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Based on traditionally accepted hypothesis and verified by existing data, an expression is derived to calculate response spectrum at the ground surface if the response spectrum at the basement rock is known. The fundamental assumptions are with regards to the form of variation of the exceedance rates of spectral accelerations in the basement rock, and based also on the usual (lognormal) distribution forms of the uncertainties associated with the spectral amplification function. The resulting approach multiplies the mean of the amplification function in order to consider in a rigorous way its uncertainty.

Keywords: Soil-rock spectra ratios; uniform hazard spectra; surface level.

1. Introduction

In the definition of response spectra for seismic design the following practice has been the common approach; first the spectra for uniform hazard at the rock basement level is estimated, this spectrum is then scaled by an amplification-function type of factor to arrive at the uniform hazard response spectrum at the ground surface and with the same exceedance rate as the one corresponding to a rock. This amplification-function should be more appropriate referred to like, Ratio of Response Spectra (RRS), since it will become apparent shortly that such a function is not really a transfer function within the classical structural dynamics context (see for instance Clough and Penzien [1993] for a standard definition of a transfer function). The common approach described above is not completely rigorous and has been mainly motivated and justified by the standard state of practice as explained below. In the current seismic design methodologies it is usual to define the seismic action in terms of response spectra. Such an approach implicitly assumes that the soil can be fully characterised by an RRS factor. In the case of a transfer function (TF) and linear elastic soil response the only relevant parameters are the soil properties and such an approach is clearly valid. However, this is not the case if one attempts to describe the soil in terms of a simple RRS factor since in this latter case there is also a dependence on the time histories at the rock basement. Moreover, such a dependence is strongly apparent at high frequencies and less important at low frequencies, Roesset [1969]. Clearly, the current seismic design procedures are the ones promoting the use of amplification functions to characterise the response of soil deposits.

Although there are several methods to convert Fourier spectra into acceleration response spectra, like those based on random vibration theory, all of them are too complex to be used in everyday applications. A more compelling reason to preclude the use of such methods is the fact that the Fourier spectra is not readily available at the rock basement level.

The present contribution presents and describes a methodology that retains the simplicity of the approach based on RRS factors while it incorporates in a rigorous manner the uncertainty in the amplification function to estimate the uniform hazard response spectra at the ground surface level.

As was already pointed out, in order to determine the uniform hazard spectra at the ground surface level associated to a given return period it is necessary to have access to the uniform hazard spectra at the rock basement level and the amplification function RRS and its uncertainty. Additionally one also needs to have access to some set of parameters defining the sesimicity of the involved sources, a parameter that characterises the attenuation equations for the seismic waves as they propagate through the earth structure and a parameter that defines the dependence of the amplification function RRS on the spectral acceleration at the rock basement level.

2. Proposal

In what follows it is considered that the amplification function or RRS factor is expressed like:

$$\mathrm{RRS} = \overline{\mathrm{RRS}} \cdot \varepsilon. \tag{1}$$

In Eq. (1) $\overline{\mathrm{RRS}}$ is the mean of the amplification function, RRS results from the ratio between the acceleration response spectra at the ground surface and that one at the basement rock, ε is a random variable which is log-normally distributed with a mean of 1.0 and deviation denoted by σ_ε. This relationship exhibits the fact that the uncertainty in the amplification function increases with the mean value of the amplification.

To consider the dependence of the function on the spectral acceleration at the basement rock the following relationship is proposed:

$$\breve{\mathrm{RRS}} = \alpha a_b^{K_{AF}} \quad (K_{AF} < 0), \tag{2}$$

where $\breve{\mathrm{RRS}}$ is the median of RRS. Furthermore, it is suggested herein that the exceedance rate corresponding to the spectral ordinates $\nu(a_b)$ for the rock basement varies as suggested by Esteva [1970]:

$$\nu(a_b) = \nu_0 a_b^{K_H}, \quad (K_H < 0). \tag{3}$$

The exceedance rate for the spectral ordinates at the ground surface $v(a_s)$ is determined from:

