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Properties of speckle patterns generated through multiaperture pupils

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Abstract

The characteristics of the image speckles obtained through multiple aperture pupils are theoretically analyzed in terms of the parameters defining the pupils. The possibility of interpreting and synthesizing the image speckle distribution in terms of rather elementary structures is considered, based on the Fourier optics analysis. Then, first and second order statistical properties of the speckle patterns are studied by evaluating both the mutual intensity and the auto-correlation intensity of speckle distributions obtained by means of pupils consisting of identical apertures. Experimental results are presented to confirm the analysis. © 2001 Published by Elsevier Science B.V.

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1. Introduction

The spectral components of input images usually concentrate in the low-frequency region of the Fourier plane, which in general makes it difficult to filter the spectrum and to process the images [1]. Considerable interest has been devoted to image processing and metrology applications based on the introduction of a spatial carrier through speckle patterns [2,3] to spread out the spectral content into the high-frequency region.

Some techniques have been implemented on the basis of internal modulation of the speckle grains in the image field, which can be achieved by placing a mask with several apertures immediately behind the imaging lens. The main advantage of the internal speckle modulation is that it allows for selective concentration of the spectral image components into isolated high-frequency regions of the Fourier plane [4,5].

Many optical systems have been conceived based on double aperture pupils. These include optical arrangements for binary signal encoding, gray level and color image storage, pseudo-coloring operations, optical image subtraction, speckle photography and speckle pattern interferometry

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[6–12]. However, the advantage of employing multiple aperture pupils has been demonstrated by solving specific problems in image multiplexing [5], speckle photography [4,13] and speckle-shearing interferometry [14].

It should be pointed out that double aperture systems allow the realization of some proposals that cannot be implemented through a single aperture pupil. Furthermore, several applications using multiple aperture pupils demonstrate the possibility of carrying out a variety of experiments we cannot implement otherwise. Thus, the internal modulation of speckles obtained through multiple aperture pupils not only exhibits more complexity but it offers larger capabilities in terms of potential applications. In contrast, other applications in this field are not as wide and diverse as expected. In addition, the characteristics of modulated speckles are scarcely treated in, for instance, Refs. [4,14–19] concerning specific applications of optical systems employing multiple apertures.

In this paper, we present a contribution devoted to study the features of speckle patterns generated by using optical systems whose pupils consist of multiple apertures. The aim is to provide a suitable framework that could contribute to understand both actual and potential applications based on the internal modulation of speckles. In Section 2, the possibility of synthesizing the images in terms of quite elementary speckle patterns is theoretically analyzed. Subsequently, the first and the second order statistical properties of the speckle patterns are investigated through the mutual intensity and the auto-correlation of the intensity. In Section 3 the experimental results are presented.

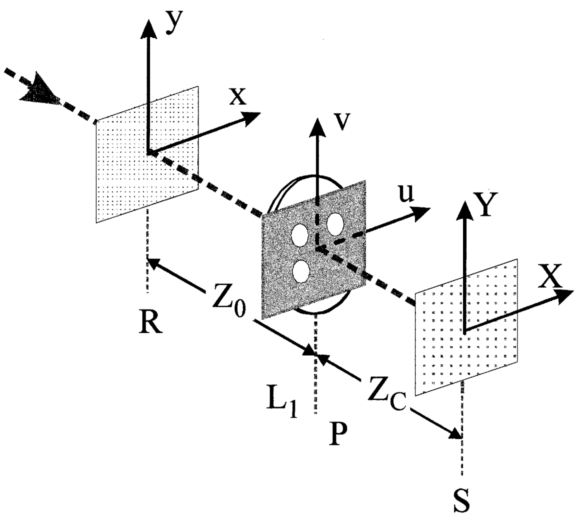

Fig. 1. Experimental setup: R, diffuser; L1, imaging lens; P, pupil mask; S, image plane.

2. Theoretical analysis

The experimental setup is sketched in Fig. 1. There, the diffuser R is coherently illuminated by using an expanded laser beam of wavelength λ and the lens L images the diffuser in the X–Y plane. Besides, a mask with multiple apertures is located immediately behind the lens in the u–v plane. The distances from the diffuser to the lens and from the lens to the image plane are Z_0 and Z_C, respectively.

The pupil function can be defined as:

$$P(u,v) = \sum_{n=1}^{N} a_n(u,v) \tag{1}$$

where $a_n(u,v)$ represents the amplitude transmission function corresponding to the nth aperture ($n = 1, 2, \ldots, N$). Each $a_n(u,v)$ is a real-valued function, which equals zero outside the respective aperture. Because the apertures are separated from each other, it follows that $a_n(u,v)a_m(u,v) \equiv 0$ for $n \neq m$. Hence, the following equation holds:

$$|P(u,v)|^2 = \sum_{n=1}^{N} |a_n(u,v)|^2 \tag{2}$$

2.1. Intensity distribution features

Let $\alpha_0(x,y)$ represents the complex amplitude of the wave in the diffuser exit plane. Then the amplitude in the image plane is given by [1]:

