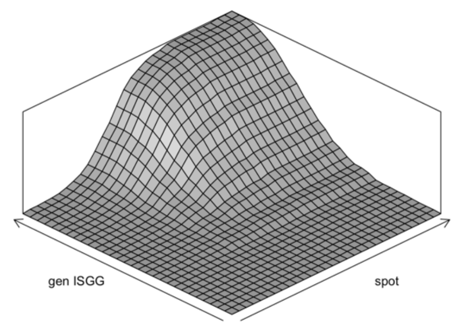
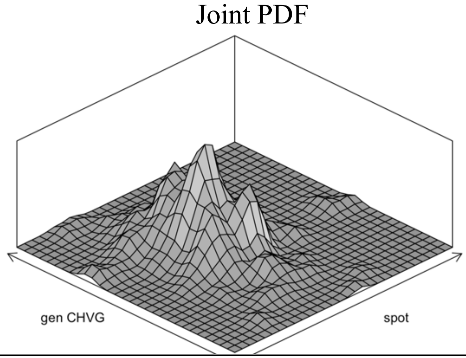


ϵ_t^p of Spot Price *vs.* $\epsilon_t^{g_{CHVG}}$ of CHVG Energy Generation

Joint PDF



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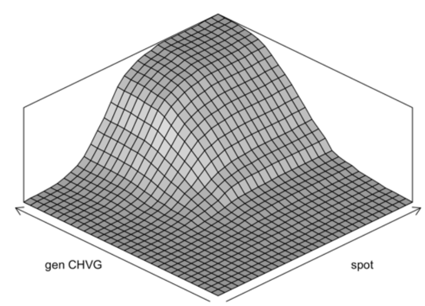
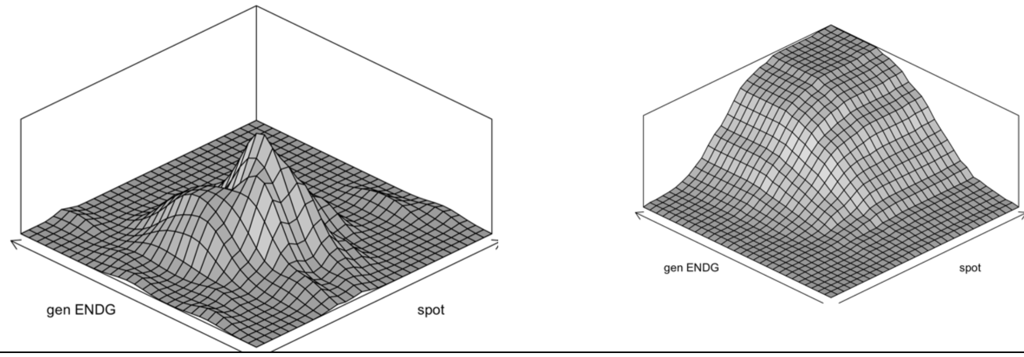


ϵ_t^p of Spot Price *vs.* $\epsilon_t^{g_{ENDG}}$ of ENDG Energy Generation

Joint PDF

Joint CDF


gen ENDG
spot
gen ENDG
spot

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