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Analysis of the procedure for suspension evaluation of civil armoured vehicles: reliability and safety driving criteria

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Abstract: The present study exposes an analysis of the suspension system technical state evaluation for civil vehicles that have been subject to armouring processes. Such evaluation is performed through a mechanised revision established by state regulation and is based on the method defined by EuSAMA. The development of this analysis focuses on establishing the existing relation between the ballistic resistance integrated to a vehicle and the dynamic effect exercised for the modification of its mass, according to two reliability/safety driving measurement criteria: (i) tyre-road adhesion index and (ii) tyre excitation phase angle. The study proposes new elements to the current procedure established to evaluate the suspension of civil armoured cars considering the two measurement criteria, which can be acquired by a standard commercial suspension tester machine.

Keywords: adhesion index; armoured vehicle; EuSAMA; phase angle; safety driving criterion; suspension system; viscous damping ratio.

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1 Introduction

State regulation of the members of the European Union, United States, Japan, Colombia, amongst others, establishes that there must exist a periodic technical revision for the different systems that vehicles possess (see Table 1): (i) suspension system, (ii) state of the bodywork, (iii) gas emission level, (iv) functioning of mechanical, electrical and optical assembly systems, (v) efficiency of the internal combustion system, and (vi) state of security elements, tyres and brakes system (STTB, 2002). The regulation establishes two types of periodic revision for the suspension system (NTC 5375, 2006):

1 visual inspection, examines the state of: (a) fixations and suspension components, (b) presence of fissures, (c) corrosion symptoms, (d) existence of welded repairs and (e) presence of damages, deformations and oil leakage (MITC, 2006); and

2 mechanised revision, identifies the suspension state according to the method denominated EuSAMA (EuSAMA TS-02-76, 1976), which has been a base document to studies from equipments for the evaluation of vehicles suspension system conditions (SAE 960735, 1996; Balsarotti and Bradley, 2000); and national standards (NTC 5375, 2006; NTC 5385, 2006).

Whilst there has been a significant increase in the amount of consumer interest in the driving safety performance of privately owned vehicles, the role that it plays in consumers' purchase decisions is poorly understood (Koppel et al., 2008), especially relative to Civil Armoured Vehicles (CAVs). This work develops a study of the results obtained through the mechanised revision established by state regulation, focused on the evaluation of the suspension system for commercial vehicles that have been subject to armouring processes (glass and bodywork reinforced with ballistic type material), incorporating additional mass (m_a) to the design mass of the vehicle (m_v) and therefore causing an increase in the total mass of the vehicle (m_t), where $m_t = m_v + m_a$, which affects the reliability/safety driving criteria.

Table 1 State regulation for vehicle revision

Country/region	*Periodicity*	*Regulatory entity*	*Issue date*
European Union	Biennial	Council policy 96/96/CE	1996
USA	Biannual/Annual*	Office of the Law Revision Counsel	2006
Japan	Biennial	National Agency of Vehicle Inspection	–
Colombia	Biennial	Transit and Transport Ministry	2002

Note: * Varies according to the laws of every state.

The current evaluation of CAVs suspension system is performed following the guidelines of the method established by EuSAMA, which uses the criteria in function of the vertical oscillation frequency of the tyre (ω_3), denoted adhesion index[1] $A(\omega)$, where $A(\omega) \propto m_t$. The need of including an additional criterion for the safety state and comfort evaluation in CAVs has been identified. The present study proposes to include the phase angle[2] $\Psi(\omega)$ criterion, where $\Psi(t) \propto \frac{1}{m_t}$. Such method has been established by SAE (SAE 960735, 1996).

2 Description of the object of study

The amount of CAVs has an annual growth rate in the countries of the American continent, especially in Colombia (see Table 2), and its armour level (*BR*) is defined according to the ballistic resistance of the glasses, establishing a range of five levels, $BR = \{I, II, III, IV, V\}$, V being the highest ballistic resistance (NTC 5501, 2007). The magnitude of additional mass incorporated to CAVs is found in the range $m_a = \{100,\ldots,1000\}$ kg, where $BR \propto m_a$. Therefore, it is possible to establish the following relations (see Table 3):

1 given an armour level $BR = I \therefore m_a \approxeq 100$ kg,

2 given an armour level $BR = V \therefore m_a \approxeq 1000$ kg, and

3 given an armour level $BR = IV \therefore m_a \gtrsim 500$ kg, where the m_a value exceeds the maximum design load (C_{max}) of a standard vehicle, this is $m_a \gg C_{max}$, thus requiring modifying the suspension system to conserve the design safety standards.