$$A(X,Y) = C \int\int \alpha_0(x,y)P(u,v) \times \exp\left\{ -\mathrm{i}\frac{2\pi}{\lambda}\left[\left(\frac{x}{Z_0}+\frac{X}{Z_C}\right)u + \left(\frac{y}{Z_0}+\frac{Y}{Z_C}\right)v\right]\right\} \mathrm{d}u\,\mathrm{d}v\,\mathrm{d}x\,\mathrm{d}y \tag{3}$$

where $C = 1/\lambda^2 Z_0 Z_C$. Then, the image intensity distribution $I(X,Y) = |A(X,Y)|^2$ is:

$$I(X,Y) = C^2 \int\int \alpha_0(x,y)\alpha_0^*(x',y') \times \int\int P(u,v)P(u',v') \times \exp\left\{-\mathrm{i}\frac{2\pi}{\lambda}\left[\left(\frac{x}{Z_0}+\frac{X}{Z_C}\right)u + \left(\frac{y}{Z_0}+\frac{Y}{Z_C}\right)v\right]\right\} \times \exp\left\{\mathrm{i}\frac{2\pi}{\lambda}\left[\left(\frac{x'}{Z_0}+\frac{X}{Z_C}\right)u' + \left(\frac{y'}{Z_0}+\frac{Y}{Z_C}\right)v'\right]\right\} \times \mathrm{d}u\,\mathrm{d}v\,\mathrm{d}u'\,\mathrm{d}v'\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y' \quad (4)$$

On the other hand, from Eq. (1) we obtain:

$$P(u,v)P(u',v') = \sum_{n=1}^{N} a_n(u,v)a_n(u',v') + \sum_{\substack{n,m=1\\ n\neq m}}^{N} a_n(u,v)a_m(u',v') \quad (5)$$

By replacing this result in Eq. (4), it is easy to demonstrate that

$$I(X,Y) = \sum_{n=1}^{N} |A_n(X,Y)|^2 + 2\sum_{\substack{n,m=1\\ n<m}}^{N} \mathrm{Re}\{A_n(X,Y)A_m^*(X,Y)\} \quad (6)$$

where Re refers to the real part of the expression in brackets and

$$A_n(X,Y) = C\int\int \alpha_0(x,y)a_n(u,v) \times \exp\left\{-\mathrm{i}\frac{2\pi}{\lambda}\left[\left(\frac{x}{Z_0}+\frac{X}{Z_C}\right)u + \left(\frac{y}{Z_0}+\frac{Y}{Z_C}\right)v\right]\right\}\mathrm{d}u\,\mathrm{d}v\,\mathrm{d}x\,\mathrm{d}y \quad (7)$$

$A_n(X,Y)$ represents the complex amplitude associated with the image obtained only by the aperture $a_n(u,v)$, for $n = 1,2,\ldots,N$. Then, the first term in the right-hand side of Eq. (6) refers to the incoherent superposition of the speckle patterns individually generated through the different apertures. The second term stands for the interference of the waves traveling from the apertures to the image plane. As expected, if a double aperture pupil is employed, then Eq. (6) reduces to $I(X,Y) = |A_1(X,Y) + A_2(X,Y)|^2$.

Note that the interference term in Eq. (6) includes an individual contribution $2\,\mathrm{Re}\{A_n(X,Y) \times A_m^*(X,Y)\}$ associated with each pair of different apertures $a_n(u,v)$ and $a_m(u,v)$, where $n,m = 1,2,\ldots,N$ and $n < m$. Thus, it is possible to identify the internal speckle modulation with a set of elementary fringe systems individually generated by different pairs of apertures in the pupil.

By adding and subtracting $(N-1)\sum_{n=1}^{N} \times |A_n(X,Y)|^2$ in the right-hand side of Eq. (6) and rearranging terms, it results:

$$I(X,Y) = \sum_{\substack{n,m=1\\ n<m}}^{N} |A_n(X,Y) + A_m(X,Y)|^2 - (N-2)\sum_{n=1}^{N} |A_n(X,Y)|^2 \quad (8)$$

Notice that $A_n(X,Y) + A_m(X,Y)$ is the coherent superposition of the waves from the apertures $a_n(u,v)$ and $a_m(u,v)$, where n, $m = 1,2,\ldots,N$ and $n \neq m$. Then, $|A_n(X,Y) + A_m(X,Y)|^2$ represents the intensity distribution of the modulated speckle pattern obtained by using those apertures, i.e. by employing the double aperture pupil $P_{nm}(u,v) \equiv a_n(u,v) + a_m(u,v)$. This intensity distribution specifies the interference of waves from the nth and the mth apertures and stands for an elementary fringe system f_{nm} whose orientation and period are determined by the relative position of the apertures. Moreover, the first term in the right-hand side of Eq. (8) refers to the incoherent superposition of the modulated speckle patterns independently generated by the pupil aperture pairs.

The second term in Eq. (8) is proportional to the incoherent superposition of the speckle patterns individually formed by using each aperture pupils. However, there is no internal modulation in these patterns.

