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A Revision to the Theory and the Practice of Development Accounting

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Abstract

Practitioners of development accounting assume that the private return on schooling equals its social marginal product. I show that this assumption is inconsistent with the mathematical structure of a multiplicative production function, which specifies that the private return is only a fraction of the social marginal product. I then show that the empirical results from development accounting in the literature substantially underestimate human capital's contribution to national output and overestimate national productivity differences.

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I. Introduction

Robert Solow [1957] showed that in a production function with constant economies of scale, the growth of national output can be estimated as the sum of the growth in productivity due to technological change and the product of the (social) marginal product of the factors of production and the growth in these factors. When this relationship is employed across countries to determine the share of national output that can be explained by factors of production, it is known as development accounting, and the unexplained share of output is attributed to differences in national productivity.

Hall and Jones [1999] use a development accounting methodology with a Cobb-Douglas production function to examine whether differences in physical capital/worker (K/L) and human capital/worker (H/L) can explain output/worker (Y/L) across countries. They find that residual differences in national productivity (A) explain at least half the output differences. Caselli [2005] claims this methodology and these findings represent the consensus view in the development accounting literature.

In this article I show that Hall and Jones' methodology and findings are flawed because their assumption that factors are paid their social marginal product is inconsistent with the mathematical structure of the Cobb-Douglas production function. I show that the factors of production in this function receive their social marginal product, but they are paid only a fraction of this product. The rest of their income accrues to them as an external effect of the other factors of production. I also show that Hall and Jones' treatment of physical capital and human capital is inconsistent, since in their calculations they assume that physical capital contributes its social marginal product to national output, while human capital contributes only the private share of its social marginal product. As a result of this methodological error and poor

assumptions about the private returns to schooling across countries, the consensus view in the development accounting literature substantially underestimates the contribution of human capital to national output and overestimates national productivity differences across countries.

II. Development Accounting in Theory

Mankiw, Romer, and Weil [1992] created a production function that includes physical capital, human capital, and labor as multiplicative factors of production:

$$1) \quad Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$

In this function Y is national output, and K and H are stocks of physical and human capital estimated from historic investment and depreciation rates. The partial derivative of output with respect to each factor of production provides its social marginal product, which includes any external effect(s) of the factor on output [Barro and Sala-i-Martin, 2004]:

$$2) \quad r_k = MPK = \partial Y / \partial K = \alpha Y / K$$

$$3) \quad r_h = MPH = \partial Y / \partial H = \beta Y / H$$

$$4) \quad w = MPL = \partial Y / \partial L = (1-\alpha-\beta)Y/L$$

If the factors of production are paid their social marginal product, each factor receives a share of output equal to its exponent, and the sum of these shares is total output:

$$5) \quad Y = r_k K + r_h H + wL = (\alpha Y / K)K + (\beta Y / H)H + ((1-\alpha-\beta)Y/L)L = \alpha Y + \beta Y + (1-\alpha-\beta)Y$$

This result appears to validate the assumption that each factor is paid its social marginal product.

But this assumption is inconsistent with the mathematics of the Cobb-Douglas function and with any production function with multiplicative factors. In these functions when a factor of production increases, its social marginal product declines and the social marginal product of the other factors increases. As a result, each factor's social marginal product accrues to all three

factors of production, in shares determined by the exponent on each factor [Breton, 2013]. As a consequence, the payment to each factor is only a fraction of its social marginal product.

In the case of physical capital, the production function specifies that the owners of K are paid the α share of r_k :

$$5) \quad \text{Private } r_k = \partial(r_k K)/\partial K = \partial((\alpha Y/K) K)/\partial K = \alpha \text{ MPK} = \alpha^2 Y/K$$

Although not shown, the production function specifies that the β and $1-\alpha-\beta$ shares of r_k accrue to the owners of K and to L as an external effect.

In the case of human capital, the production function specifies that the owners of H are paid the β share of r_h :

$$6) \quad \text{Private } r_h = \partial(r_h H)/\partial H = \partial((\beta Y/H) H)/\partial H = \beta \text{ MPH} = \beta^2 Y/H$$

The function species that the $1-\alpha-\beta$ share of r_h accrues to workers (L) as an external effect:

$$7) \quad \text{Share of } r_h \text{ for L} = \partial(wL)/\partial H = \partial(((1-\alpha-\beta)Y/L) L)/\partial H = (1-\alpha-\beta) \text{ MPH} = (1-\alpha-\beta)\beta Y/H$$

Since workers receive both the private share of r_h and the external share accruing to L, it is not clear whether in practice the observed return to schooling estimated from salary increases includes only the β share of r_h , or also the $(1-\alpha-\beta)$ external share.

