Mining and Energy Boom, Dutch Disease and Informality in Colombia: a DSGE Approach

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Abstract

The paper develops a Dynamic Stochastic General Equilibrium (DSGE) model, which assesses the macroeconomic and labor market effects derived from simulating a positive shock to the stochastic component of the mining-energy sector’s productivity. Calibrating the model for the Colombian economy, this shock generates an overall increase in formal wages and a raise in tax revenues, expanding total consumption of the household members. These facts increase non-tradable goods prices relative to tradable goods prices, then real exchange rate decreases (appreciation) and there occurs a displacement of productive resources from the tradable (manufacturing) sector to the non-tradable sector, followed by an increase in formal GDP and formal job gains. This situation makes the formal sector to absorb workers from the informal sector through the non-tradable formal subsector, which causes informal GDP to go down. As a consequence, net consumption falls for informal workers, which leads some members of the household not to offer their labor force in the informal sector and instead they prefer to remain unemployed. Therefore, the final result on the labor market is a decrease in the number of informal workers, where one part of the non-informal workers is in the formal sector and the remaining part is unemployed.

JEL classification: E0, E1, E2, E3.

Keywords: Mining and energy boom, dutch disease, formal and informal sectors, unemployment, DSGE model.

1. Introduction

Between 2004 and 2014, numerous capital flow entries occurred in Colombia, which improved the current account of the balance of payments and the economy in general. These capital inflows represented in the form of Foreign Direct Investment (FDI), were mainly directed to the mining and energy sector in order to finance exploration activities and heavy cargo transportation (López, Montes, Garavito and Collazos, 2013). Increased capital inflows aimed at the mining and energy activities, were mainly explained by high commodity prices such as oil, coal and nickel.

As a result of this phenomenon, oil production was steadily increasing during this period. This meant that the mining and energy activities had become more important within the economy in representing about 40% of fiscal revenues, 60% of total exports, and 60% of the current account in the balance of payments (Cano, 2010).

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High international commodity prices coupled with large foreign capital inflows, led to the possibility of the Dutch disease phenomenon having become present in Colombia. On this subject, some empirical studies concluded that in fact the Dutch disease phenomenon was presented in Colombia (Clavijo, Vera, and Fandiño, 2012; Goda and Torres, 2015). Additionally, in Colombia papers such as Suescun (1997) and Ojeda, Parra and Vargas (2014), analyze the potential macroeconomic effects and the fiscal policy responses that the Colombian economy would experience when facing positive productivity shocks in commodity sectors such as coffee, minerals and oil. In particular, Ojeda, et al. (2014) studied the possible effects on prices, wages, real exchange rate, product and employment derived from positive productivity shocks to the Colombian mining and energy sector through a Dynamic Stochastic General Equilibrium (DSGE) model. Their results are in line with the effects found in the conventional literature about the Dutch disease phenomenon, i.e., consumption increases, the real exchange rate is appreciated, and occurs a shift of resources from tradable no-boom sectors to non-tradable sectors such as building, real estate activities and services. This causes a weakening of production, employment and capital stock in the tradable manufacturing sector, then both employment and output in non-tradable goods sector increase.

However, since the first half of 2014 international prices of these primary goods have been falling rapidly and persistently, which leads to the conclusion that the energy and mining boom in Colombia no longer exist. Meanwhile, government and Colombian fiscal policies were prepared since 2011 for an energy and mining boom that would long 10 more years, but actually oil and mining boom has been vulnerable to capital and international prices fluctuations, factors that are unpredictable for a small open economy such as the Colombian.

In general, it is relevant to question what effects brought to the Colombian economy this mining-energy boom that in principle was expected to be permanent, but in fact was transitory. In particular, since the tradable goods producing sector (apart from the mining and energy sector) and the non-tradable producing sectors are both intensive in the use of labor, it is relevant to investigate the labor market adjustment to the transitory mining-energy boom. Specifically, since non-tradable sectors such as building, real estate activities and services are intensive in unskilled labor, the hypothesis that the Dutch disease not only reallocates aggregate capital resources and employment from manufacturing (tradable) to non-tradable sectors, but it is also possible that some productive non-qualified resources get moved from informal sector towards non-tradable formal activities (Arguello, Jiménez, Torres and Cesca 2014). In that direction, data from the Great Integrated Household Survey (GEIH) show that the percentage ratio between the number of informal and formal employees (measured as the natural logarithm of the ratio of these two variables) reached a maximum 9.8% in March 2008, which progressively declined to 5.1% in January 2014, in a context where the unemployment rate showed a negative trend. In order to rigorously study the above elements, the main objective of this paper is to develop a macroeconomic dynamic stochastic general equilibrium (DSGE) model that incorporates formal and informal employment, in order to analyze the macroeconomic effects derived from simulating a positive shock to the stochastic component of productivity in the mining and energy sector, and consequently observe the dynamic adjustment of the labor market, particularly in informal employment, formal employment and unemployment.