Then, the internal modulation of individual speckles obtained with a multiple aperture pupil is determined only by the incoherent superposition of all those elementary fringe systems corresponding to the different aperture pairs in the pupil. Besides, this superposition does not adequately describe the observed intensity $I(X, Y)$. To emphasize this point, let us combine Eqs. (6) and (8):

$$2\sum_{\substack{n,m=1\\ n<m}}^{N} \mathrm{Re}\left\{A_n(X,Y)A_m^*(X,Y)\right\} = \sum_{\substack{n,m=1\\ n<m}}^{N} |A_n(X,Y)+A_m(X,Y)|^2 - (N-1)\sum_{n=1}^{N}|A_n(X,Y)|^2 \quad (9)$$

This equation confirms us that the interference of the N waves going through the apertures does not coincide with the incoherent superposition of all the speckle patterns generated by the different aperture pairs. Notice that the contribution due to the aperture pairs responsible for the speckle formation include $N-1$ times the contribution associated with each one of the apertures, as confirmed by the second term of the right-hand side in Eq. (9).

2.2. Statistical properties

In this section, we study the statistical properties of the speckle pattern generated through multiple aperture pupils. We evaluate the autocorrelation functions both for the complex amplitude and the intensity of the speckle pattern.

The speckle patterns obtained by using multiple aperture pupils fulfill the circular Gaussian random statistics with zero mean in the complex plane because the amplitude of these patterns results from the superposition of the complex amplitudes of light from the individual apertures satisfying the same statistics [20]. On the other hand, the complex amplitudes of waves going through different apertures are statistically independent from each other, because different components of the angular spectrum of scattered light are accepted by the apertures. In fact, it can be demonstrated [13] that $\left\langle A_n(X,Y)A_m^*(X,Y)\right\rangle \equiv 0$ for n, $m = 1, 2, \ldots, N$ and $n \neq m$. On this basis, Eq. (6) implies that the average intensity in the image plane is given by:

$$\langle I(X,Y)\rangle = \sum_{n=1}^{N}\left\langle |A_n(X,Y)|^2\right\rangle \quad (10)$$

Thus, Eq. (10) implies that the interference term in Eq. (6) has zero mean.

When considering the incoherent superposition of the speckle patterns obtained through the different aperture pairs in the pupil, it is apparent that the first order statistical properties of this pattern do not agree with those of the speckle pattern generated by using all the apertures. Two reasons explain this behavior: (a) The contrast of a speckle pattern resulting from the addition of intensities diminishes as the correlation between the adding speckle patterns decreases [20]. (b) The magnitude of the correlation between two speckle patterns associated with different pupils depends on the transmission area the respective pupils have in common [13]. In our case, the contrast of the synthesized speckle pattern does not reach the optimum value because we are adding the intensities of speckle patterns whose correlation is not complete. The speckle patterns obtained through two different double aperture pupils $P_{nm}(u,v) \equiv a_n(u,v) + a_m(u,v)$ and $P_{rs}(u,v) \equiv a_s(u,v) + a_r(u,v)$ are correlated only if there is one aperture these pupils have in common. Otherwise the correlation between the respective patterns is null.

Let us consider now the second order statistics of the modulated speckle patterns. The mutual intensity or the auto-correlation function of the complex amplitude of the image field can be expressed as [20]:

$$J_A(\Delta X, \Delta Y) = \langle A(X_1,Y_1)[A(X_2,Y_2)]^*\rangle = \frac{\kappa}{(\lambda Z_C)^2}\Im\left\{|P(\xi,\eta)|^2\right\}\left(\frac{\Delta X}{\lambda Z_C}, \frac{\Delta Y}{\lambda Z_C}\right) \quad (11)$$

where $(\Delta X, \Delta Y) = (X_2 - X_1, Y_2 - Y_1)$, κ is a constant factor and $\Im$ denotes a bidimensional Fourier transform. Then, the average intensity of the image speckle pattern is:

$$\langle I\rangle \equiv J_A(0,0) = \frac{\kappa}{(\lambda Z_C)^2}\Im\left\{|P(\xi,\eta)|^2\right\}(0,0) \tag{12}$$

By considering that the amplitude field is a circular complex Gaussian random variable with zero mean, the auto-correlation function of the speckle intensity is:

$$R_I(\Delta X,\Delta Y) = \langle I\rangle^2 + |J_A(\Delta X,\Delta Y)|^2 \tag{13}$$

To evaluate these magnitudes, we assume that the speckle pattern is formed by using a pupil that consists of identical apertures. In this case, every transmission function $a_n(u,v)$ is defined in terms of a single function $a(u,v)$ which is translated to different positions in the pupil plane, i.e. $a_n(u,v) = a(u-u_n, v-v_n)$, where (u_n,v_n) is a constant vector that specifies the position of the nth aperture, $n = 1,2,\ldots,N$. Note that, for a given function $a(u,v)$, the point (u_n,v_n) determines the geometrical locus of the aperture $a_n(u,v)$. In this sense we say that the aperture $a_n(u,v)$ is located at the point (u_n,v_n) (see Fig. 2).

