Paper

Hessian Eigenfunctions for Triangular Mesh Parameterization

Topics: CAGD/CAD/CAM Systems; Geometric Computing; Texture Models, Analysis, and Synthesis

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Hessian Eigenfunctions for Triangular Mesh Parameterization

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Keywords: Applied Differential Geometry, Dimensionality Reduction, Hessian Locally Linear Embedding, Manifold Learning, Mesh Parameterization.

Abstract: Hessian Locally Linear Embedding (HLLE) is an algorithm that computes the nullspace of a Hessian functional $\mathcal{H}$ for Dimensionality Reduction (DR) of a sampled manifold M. This article presents a variation of classic HLLE for parameterization of 3D triangular meshes. Contrary to classic HLLE which estimates local Hessian nullspaces, the proposed approach follows intuitive ideas from Differential Geometry where the local Hessian is estimated by quadratic interpolation and a partition of unity is used to join all neighborhoods. In addition, local average triangle normals are used to estimate the tangent plane T_xM at $x \in M$ instead of PCA, resulting in local parameterizations which reflect better the geometry of the surface and perform better when the mesh presents sharp features. A high frequency dataset (*Brain*) is used to test our algorithm resulting in a higher rate of success (96.63%) compared to classic HLLE (76.4%).

1 INTRODUCTION

Dimensionality Reduction (DR) takes a d-manifold $M \subset \mathbb{R}^D$ and computes a map $\mathbf{h}: M \to \mathbb{R}^d$ such that: 1) $\mathbf{h}$ is bijective and 2) $\mathbf{h}$ and $\mathbf{h}^{-1}$ are continuous. Therefore, $\mathbf{h}$ is an homeomorphism and the image of M under $\mathbf{h}$ is a DR of M.

Mesh Parameterization can be seen as a particular case of DR where $M \subset \mathbb{R}^3$ is a triangular mesh of a 2-manifold (i.e. $D = 3$ and $d = 2$). Triangular meshes are very common data structures in CAD CAM CAE applications and parameterization of such meshes is relevant for areas such as: reverse engineering, tool path planning, feature detection, etc.

A natural way to handle Mesh Parameterization is to attack the problem from the point of view of DR. Classic HLLE (Hessian Locally Linear Embedding) (Donoho and Grimes, 2003) is an algorithm which proposes to compute a DR of M by computing the eigenvectors of a Hessian functional. This article proposes a modification for the classic HLLE which can be applied to triangular meshes. Our proposed approach computes a partition of unity on M and estimates the tangent Hessian on each neighborhood N_i of M by interpolating any function f with second degree polynomials. In addition, local average triangle normals are used to compute the tangent local plane T_xM of M which is more consistent than Principal Component Analysis (PCA) specially for surfaces with sharp features.

The remainder of this article is organized as follows: Section 2 reviews the relevant literature. Section 3 describes the implemented methodology. Section 4 discusses and compares the results of the proposed approach against classic HLLE. Section 5 concludes the paper and introduces what remains for future work.

2 LITERATURE REVIEW

Given a set of points $X = [x_1, x_2, \ldots, x_N] \subset \mathbb{R}^D$ lying on a d-manifold M, DR seeks a homeomorphic function $\mathbf{h}: M \to \mathbb{R}^d$ such that the set of points $[\mathbf{h}(x_1), \mathbf{h}(x_2), \ldots, \mathbf{h}(x_N)] \subset \mathbb{R}^d$ compose a DR of X. For the rest of the article we assume $D = 3$ and $d = 2$, turning the DR problem into a Mesh Parameterization one.

The most popular algorithm for DR is the Principal Component Analysis (PCA). PCA is a linear algorithm which parameterizes M by projecting X onto a plane, which is only a valid parameterization if $\mathbf{h}$ is linear. However, this assumption limits the algorithm making it useful only for trivial cases.

For nonlinear manifolds, other approaches have been proposed in the literature. For example, Isomap (Tenenbaum et al., 2000) attempts to compute an isometric parameterization of M by computing the geodesic distances in M and reproducing them in the

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