Table 2 Quantity of CAVs in Colombia

Year	*Authorised armours*	*Nr. accumulated authorisations*	*Authorised disarmours*	*Total armoured vehicles*
2004	–	8,753	–	8,753
2005	827	9,580	14	9,566
2006	911	10,491	36	10,455
2007	916	11,407	25	11,382
2008	795	12,202	72	12,130
2009	963	13,165	35	13,130
2010	993	14,158	50	14,108
2011*	1,408	15,566	97	15,469

Note: * To 8th November 2011.

Table 3 Relation of the armour level in civil vehicles (CEN/BS/EN 1063, 2000)

BR	*Weapon*	*Calibre*	*Materials*	*m_a [kg]*
I	Handgun/Rifle	0.22 LR	Aramid	{90,…,110}
II	Handgun	9 × 19 mm Para.	Aramid	{90,…,110}
III	Handgun	0.357 Magnum	Aramid, balistic steel	{110,…,180}
IV	Handgun	0.44 Magnum	Steel, aluminium, dyneema	{200,…,450}
V	Rifle	7.62 × 51mm NATO	Special steel, aluminium, dyneema, aramid, ceramic	{500,…,1000}

3 Proposed procedure to evaluate CAVs suspension

The proposed procedure to evaluate the CAVs suspension state considers two measurement criteria: (i) ${}^{k}A_{\min}$ and (ii) ${}^{k}\Psi_{\min}$, which can be acquired by a standard commercial suspension tester machine.

${}^{k}A_{\min}$ is defined as the ratio between the minimum vertical force $\min({}^{k}F_{23})$ in the contact surface of the *k*-th tyre (unsprung mass ${}^{k}m_2$), and the static load ${}^{k}P$ exercised by the corresponding unsprung mass ${}^{k}m_2$ (SAE 960735, 1996), this is:

$$ {}^{k}A_{\min}[\%] = 100\frac{\min({}^{k}F_{23})}{{}^{k}P}. \tag{1}$$

${}^{k}\Psi_{\min}$ is defined as the minimum angular difference between the vertical position of the excitation platform (X_3) and the vertical position of ${}^{k}m_2$ in relation to the platform X_{23} (SAE 960735, 1996). X_3 is expressed as a sinusoidal function based on the movement equation

$$X_3(t) = a \cdot \sin(\omega_3 \cdot t + \Phi_3), \tag{2}$$

where *t* is the instant in time of the test; *a* the amplitude of the platform displacement; ω_3 the platform excitation frequency at instant *t* and Φ_3 the phase.

X_{23} is indirectly found using the magnitude of the tyre–platform contact force ${}^{k}F_{23}(t)$, expressed as a sinusoidal function

$$ {}^{k}F_{23}(t) = F_0(t) \cdot \sin(\omega_{23} \cdot t) \tag{3}$$

where $F_0(t)$ is the amplitude of the force for every instant *t*; and ω_{23} the response frequency of the unsprung mass.

Expressing the platform displacement angle as $\Psi_3(t) = \omega_3 \cdot t$, and the displacement angle of ${}^{k}m_2$, as $\Psi_{23}(t) = \omega_{23} \cdot t$; the equations (2) and (3) can be expressed as the following:

$$\Psi_3(t) = sen^{-1}\left(\frac{X_3(t)}{a}\right); \quad \Psi_{23}(t) = sen^{-1}\left(\frac{{}^{k}F_{23}(t)}{F_0(t)}\right) \tag{4}$$

Therefore, the phase angle for the *k*-th unsprung mass ${}^{k}m_2$ can be expressed as ${}^{k}\Psi(t)[\text{deg}] = {}^{k}\Psi_3(t) - {}^{k}\Psi_{23}(t)$.

The evaluation is performed independently to each unsprung mass (to each tyre of the CAV) with a suspension tester machine where (SAE 960735, 1996):

(i) the machine registers the static load of the *k*-th tyre ${}^{k}P$;

(ii) the platform exercises initial oscillation frequency $\omega_{3,o} = 0$ Hz increasing the frequency to $\omega_{3,f} = 25$ Hz, with a constant amplitude *a* = 6 mm and

(iii) the equipment registers ${}^{k}F_{23}(t)$, and the position of the platform $X_3(t)$.