On this basis and taking into account Eq. (2) we obtain:

$$|P(u,v)|^2 = \sum_{n=1}^{N}|a(u-u_n,v-v_n)|^2 \tag{14}$$

Then, by replacing Eq. (14) in Eq. (11) and by using the Fourier transform properties, it follows that:

$$J_A(\Delta X,\Delta Y) = J_0(\Delta X,\Delta Y)\sum_{n=1}^{N} \times \exp\left[-\mathrm{i}\frac{2\pi}{\lambda Z_C}(u_n\Delta X + v_n\Delta Y)\right] \tag{15}$$

where

$$J_0(\Delta X,\Delta Y) \equiv \langle A_0(X_1,Y_1)A_0^*(X_2,Y_2)\rangle = \frac{\kappa}{(\lambda Z_C)^2}\Im\left\{|a(u,v)|^2\right\}\left(\frac{\Delta X}{\lambda Z_C},\frac{\Delta Y}{\lambda Z_C}\right) \tag{16}$$

In this equation, $A_0(X,Y)$ represents the complex amplitude of the speckle field that would be obtained at (X,Y) if a single aperture $a(u,v)$ were employed. Then, the function $J_0(\Delta X,\Delta Y)$ defines the correlation of the complex amplitude of waves propagating from the aperture of the generic form $a(u,v)$ to the points (X_1,Y_1) and (X_2,Y_2) in the image plane.

Besides $A_n(X,Y)$, for $n = 1,2,\ldots,N$, represents the amplitude of the image formed by light passing exclusively through the nth aperture. Then, the amplitude field at a point (X,Y) in the image plane is obtained by the coherent superposition

$$A(X,Y) = \sum_{n=1}^{N} A_n(X,Y) \tag{17}$$

If we replace this result in the equation $J_A(\Delta X,\Delta Y) \equiv \langle A(X_1,Y_1)A^*(X_2,Y_2)\rangle$ and take into

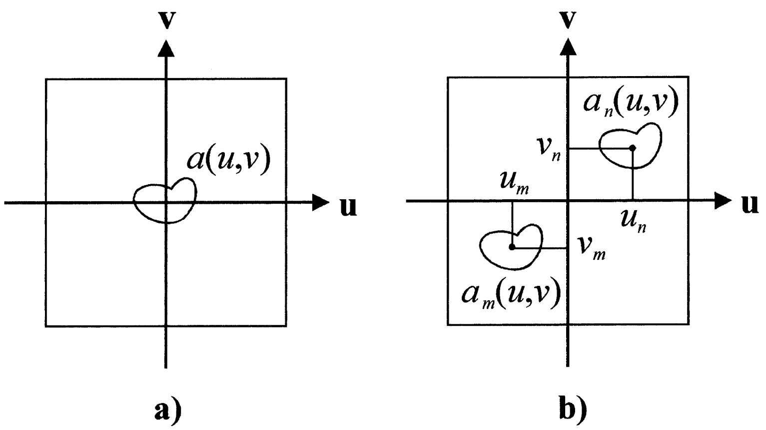

Fig. 2. Scheme of (a) a single aperture pupil and (b) a pupil with two identical apertures located at (u_n,v_n) and (u_m,v_m).

account that $\langle A_n(X_1, Y_1)A_m^*(X_2, Y_2)\rangle \equiv 0$ for $n \neq m$, then we find:

$$J_A(\Delta X, \Delta Y) = \sum_{n=1}^{N} \langle A_n(X_1, Y_1)A_n^*(X_2, Y_2)\rangle \quad (18)$$

By comparing Eqs. (18) and (15) it can be inferred that:

$$\langle A_n(X_1, Y_1)A_n^*(X_2, Y_2)\rangle = J_0(\Delta X, \Delta Y) \times \exp\left[-\mathrm{i}\frac{2\pi}{\lambda Z_C}(u_n \Delta X + v_n \Delta Y)\right] \quad (19)$$

The exponential function in Eq. (19) can be interpreted as follows. For a single point source located at the origin of the u–v plane we can associate certain phase delay $\Delta\phi_0$ with the complex amplitudes at points (X_1, Y_1) and (X_2, Y_2). Nevertheless, if this source were located at (u_n, v_n), then the incident waves at the points (X_1, Y_1) and (X_2, Y_2) would experience an additional phase delay given by $\Delta\phi_n = -\mathrm{i}(2\pi/\lambda Z_C)(u_n\Delta X + v_n\Delta Y)$. Then, the phase factor $\exp(-\mathrm{i}\Delta\phi_n)$ in Eq. (19) represents the effect associated with the aperture translation from the origin to the point (u_n, v_n) in the u–v plane.

By using Eqs. (18) and (19), it results that the average intensity is proportional both to the number of apertures and the $J_0(0,0)$ value:

$$\langle I\rangle = J_A(0,0) = NJ_0(0,0) = N\langle I_0\rangle \quad (20)$$

Note that this is a particular case of Eq. (10), in which the average intensity of speckles generated by each aperture in the pupil has the same value $\langle I_0\rangle = J_0(0,0)$.

Finally, let us evaluate the auto-correlation function of the intensity distribution. From Eq. (13) and by using Eqs. (15) and (20):

$$R_I(\Delta X, \Delta Y) = N^2\langle I_0\rangle^2 + |J_0(\Delta X, \Delta Y)|^2 \times \left[N + 2\sum_{\substack{n,m=1\\ n<m}}^{N} \cos\left\{\frac{2\pi}{\lambda Z_C}[(u_m - u_n)\Delta X + (v_m - v_n)\Delta Y]\right\}\right] \quad (21)$$

This equation allows us to conclude the following:

(i) The average transversal dimensions of speckles are determined by the mean width of the function $J_0(\Delta X, \Delta Y)$, which depends only on the generic intensity transmission function $|a(u,v)|^2$, irrespective of the location of the apertures in the pupil plane.