$$\nu(a_s) = -\int_0^{\infty} pr(A_s > a_s|a_b)\frac{d\nu(a_b)}{da_b}da_b, \tag{4}$$

with

$$A_s = \mathrm{RRS} \cdot a_b. \tag{5}$$

Evaluating Eq. (4) and considering Eqs. (2) and (3) it can be shown that the exceedance rate for the spectral accelerations at the ground surface level can be written as:

$$\nu(a_s) = \nu_0\left(\frac{a_s}{\alpha}\right)^{\frac{K_H}{K_{AF}+1}}(CV_{\mathrm{RRS}}^2+1)^{\frac{K_H^2}{2(K_{AF}+1)}}, \tag{6}$$

where CV_{RRS} is the coefficient of variation for the RRS amplification function. If one fixes the exceedance rate, ν_{pr}, associated to a given return period then, the spectral acceleration at the ground surface level a_{spr}, can be expressed in terms of the spectral acceleration at the rock basement level a_{bpr} associated to the same exceedance rate as:

$$a_{spr} = \breve{\mathrm{RRS}} \cdot a_{bpr}(CV_{\mathrm{RRS}}^2+1)^{\frac{K_H}{2(K_{AF}+1)}}. \tag{7}$$

Given the fact that

$$(CV_{\mathrm{RSS}}^2+1) = \left(\sigma_\varepsilon^2+1\right) = \exp\left(\sigma_\delta^2\right), \tag{8}$$

where σ_δ^2 is the variance of $\ln(\varepsilon)$ or equivalently, the variance of ln(RRS), allows one to express Eq. (7) as:

$$a_{spr} = \breve{\mathrm{RRS}} \cdot a_{bpr}\exp\left(-\frac{K_H\sigma_\delta^2}{2\,(K_{AF}+1)}\right). \tag{9}$$

Noticing that Eq. (9) has been written in the same form as the original proposal by McGuire [2004] and the later corrections in McGuire [2005] the only difference relies on the fact that what McGuire refers to like the media of the RRS function should be correctly called the median.

Now, knowing that:

$$\breve{\mathrm{RRS}} = \overline{\mathrm{RRS}}\exp\left(-\frac{\sigma_\delta^2}{2}\right), \tag{10}$$

Eq. (7) can be further written like:

$$a_{spr} = \overline{\mathrm{RRS}} \cdot a_{bpr}(CV_{\mathrm{RRS}}^2+1)^{-\frac{(K_H+1)}{2(K_{AF}+1)}}. \tag{11}$$

In what follows, the function RRS_{eq}, defined like the equivalent amplification function, will allow one to find the spectral acceleration at the ground surface level after scaling the spectral acceleration at the rock basement level and when both are associated to the same exceedance rate:

$$\mathrm{RRS}_{eq} = \overline{\mathrm{RRS}}(CV_{\mathrm{RRS}}^2 + 1)^{-\frac{(K_H+1)}{2(K_{AF}+1)}}. \tag{12}$$

Equation (12) reveals that the relevance of the uncertainty of the soil amplification function described in terms of CV_{RRS} in the equivalent amplification function is through the exponent K_H. As will be shown shortly, this factor is a function of characteristic features of the seismic source and characteristic of the seismic motion in terms of the magnitude. The steepest slope that defines the exceedance rate for the spectral acceleration in the rock basement has the strongest influence of uncertainty in the amplification function for the spectral ordinates at the ground surface.

Theoretically, for values of K_H corresponding to -1.0, the uncertainty in the soil amplification function, regardless of its magnitude, is not relevant at all during the estimation of the exceedance rates for the spectral accelerations at the ground surface level; these can be estimated assuming that the amplification function is known without uncertainty and with a value corresponding to its mean. For values of K_H greater than -1.0 one would have an attenuation effect over $\overline{\mathrm{RRS}}$ at the time of estimating RRS_{eq}.