Previous studies (Tsymberov, 1994; SAE 960735, 1996) have established four acceptance states that clearly qualify the evaluation of vehicle suspension systems, according to the values obtained for minimum adhesion index and minimum phase angle, ${}^{k}A_{\min}$ and ${}^{k}\Psi_{\min}$ (see Table 4).

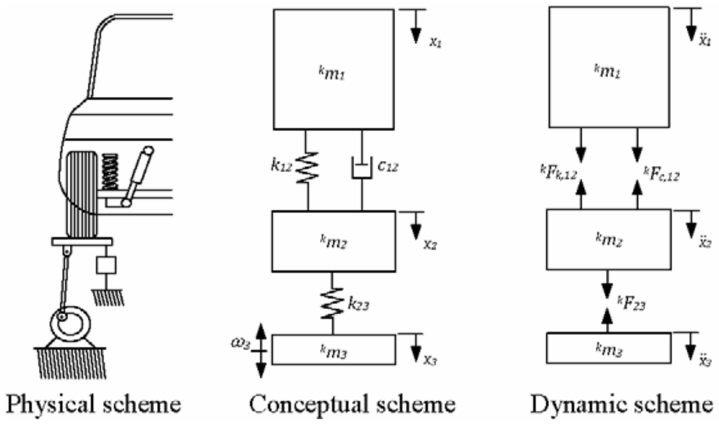
Table 4 Evaluation of vehicle suspension systems (Tsymberov, 1994; SAE 960735, 1996)

Criterion	*Evaluation of vehicle suspension systems*			
	Excellent	*Good*	*Fair*	*Deficient*
${}^{k}A_{\min}$ [%]	{60,…,100}	{40,…,60}	{20,…,40}	{0,…,20}
${}^{k}\Psi_{\min}$ [deg]	{60,…,180}	N.A.	{30,…,60}	{0,…,30}

4 Development of the numeric model

The developed model represents the dynamic behaviour equivalent to quarter-car (Gáspár et al., 2007) see Figure 1, through two Degrees of Freedom (DoF) and considering a system of three masses (Haroon and Adams, 2008; Pourqorban et al., 2010): (i) platform ${}^{k}m_3$; (ii) tyre or unsprung mass ${}^{k}m_2$ and (iii) sprung mass ${}^{k}m_1$. The relation of the masses is $m_t \cong \sum_{k=1}^{4}\sum_{r=1}^{2} {}^{k}m_r$.

Figure 1 Model of one fourth of a vehicle with 2 DoF



The development of the model has been structured in three stages:

1 model with mechanical properties of a commercial vehicle, denoted reference model (see Table 5);

2 modelling of CAVs with armour levels $BR = \{I,\ldots,V\}$, conserving the characteristics of the standard commercial vehicle suspension system and

3 modelling of CAVs with armour levels $BR = \{I,\ldots,V\}$, and with modification of the properties of the suspension system: (i) loss of properties (ageing of the elements) and (ii) improvement of the damping properties.

The numeric model has been subject to a validation process regarding previous experimental studies (Arbeláez et al., 2007), which develop a study through a test bench to analyse the adhesion in the evaluation of light vehicle suspensions.

Table 5 General parameters of the model

${}^k m_1$ [kg]	${}^k m_2$ [kg]	k_{12} [kN/m]	k_{23} [kN/m]	C_{12} [kNs/m]	x_3 [m]	ω_3 [Hz]
173.000	35.000	18.709	127.200	1.300	6×10^{-3}	{0,…,25}