(ii) The individual speckles are internally modulated by fringes. In general, this is a complex system of fringes whose statistical characteristics are determined by:

$$N + 2\sum_{\substack{n,m=1\\ n<m}}^{N} \cos\left\{\frac{2\pi}{\lambda Z_C}[(u_m - u_n)\Delta X + (v_m - v_n)\Delta Y]\right\}.$$

(iii) Each pair of apertures $a(u - u_n, v - v_n)$ and $a(u - u_m, v - v_m)$, with n, $m = 1, 2, \ldots, N$ and $n \neq m$, can be associated with an elementary fringe system f_{nm}, specified by $\cos\{(2\pi/\lambda Z_C)[(u_m - u_n) \times \Delta X + (v_m - v_n)\Delta Y]\}$. The fringes of the f_{nm} system are perpendicular to the line joining the points (u_n, v_n) and (u_m, v_m). These fringes form an angle $\alpha_{nm} = \tan^{-1}[(v_m - v_n)/(u_m - u_n)]$ with the X axis. The average period of each fringe system is $\Lambda_{nm} = \lambda Z_C/d_{nm}$, where $d_{nm} = \left[(u_m - u_n)^2 + (v_m - v_n)^2\right]^{1/2}$ represents the mean separation between the respective apertures. As a consequence of the finite width of the apertures and due to the random variations of the field in the pupil plane, both the period and the orientation of each elementary fringe system f_{nm} vary from one speckle to another. As the dimensions of the apertures increase, the fluctuations of the speckle pattern modulation are more significant. Nevertheless, if the apertures $a_n(u,v)$ and $a_m(u,v)$ would be reduced to two point sources, then the period and fringe direction would be determined by the loci of the apertures.

(iv) The average modulation depth of each elementary fringe system f_{nm} is specified by the number of aperture pairs in the pupil that are responsible for that fringe formation. Specifically, the average magnitude of the modulation associated with a particular fringe system f_{nm} depends on the multiplicity of the pairs of apertures $a_i(u,v)$ and $a_j(u,v)$, for i, $j = 1, 2, \ldots, N$ and $i \neq j$, satisfying the relation $\pm(u_j - u_i, v_j - v_i) = (u_m - u_n, v_m - v_n)$.

Let us consider an example by employing the pupil schematized in Fig. 3. It consists of a mask

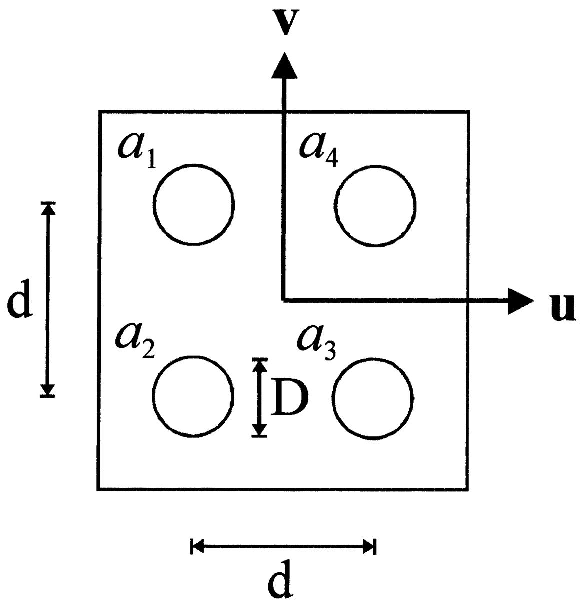

Fig. 3. Scheme of a pupil with four circular apertures of diameter D, located at the vertices of a square d on a side.

with four circular holes of diameter D and centered at the vertices of a square of side d. In this case, from Eq. (21) we obtain:

$$\begin{aligned} R_I(\Delta X, \Delta Y) = {} & 16\langle I_0\rangle^2 + |J_0(\Delta X, \Delta Y)|^2 \\ & \times \left\{ 4 + 4\cos\left(\frac{2\pi}{\lambda_W Z_C} d\,\Delta X\right) \right. \\ & + 4\cos\left(\frac{2\pi}{\lambda_W Z_C} d\,\Delta Y\right) \\ & + 2\cos\left[\frac{2\pi}{\lambda_W Z_C} d\sqrt{2}(\Delta X + \Delta Y)\right] \\ & \left. + 2\cos\left[\frac{2\pi}{\lambda_W Z_C} d\sqrt{2}(\Delta X - \Delta Y)\right] \right\} \end{aligned} \quad (22)$$

where according to Eq. (16):

$$J_0(\Delta X, \Delta Y) = \langle I_0\rangle \frac{2J_1\left(\dfrac{\pi D\sqrt{(\Delta X)^2 + (\Delta Y)^2}}{\lambda_W Z_C}\right)}{\dfrac{\pi D\sqrt{(\Delta X)^2 + (\Delta Y)^2}}{\lambda_W Z_C}} \quad (23)$$

and

$$\langle I_0\rangle = J_0(0, 0) = \frac{\kappa}{(\lambda Z_C)^2}\pi\left(\frac{D}{2}\right)^2 \quad (24)$$