Due to the mathematical properties of the Cobb-Douglas function, if each factor is paid the share of its social marginal product equal to its own exponent and receives this share of the social marginal product of the other factors as an external effect, in total each factor receives the share of national output equal to its exponent. This can be shown with physical capital:

$$7) \quad \alpha(r_k K) + \alpha(r_h H) + \alpha(1-\alpha-\beta)(wL) = \alpha(\alpha Y) + \alpha(\beta Y) + \alpha(1-\alpha-\beta)Y = \alpha Y$$

So even though each factor is paid only a fraction of its social marginal product, it still receives its entire social marginal product through the direct payment and the external effect(s).

II. Development Accounting in Practice

Hall and Jones estimate a restricted version of Mankiw, Romer, and Weil's model in which $\beta = 1 - \alpha$:

$$8) \quad Y = K^{\alpha} (AH)^{1-\alpha}$$

In this model an increase in L (number of workers) has no effect on national income.¹ Hall and Jones estimate national productivity (A) as a residual by estimating the stocks of physical capital and human capital and assuming that $\alpha = 0.33$. Bernanke and Guykarnak [2001] estimate that across countries 0.35 of national output accrues to owners of physical capital. So implicitly Hall and Jones assume that physical capital receives its social marginal product.

They estimate H using average years of schooling and their estimate of the *private* return to schooling:

$$9) \quad H/L = e^{\varphi \text{ years}}$$

In this calculation they assume that φ decreases from 0.134 in countries with 0-4 years of schooling to 0.10 in countries with 4-8 years of schooling, and to 0.068 in countries with 8+ years of schooling.

Since $1 - \alpha - \beta = 0$, human capital has no external effect on L, but it has an external effect on K. Since they have not included this effect in their calculation, equation (7) shows that Hall and Jones have assumed that the $1 - \alpha$ share of the social marginal product is the full marginal product. As a consequence, they underestimate the effect of human capital on national income by 1/3.

Their assumption that average years of schooling have a substantial diminishing effect on national output further underestimates the effect of human capital. Psacharopoulos and Patrinos [2004] present data showing that an additional year of schooling raises national income by about 0.10 across countries and that this effect does not diminish with additional years of schooling.

¹ Breton [2013] estimates the model in (1) using historic investment in schooling to estimate the stock of human capital, and he finds that $\alpha \approx \beta \approx 0.35$, which indicates that Hall and Jones' assumption that $\beta = 1 - \alpha$ is incorrect.

As a consequence of their assumptions, Hall and Jones [1999], Caselli [2005], and others using their methodology underestimate the effect of human capital on national income by a factor of three and substantially overestimate residual productivity differences.

III. Consistency of Private Returns to K with the Model's Estimates

The demonstration that the factors of production in a Cobb-Douglas function are paid only a fraction of their social marginal product does not indicate whether this assumption is representative of reality. The assumed multiplicative relationship between physical capital and human capital in the function could be incorrect. As a test of the validity of this assumption, I compare the model's estimate of the social marginal product and the private share of K to an independent estimate of the private return on K in the U.S.

In a competitive market economy, the real private return should be equal to the real cost of financial capital, which is the real weighted average cost of capital (WACC) to private companies. McGrattan and Prescott [2003] estimate that between 1960 and 2002 the real return on equity in the U.S. averaged about 5 percent and the real return on corporate debt averaged about 3.8 percent. Assuming a 1.5 debt/equity ratio, the average WACC during this period was 4.3 percent.

The social marginal cost r_k can be estimated from the U.S. K/Y ratio, calculated from the data on `ck` and `rgdpo` in Penn World Table 8.0 [Feenstra, Inklaar, and Timmer, 2013]. This ratio averaged 2.9 from 1980 to 2005, which with $\alpha = 0.35$ yields a U.S. social marginal product of 12.1%. The α share of this product is 4.2%, which is similar to the average WACC of 4.3%.

One problem with this comparison is that the social marginal product is biased upward because the estimate includes the income from non-reproducible K (e.g., land and natural resources), but it does not include the stock of non-reproducible K. But excluding the non-

reproducible income only lowers the social marginal product (in 2000) to 9% [Caselli and Feyrer, 2007], which is still much higher than the estimated private return. So the results of this test provide evidence that the Cobb-Douglas function in (1) is a valid model of national output and that the current assumption in development accounting that factors of production are paid their social marginal product is incorrect.²

IV. Conclusions

The consensus in development accounting is that differences in productivity unrelated to stocks of capital explain at least half the differences in national income across countries [Casselli, 2005]. In this article I show that the calculations used to reach this conclusion are flawed because they utilize the private return on schooling to estimate the social marginal product and they assume this return exhibits diminishing returns. Neither of these assumptions is correct. As a consequence, the consensus finding in development accounting substantially underestimates the effect of differences in human capital/worker across countries and overestimates the differences in income/worker due to differences in national productivity.

² Breton [2013] presents the results for a similar test for human capital. He shows that the private return on schooling is much lower and consistent with the social marginal product for the model in equation (1).

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