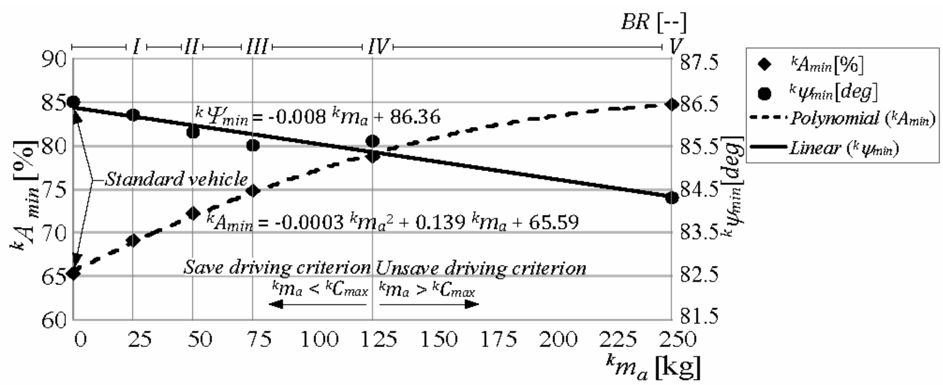
5 Analysis of the results of the numeric model

The response of the numeric model is registered in each of the three stages of the development of the model, the obtained data are processed then with a set of algorithms structured according to the proposed procedure for the evaluation of CAVs suspension (described in Section 3), allowing to find ${}^k A_{\min}$ and ${}^k \Psi_{\min}$.

5.1 Stages 1–2: CAVs with standard vehicle suspension

The analysis represents the dynamic behaviour of the vehicle with the added mass due to armouring m_a, conserving the characteristics of the standard commercial vehicle suspension system. A set of simulations are performed with different armour levels $BR = \{I, \ldots, V\}$, defining as the parametric variable the sprung mass ${}^k m_1$, to simulate the overload of the vehicle due to the mass m_a. Assuming that m_a is evenly distributed in the vehicle (Martinod et al., 2012), it is possible to state that the value ${}^k m_a = \frac{m_a}{4}$ is added to the sprung mass ${}^k m_1$. The parametric space of the added mass from the armouring process is ${}^k m_a$ = (25, 50, 75, 125, 250} kg, equivalent to incorporating armour levels $BR = \{I, \ldots, V\}$. Figure 2 exposes the dependence of criteria ${}^k A_{\min}$ and ${}^k \Psi_{\min}$ to the variation of ${}^k m_a$ in the armouring process.

Figure 2 Development of regressive models ${}^k A_{\min}$ and ${}^k \Psi_{\min}$ in function of ${}^k m_a$



5.1.1 Analysis of results of criterion ${}^{k}A_{\min}$

${}^{k}A_{\min}$ possess a directly proportional tendency, obtaining a second order polynomial regressive model with a correlation coefficient of $\sqrt{R^2} > 0.99$,

$$ {}^{k}A_{\min}[\%] = -0.0003{}^{k}m_a{}^2 + 0.1394{}^{k}m_a + 65.598. \tag{5}$$

The regressive model is considered valid given that the $\sqrt{R^2}$ value represents the association measure of the statistic model with the obtained data (Grant, 1964), which have an acceptable level for the scope of this study.

The maximum design load equivalent to ¼-standard-vehicle is ${}^{k}C_{\max} \cong \frac{500}{4}$ kg, which represents the limit value to which the vehicle can be loaded with added weight without requiring modification of the suspension system. However, the following relation is observed if ${}^{k}m_a = \{{}^{k}C_{\max},\ldots,250\}$ kg $\therefore$ ${}^{k}m_{\min} \geq 80\%$, such relation expresses that: the evaluation criterion for the evaluation of the suspension state ${}^{k}A_{\min}$ qualifies as *excellent* the behaviour of CAVs standard suspension, even in cases in which the maximum design load ${}^{k}C_{\max}$ is exceeded; furthermore, the criterion ${}^{k}A_{\min}$ signals that the suspension state for CAVs improves indefinitely with the increase of extra mass ${}^{k}m_a$. Therefore, it is possible to affirm that the criterion ${}^{k}A_{\min}$ is not enough for evaluating CAVs suspension state.

5.1.2 Analysis of the results of criterion ${}^{k}\Psi_{\min}$

${}^{k}\Psi_{\min}$ presents an inversely proportional behaviour, obtaining a linear regressive model

$$ {}^{k}\Psi_{\min} = -0.0082{}^{k}m_a + 86.363, \tag{6}$$

with correlation coefficient of $\sqrt{R^2} > 0.97$, which is considered valid for the scope of this study.