In Eq. (22) we can identify four different elementary fringe systems, one running parallel to the Y axis, other running parallel to the X axis and others forming angles of 45° and 135° with respect to the X axis, respectively. The period of the diagonal fringes is less than the period of the other fringe systems, which is in agreement with the distances between the apertures that contributes to form the respective fringe systems. Nevertheless, the coefficient associated with the diagonal fringe system is a half of the other coefficients, because there is only one pair of apertures associated to each diagonal fringe system formation, meanwhile there are two aperture pairs in the pupil responsible for the vertical and the horizontal fringe formation. The fringe generated by using the apertures a_1 and a_2 has the same period and orientation as the fringe system formed by the apertures a_3 and a_4. The respective speckle patterns are statistically independent from each other, because the two pupils $P_{12} = a_1 + a_2$ and $P_{34} = a_3 + a_4$ have no transmission areas in common. A similar analysis could be applied for the speckle patterns whose fringes run vertically. Finally, the magnitude of the average intensity of the speckle pattern is proportional to the area of one aperture times the number of apertures, i.e. proportional to the total transmission area of the pupil, irrespective of the particular shape of the apertures.

3. Experimental results

To illustrate the theoretical analysis, we present different speckle patterns obtained with the experimental arrangement of Fig. 1. The parameters employed are $\lambda = 514$ nm, $Z_0 = 138$ mm and $Z_C = 382$ mm. The experimental images were recorded by using a CCD camera with a zoom-microscope focused on the observation X–Y plane. Each image shows the intensity distribution corresponding to the same region whose actual dimensions are 0.83 mm × 0.63 mm. We used the pupil mask schematized in Fig. 3, with circular holes of diameter $D = 4.8$ mm and centered at the vertices of a square $d = 9.2$ mm on a side. Under these conditions, the average diameter of imaged speckles was $\delta t = 1.2(\lambda Z_C/D) \approx 49$ μm.

Fig. 4 depicts the speckle patterns generated by light passing through each pupil aperture. These speckle patterns are statistically independent because each aperture accepts a distinct region of the angular spectrum of the diffuser. In fact, there is no apparent relation among these images.

Fig. 5 shows the speckle patterns obtained by using a mask formed with different aperture pairs of the pupil schematized in Fig. 3. The images correspond to pupil masks with the apertures: (a) a_2 and a_3; (b) a_1 and a_2; (c) a_1 and a_3; (d) a_1 and a_4;

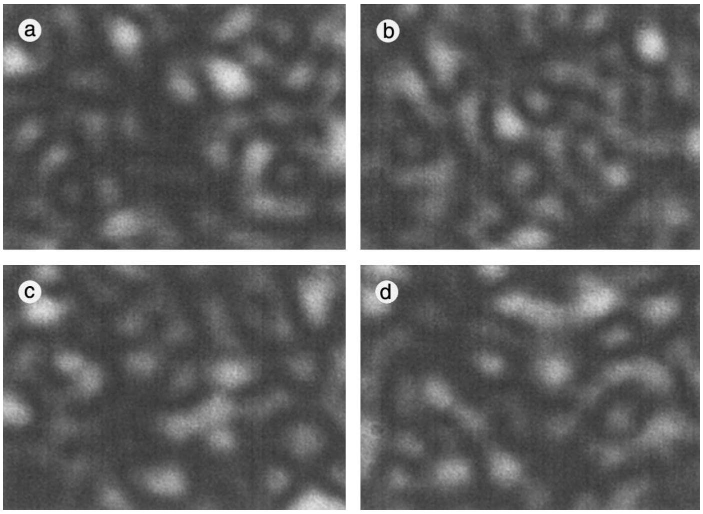

Fig. 4. Speckle patterns obtained by employing different single aperture pupils. By referring to Fig. 3, light goes through (a) a_1, (b) a_4, (c) a_2 and (d) a_3.

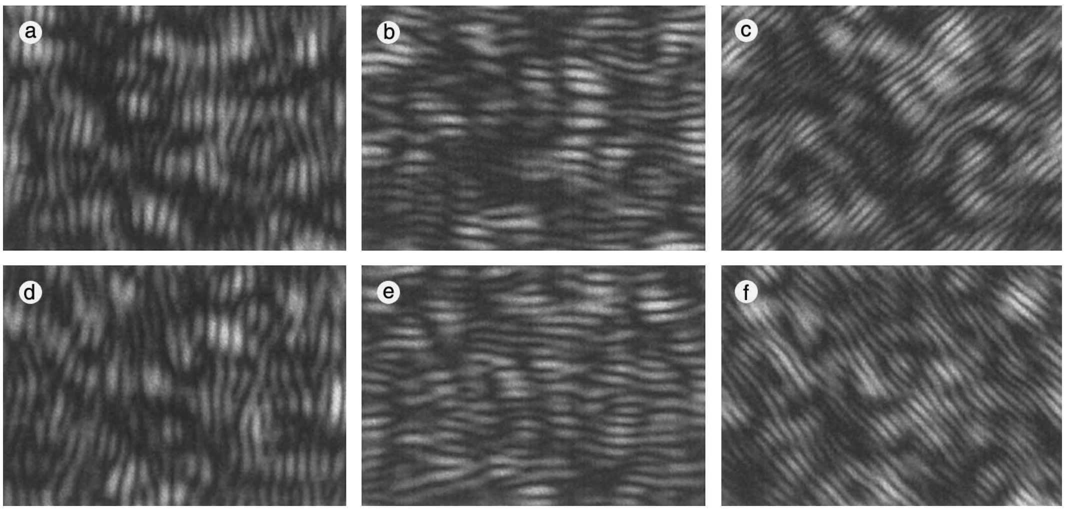

Fig. 5. Speckle patterns obtained by using only two apertures of the pupil schematized in Fig. 3. Scattered light goes through the apertures (a) a_2 and a_3, (b) a_1 and a_2, (c) a_1 and a_3, (d) a_1 and a_4, (e) a_3 and a_4, (f) a_2 and a_4, respectively.