At this point it is relevant to study what are reasonable values for the exponent K_H as well as its dependence. If one assumes that the exceedance rate of magnitudes, M for each one of the sources i considered independent from each other and affecting a given site is given by:

$$\nu_i(M) = \lambda_i e^{-\beta M}, \tag{13}$$

and that the attenuation law for the spectral ordinates in the rock basement, a_b in terms of the magnitude M and the distance between the source i and the site R_i is given by:

$$\ln(a_b) = c_0(R_i) + c_1 M + \gamma, \tag{14}$$

where γ is a random variable normally distributed with a mean of 0.0 and deviation equal to σ_γ, then it can be shown that the exceedance rate of the spectral accelerations in rock can be written as:

$$\nu(a_b) = \left(\sum_i \lambda_i \exp\left(\frac{\beta}{c_1} c_0(R_i)\right)\right) \exp\left(\frac{\beta^2 \sigma_\gamma^2}{2c_1^2}\right) a_b^{-\frac{\beta}{c_1}}, \tag{15}$$

which according to Eq. (3) yields:

$$K_H = -\frac{\beta}{c_1}. \tag{16}$$

From seismic source characterisation and attenuation laws widely recognised throughout the world it is concluded that two reasonable values for β and c_1 are 2.0 and 0.8 respectively, which results in a value of K_H equal to -2.5. A reasonable value for K_{AF} defining the slope in the relationship between the logarithm of the

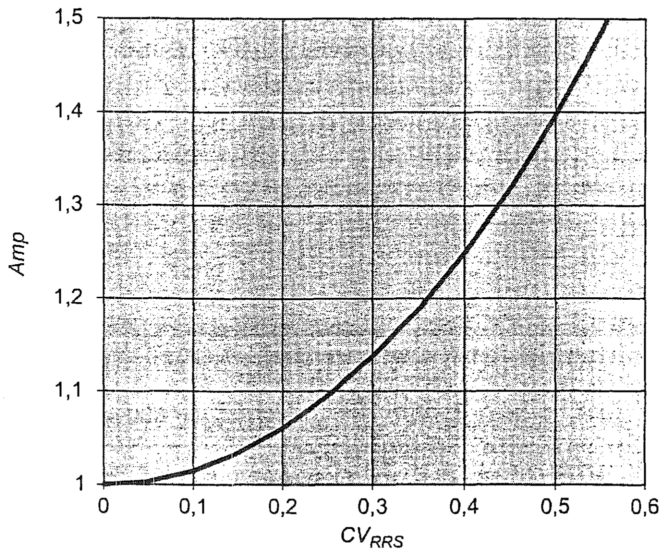

Fig. 1. Amplification factors due to uncertainties in $\overline{\text{RRS}}$.

amplification function and the logarithm of the spectral acceleration in rock is equal to −0.5 [McGuire, 2004].

Figure 1 shows the amplification factor over $\overline{\text{RRS}}$ described in Eq. (12) as a function of the uncertainty in the amplification function, CV_{RRS}, and for the proposed values of K_H and K_{AF}.

For instance from Fig. 1 it can be seen that for a site in which the value of K_H is −2.5 and the value for K_{AF} is equal to −0.5 then the scaling factor for the mean of the amplification function due to its uncertainty is 1.40 when the latter is equal to 0.50.

3. Conclusions

The present work introduces and describes an expression that incorporates in a rigorous manner the uncertainties in the spectral amplification function used to estimate the spectral ordinates at the ground surface in terms of the corresponding ones at the basement rock. The approach is based on traditionally accepted and verified assumptions related to the shape and variation of the exceedance rates for the spectral accelerations at the rock basement level. It is also based in a commonly used (lognormal) form of the distribution of the uncertainties in the amplification function for the spectral accelerations and a decreasing exponential law relating the rock spectral acceleration and the amplification function.

The found scaling factors range between a lower bound of 1.0 corresponding to null uncertainty, as expected, and values as high as 1.4 for coefficients of variation due to uncertainties in the amplification function equal to 0.5. The exponential form for the scaling factor due to uncertainty is also pointed out.

Having a metric that incorporates the effect of the uncertainty and deals with it in an equivalent deterministic way, not only facilitates the estimations but most importantly calls the attention about its relevance and warns about its consequences.

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