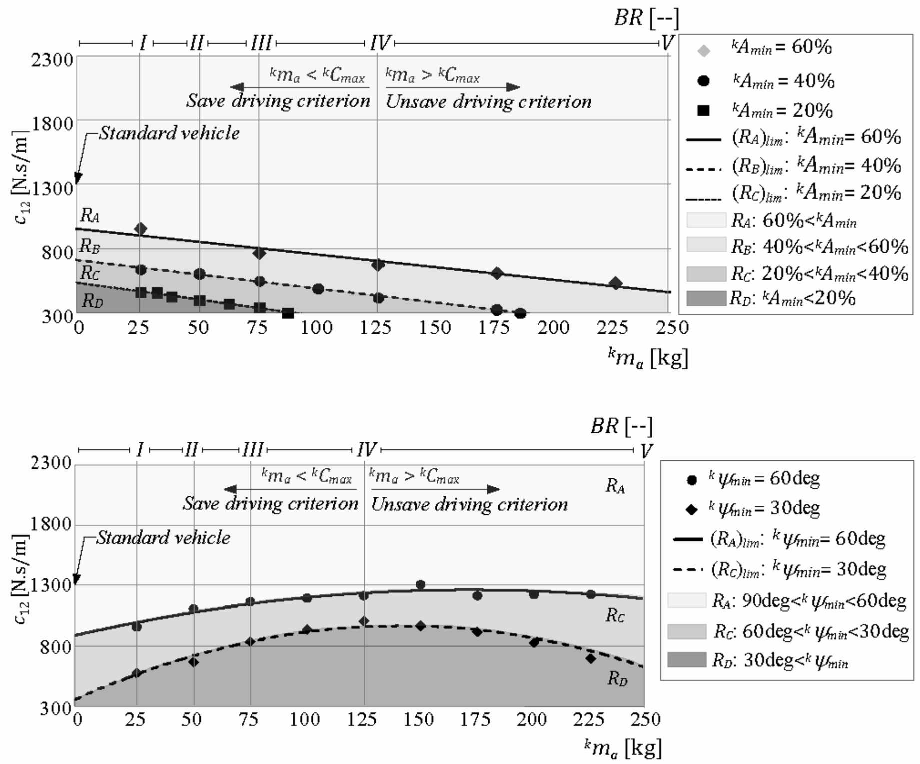
The evaluation criterion for the suspension behaviour ${}^{k}\Psi_{\min}$ presents a coherent relation with CAVs suspension state, where it is possible to propose as limit evaluation value of suspension state for CAVs $\left({}^{k}\Psi_{\min}\right)_{\lim} = 85.5$ deg, this limit value is highly sensitive to the maximum design load, for which it is enough for CAVs evaluation of suspension state.

5.2 Stage 3: CAVs with variation of suspension properties

The parameter ${}^{k}m_a$ is defined by the armour levels $BR = \{I,\ldots,V\}$, this is it has parametric space ${}^{k}m_a$ = {25, 50, 75, 125, 250} kg. The parameter c_{12} has parametric space c_{12} = {300,…,2300} Ns/m, this damping range consider the extreme damping state values: defective and excessively rigid.

Figure 3 exposes the dependency of criteria ${}^{k}A_{\min}$ and ${}^{k}\Psi_{\min}$ to the variation of parameters ${}^{k}m_a$ and c_{12} in the armouring process. Four areas that classify the suspension state are observed, $RE = \{R_A, R_B, R_C, R_D\}$, which coincide with the evaluation of vehicle suspension systems (defined in Table 4), where R_A: Excellent, R_B: Good, R_C: Fair, R_D: Deficient.

Figure 3 Development of regressive models ${}^kA_{\min}$ and ${}^k\Psi_{\min}$ in function of ${}^km_a$ and c_{12}



5.2.1 *Analysis of the results of criterion* ${}^kA_{\min}$

Each area $RE = \{R_A, R_B, R_C, R_D\}$ has a boundary parametric function, which possess inverse relation $C_{12} \propto \frac{1}{{}^km_a}$:

$$\text{for } (R_A)_{\lim} \to C_{12} = -1.98\ {}^km_a + 955.46, \text{ with } \sqrt{R^2} = 0.97;$$

$$\text{for } (R_B)_{\lim} \to C_{12} = -2.19\ {}^km_a + 708.91, \text{ with } \sqrt{R^2} = 0.99;$$

$$\text{for } (R_C)_{\lim} \to C_{12} = -2.57\ {}^km_a + 533.70, \text{ with } \sqrt{R^2} = 0.99.$$

Considering the entire parametric field possible ({0,…,250}{300,…,2300}), each area *RE* possesses the following proportion: R_A: 78%, R_B: 14%, R_C: 6%, R_D: 2%. This criterion ${}^kA_{\min}$ is permissive with respect to parametric variables. Additionally, ${}^kA_{\min}$ qualifies as *excellent*, the behaviour of CAVs standard suspension (even allowing a reduction of suspension the dynamic properties), still in cases in which the maximum design load, ${}^{k}C_{\max}$, is exceeded.