(e) a_3 and a_4, and (f) a_2 and a_4, respectively. Note that the speckle patterns of Fig. 5(a) and (d), (b) and (e), (c) and (f) are statistically independent from each other. Otherwise, the correlation between the speckle patterns is partial. Besides, it is apparent that the orientation of fringes is approximately perpendicular to the line joining the centers of the respective apertures and, as can be confirmed by observing Fig. 5, the period, orientation and phase of the fringe system modulating the individual speckles vary slightly from one speckle to another. Also the average period decreases as the distance between the apertures increases. In fact, the average period of fringes belonging to the diagonals of the pupil, in Fig. 5(c) and (f), is lower than the period every other images.

The image of Fig. 6(a) corresponds to a pupil consisting of the apertures a_1, a_2 and a_3 schematized in Fig. 3. As expected, the internal modulation of speckles includes vertical, horizontal and diagonal fringes. In Fig. 6(b) we digitally added the images of Fig. 5(a) to (c). This result represents the incoherent superposition of the speckle patterns individually generated by the aperture pairs: a_1 and a_2, a_1 and a_3, a_2 and a_3. Note that the contrast of fringes in Fig. 6(b) is lesser than the contrast in Fig. 6(a) because the former pattern results from an addition of the intensities associated with partially decorrelated speckle patterns. Besides, the mean period and orientation of the fringe systems modulating the individual speckles in Fig. 6(a) and (b) coincide.

Fig. 6(c) shows the addition of the speckle patterns of Fig. 4(a), (c) and (d), which were formed through the apertures a_1, a_2 and a_3, respectively. On the other hand, Fig. 6(d) depicts the result of subtracting the image of Fig. 6(c) from the image of Fig. 6(b). The similarity of the images in Fig. 6(a) and (d) confirms us that we can synthesize the speckle pattern formed with multiple apertures by operating over more elementary speckle patterns, generated by the individual apertures and the different aperture pairs in the pupil. Note that this is in complete agreement with Eq. (8) for $N = 3$.

Now let us compare the incoherent superposition of the speckle pattern formed by the single aperture pupils of Fig. 6(c) and the speckle pattern obtained by simultaneously using all the apertures, depicted in Fig. 6(a). According to Eq. (6) these patterns differ solely in that the speckles are internally modulated in Fig. 6(a). This modulation is determined by the interference term of Eq. (6) of

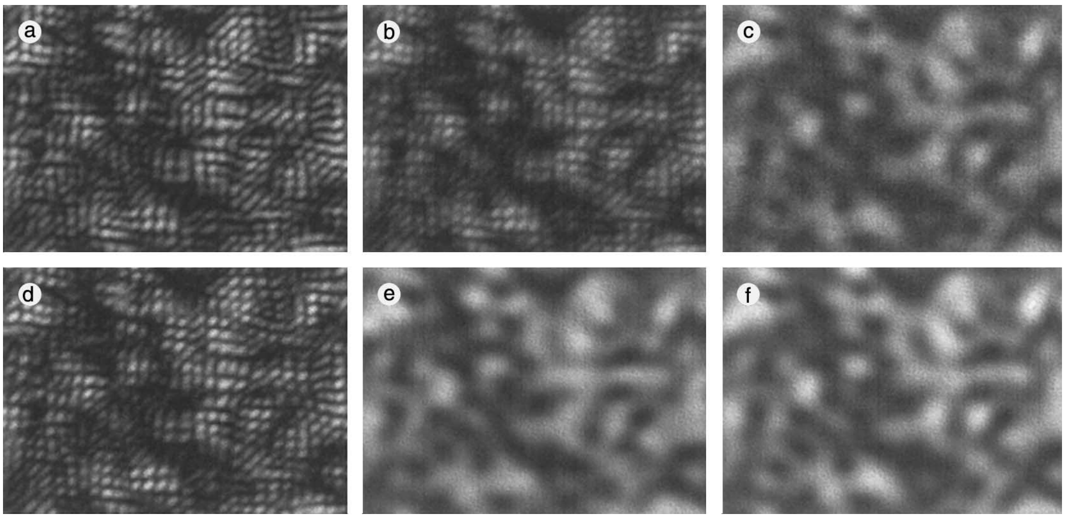

Fig. 6. Speckle patterns obtained by (a) using simultaneously the apertures a_1, a_2 and a_3; (b) adding the speckle patterns generated by the aperture pairs; (c) adding the speckle patterns generated by the apertures; (d) subtracting the image in (c) from the image in (b); (e) filtering the image in (a); (f) filtering the image in (c).