Note that criterion ${}^{k}A_{\min}$ qualifies as *excellent*, the behaviour of CAVs suspension with a damping ration c_{12} that has values lower than the 60% of the studied nominal property. This decrease of the damping ratio value is equivalent to 60% of damping wear.

Thus, once again it is possible to affirm that criterion ${}^{k}A_{\min}$ is not enough for evaluating the CAVs suspension state.

5.2.2 Analysis of the results of criterion ${}^{k}\Psi_{\min}$

Each area has a boundary parametric function of second order:

$$\text{for } (R_A)_{\lim} \to C_{12} = -0.013\ {}^{k}m_a^{\,2} + 4.37\ {}^{k}m_a + 885.97, \text{ with } \sqrt{R^2} = 0.94;$$

$$\text{for } (R_C)_{\lim} \to C_{12} = -0.029\ {}^{k}m_a^{\,2} + 8.47\ {}^{k}m_a + 359.35, \text{ with } \sqrt{R^2} = 0.96.$$

Considering the entire parametric field possible, each area *RE* possesses the following proportion: R_A: 56%, R_C: 18%, R_D: 25%. However, criterion ${}^{k}\Psi_{\min}$ is sensitive to the extra load ${}^{k}m_a$.

6 Conclusions

The present study has allowed to identify the existing relation between the CAVs armour level *BR*, with the suspension state; using two evaluation criteria for suspension condition: $A_{\min}$ and $\Psi_{\min}$.

The evaluation of the suspension system state for CAVs under $A_{\min}$ criterion established by EuSAMA regulation does not guarantee an accurate diagnostic.

The procedure proposed for the evaluation of CAVs suspension state, which considers evaluation criterion $A_{\min}$ and $\Psi_{\min}$ offers an accurate diagnostic of the state of CAVs suspension systems.

It is possible to integrate the proposed procedure for the evaluation of CAVs suspension state to current procedures used in periodic vehicle revision in countries that include vehicle revision in state regulation (European Union, United States, Japan, Colombia, within others).

Evaluation criteria $A_{\min}$, $\Psi_{\min}$, can be acquired by a standard commercial suspension tester machine, without generating additional costs to accredited entities for regulation and periodic vehicle revision of the suspension system.

The present study can be a point of reference for different cases of analysis of vehicle suspension when extra load has been added to the nominal standard design. Therefore, this methodology can be applied for other studies on suspension where the mass of the vehicle has been modified.

References

Arbeláez, J.J., Marín, J.P. and Calle, T. (2007) 'Modelado, diseño y construcción de un banco de pruebas para el análisis de la adhesión en la evaluación en suspensiones de vehículos livianos bajo la norma European Shock Absorber Manufacturers Association (EuSAMA)', *8th Iberoamerican Congress of Mechanical Engineering*, October, Vol. 1, No. 1, pp.141–150.

Balsarotti, S. and Bradley, W. (2000), 'Experimental evaluation of a non-intrusive automotive suspension testing apparatus', *SAE Technical Paper 2000-01-1329*, in SAE 2000 World Congress, March 2000, doi:10.4271/2000-01-1329.

CEN/BS/EN 1063, code (2000) 'Glass in building. security glazing – testing and classification of resistance against bullet attack', *European Committee for Standardization*.

EuSAMA TS-02-76, code (1976) 'Recommendation for performance test specification of car vehicle suspension testing system', *European Shock Absorber Manufacturers Association*.

Gáspár, P., Szabó, Z. and Bokor, J. (2007) 'Parameter identification of suspension system and road disturbance estimation', *Int. J. Vehicle Systems Modelling and Testing*, Vol. 2, No. 2, pp.128–137.