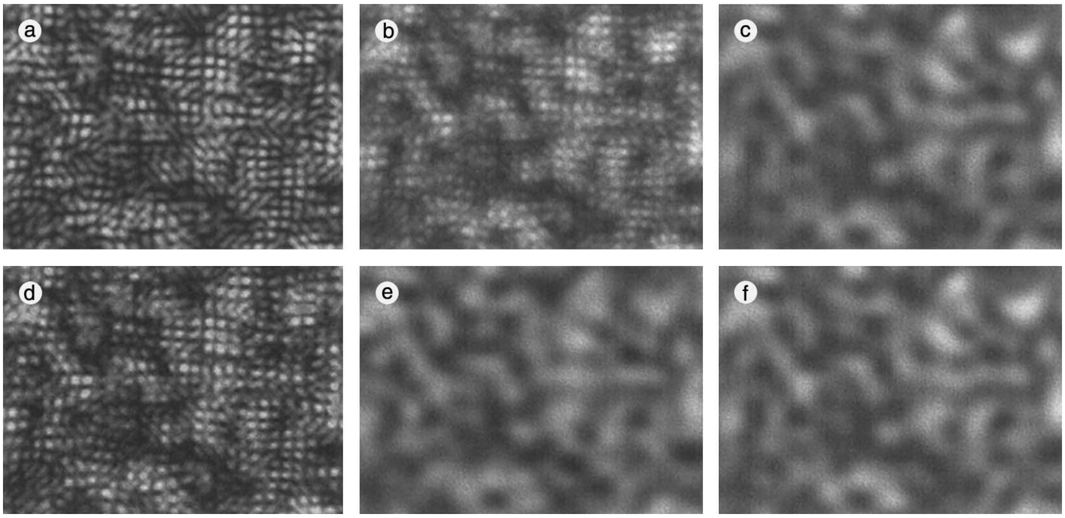

Fig. 7. Speckle patterns obtained by (a) using simultaneously the apertures a_1, a_2, a_3 and a_4; (b) adding the speckle patterns generated by the aperture pairs; (c) adding the speckle patterns generated by the apertures; (d) subtracting twice the image in (c) from the image in (b); (e) filtering the image in (a); (f) filtering the image in (c).

zero mean. As expected, by low-pass filtering the images in Fig. 6(a) and (c) we obtain the same intensity distribution, as shown in Fig. 6(e) and (f).

Finally, a similar sequence of images is presented in Fig. 7. In this case the four-aperture pupil of Fig. 3 is of concern. The image in Fig. 7(a) is obtained by using the four-aperture pupil. Fig. 7(b) corresponds to the incoherent superposition of the six images formed through the different aperture pairs in the pupil and Fig. 7(c) corresponds to the incoherent superposition of the four patterns generated by the individual apertures. The image in Fig. 7(d) was synthesized by subtracting twice the image in Fig. 7(c) from the image in Fig. 7(b), as it was established in Eq. (8) for $N = 4$. The images in Fig. 7(e) and (f) are the filtered versions of the images in Fig. 7(a) and (c), respectively. These results can be interpreted as in Fig. 6 to validate the theoretical analysis.

4. Conclusions

We studied the features of speckle distributions generated by using an optical system which has a multiple aperture pupil. It was shown that it is possible to synthesize and interpret the speckle pattern obtained through multiple aperture pupils on the basis of rather elementary speckle patterns. The intensity distribution that characterizes the internal modulation of the individual speckles can be understood in terms of the incoherent superposition of all the fringe systems independently generated by the distinct pairs of apertures in the pupil. Both the orientation and period of each one of these fringe systems depend on the relative position of the apertures. Nevertheless, to synthesize the intensity pattern corresponding to a multiple aperture pupil, we must take into account the incoherent superposition of the speckle patterns independently formed by the different apertures, including a weight factor determined by the total number of apertures.

We investigated the statistical properties of the speckle pattern obtained through multiple aperture pupils on the basis of the evaluation of the auto-correlation functions both for the complex amplitude and the intensity distribution of the modulated speckles in the image field. These functions are defined in terms of the squared modulus of the pupil function. The intensity distribution of the modulated pattern consists of two terms. One term is the result of the incoherent superposition of all the speckle patterns individually generated by the different apertures. The remaining term includes the contributions associated with all the different aperture pairs in the pupil. This term has zero mean, then the average intensity of the modulated speckle pattern coincides with the addition of the average intensities of the speckle patterns formed through the individual apertures.

For an optical system whose pupil consists of an arrangement of identical apertures, the transmission function associated with each aperture can be defined in terms of a single function shifted to different points in the pupil plane. On this basis we demonstrated that the average intensity of the modulated pattern is proportional to the number of apertures and to the average intensity of every pattern formed by a single aperture. Moreover, the mutual intensity in the image field can be expressed as the addition of the mutual intensities associated with the speckle patterns generated through the individual apertures. Each term contributing to this addition is multiplied by a characteristic phase factor that depends on the position of the respective aperture. Besides, the transversal dimensions of speckles are not affected by the aperture location, but their dimensions are determined by the squared modulus of the generic transmission function of the apertures.

Also the evaluation of the auto-correlation of the intensity in the image plane shows that fringes modulating the individual speckles can be interpreted in terms of a set of sinusoidal fringes associated with the different aperture pairs in the pupil. The average values of the period, the direction and the modulation depth of these fringe systems were determined in terms of the pupil parameters.

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