Grant, E.L. (1964) *Statistical Quality Control*, 3rd ed., McGraw Hill, USA.

Haroon, M. and Adams, D.E. (2008) 'Component restoring force identification for damage identification in vehicle suspension systems', *Int. J. Vehicle Systems Modelling and Testing*, Vol. 3, Nos. 1–2, pp.25–46.

Koppel, S., Charlton, J., Fildes, B. and Fitzharris, M. (2008) 'How important is vehicle safety in the new vehicle purchase process?' *Accident Analysis and Prevention*, Vol. 40, No. 3, pp.994–1004.

NTC 5375 (2006) 'Revisión técnico-mecánica y de emisiones contaminantes en vehículos automotores', *Instituto Colombiano de Normas Técnicas y Certificación (ICONTEC)*, 1st ed., Colombia.

NTC 5385 (2006) 'Centros de diagnóstico automotor' *Instituto Colombiano de Normas Técnicas y Certificación (ICONTEC)*, 1st ed., Colombia.

NTC 5501 (2007) 'Clasificación de los vidrios (Acristalamientos) resistentes a las balas. Ensayos balísticos' *Instituto Colombiano de Normas Técnicas y Certificación (ICONTEC)*, 1st ed., Colombia.

Martinod, R., Betancur, G. and Castañeda, L. (2012) 'Identification of the technical state of suspension elements in railway systems', *Int. J. Vehicle System Dynamics*, Vol. 50, No. 7, pp.1121–1135.

MITC, guidelines (2006) 'Manual de procedimiento de inspección de las estaciones I.T.V. ', *Ministerio de Industria, Turismo y Comercio*, 5th Rev., Spain.

Pourqorban, N., Hassanzadeh, I., Hashemzadeh, F. and Alizadeh, G. (2010) 'Optimal active suspension control based on a quarter-car model: an analytical solution' *Int. J. Vehicle Safety*, Vol. 5, No. 1, pp.1–20.

SAE 960735, code (1996) 'An improved non-intrusive automotive suspension testing apparatus with means to determine the condition of the dampers', *Society of Automotive Engineers International*.

STTB, code (2002) 'Código nacional de tránsito terrestre – Ley 769', *Secretaría de Tránsito y Transporte de Bogotá D.C.*, Colombia, p.112.

Tsymberov, A. (1994) 'Suspension tester and method', *Hunter Engineering Company*, US Patent Nr. 5369974, 6th Dec. 1994, USA.

Notes

1 Adhesion index $A(\omega)$, ratio of the vertical force exercised by a wheel with respect to the load in the contact surface of the road, during a vertical oscillation of the tyre (SAE 960735, 1996).

2 Phase angle $\Psi(\omega)$, measure of the angular difference between the contact force of the tyre and the position of the excitation platform for each instant in time (SAE 960735, 1996).

Appendix A: Notations

Abbreviations, acronyms, coefficients and constants

BR: Armour level.

*CAV*s: Civil armoured vehicles.

*EuSAM*A: European Shock Absorber Manufacturers.

NTC: Colombian Technical Standard.

SAE: Society of Automotive Engineers.

a: Platform displacement amplitude.

c: Coeficiente de amortiguación viscosa del amortiguador.

C_{max}: Maximum design load of a vehicle.

m_a: Added mass incorporated in the armouring process.

m_v: Design mass of the vehicle.

m_t: Total mass of the vehicle.

m_1: Suspended mass of the k-th tyre.

m_2: Non-suspended mass of the k-th tyre.

m_3: Platform mass.

$\min({}^kF_{23})$: Minimum vertical force in the contact surface of the k-th tyre.

${}^k\Psi_{\min}$: Minimum phase angle of the k-th tyre.

${}^kA_{\min}$: Minimum adhesion index of the k-th tyre.

${}^km_r$: r-th mass of the k-th tyre.

k_1: Rigidity coefficient of the suspension spring.

k_2: Tyre rigidity coefficient.

t_i: i-th instant in time of the test.

X_3: Vertical position of the testing platform.

X_{23}: Vertical position of ${}^km_2$ with respect to the platform.

$\omega_3(t)$: Excitation frequency of the platform.

$\omega_{3,o}$: Initial oscillation frequency of the platform in the test.

$\omega_{3,f}$: Final oscillation frequency of the platform in the test.