The model consists of a small open economy which closely follows the household modeling strategy proposed by Jaramillo, Guataqui and Obando (2015), in the sense that the representative household includes members working in formal and informal sectors and unemployed workers. Household preferences are the sum of preferences of each of its members, which means that each one of them has a different budget constraint, while each specific utility function is separable on each of its arguments. The productive sector consists of an informal and a formal sector, the latter including three productive subsectors: non-tradable goods, tradable manufactured goods and tradable mining-energy goods, based on the strategy adopted by Ojeda, et al. (2014). Additionally, formal companies demand workers and set efficiency wages such as in Alexopoulos (2004) and Jaramillo, et al. (2015), and wages in the formal sector are unique, so it allows for free movement of workers between formal subsectors. With respect to the fiscal policy, the government imposes tax rates to formal firms
incomes and government spending is unproductive, in the sense that its tax revenues are entirely transformed in lump sum transfers to household members budget constraints, as shown in Ojeda, et al. (2014). The inclusion of government is important in creating the mechanism of Dutch disease, since the higher total consumption due to the mining and energy boom is done through higher formal wages and higher public lump-sum transfers to households.

Regarding results, a positive transitory shock to the mining-energy industry productivity generates an overall increase in formal wages and tax revenues, increasing total consumption of household members. This makes non-tradable goods prices to increase in relation to tradable goods prices, reducing real exchange rate (appreciation) and causing a shift of productive resources from the formal manufacturing tradable sector to formal non-tradable sector, followed by an increase in formal GDP and net formal employment. This raise in the formal sector absorbs workers from the informal sector through the expanded non-tradable formal subsector, making informal output to decrease. Consequently, consumption of informal members decreases which encourages some household members which are not employed in the informal sector to choose staying unemployed. The final result on the labor market is a decline in the number of informal workers, where one part of the non-informal workers is on the formal sector and the remaining part is unemployed. Additionally, the results of a positive shock to the mining and energy productivity are qualitatively and quantitatively very similar to those obtained by a positive shock to international commodity prices.

The paper is structured as follows. The first section is the introduction. Section 2 exposes the related literature associated with the effects of a positive productivity shock to the mining and energy sector on i) tradable and non-tradable factor reallocation, ii) total labor market adjustment (formal, informal and unemployment rate), and iii) fiscal policy responses. Third section presents Colombian stylized facts and empirical evidence that motivates the paper, followed by the fourth section which introduces the model description. Fifth section shows the details of the calibration. Later on, sixth section shows and interprets the results of the simulations. Finally, the seventh section presents the conclusions.

2. Related Literature

The study of the effects that potentially brings to the economy a mining and energy boom calls for studying various economic relations in order to understand its impact on the reallocation of resources between tradable and non-tradable, formal and informal sectors. Of particular importance is to analyze the possible response of the price level, employment, unemployment and total output. For this purpose, the literature review integrates two aspects related to the effects that a mining and energy boom can generate on the economy. As first instead, some papers that use General Equilibrium models are addressed, which have tried to explain mining and energy boom effects on the reallocation of resources from tradable sectors towards non-tradable ones. Secondly, I consider some studies that have analyzed the labor market adjustment against mining and energy shocks through dynamic general equilibrium models.

2.1. Dutch disease in macroeconomic General Equilibrium models

One of the pioneering papers analyzing the relationship between a mining and energy boom and the other variables of the economy in a general equilibrium context is Corden and Neary (1982). Assuming free mobility of labor between sectors, raises in productivity and energy production generate a deindustrialization phenomenon reflecting a contraction in the manufacturing output and employment. Additionally, the energy boom produces an appreciation of the real exchange rate arguing that the decline of the manufacturing sector is consequence of the new balance of the economy after
the energy boom, which is characterized by having non-tradable goods with high and flexible prices, and tradable (manufacturing) goods with constant and exogenous prices.

Beverelli, Dell’Erba and Rocha (2011) developed a static General Equilibrium model including a tradable sector intensive in using natural resources (manufacturing), an energy goods productive sector and a non-tradable sector (services). These authors find that in all countries where there was a discovery of new natural resources a fall (appreciation) of the real exchange rate occurred, which supports the dominance of the spending effect over the reallocation effect.

In Colombia, Lopez, et al. (2013) shows that in 2011 the 70% of domestic oil production was recorded as exports, i.e., in Colombia the vast bulk of oil produced is exported. In this regard, Ojeda, Parra and Vargas (2014) developed a DSGE model whose qualitative results of the impulse-response functions derived from a positive mining and energy productivity shock are in the conventional direction: household consumption raises which make the aggregate demand and the general price level to increase, then the real exchange rate appreciates, GDP and manufacturing employment weaken, and this causes a shift of productive resources from manufactures to the non-tradable sector. This phenomenon increases the total household welfare, regardless of whether the fiscal rule adopted by the government is either procyclical or countercyclical.

Finally, Orrego and Vega (2014) developed and estimated a DSGE model for the peruvian economy that is very similar to Ojeda, et al. (2014). A key difference between these two studies is that Orrego, et al. (2014) provide a different functional form and parameterization for the fiscal rule. However, their results are qualitatively identical.

2.2. Dutch disease effects on labor markets

Up to now the literature has focused on the development of DSGE models with the aim of studying the effects of productivity and foreign investment booms in sectors with intensive use of natural resources, but these works have not considered the possibility of incorporating a holistically labor market in their modeling strategy. In this regard, recent papers such as Gonzalez, Lopez, Rodriguez and Tellez (2013), Ojeda, et al. (2014) and Orrego, et al. (2014), have focused on considering the employment variable in the labor market adjustment, when positive productivity shocks on the mining and energy sector are present. These papers address the Dutch disease phenomenon in DSGE models for small open economies, to reach the general conclusion that the employment rate in the sector producing non-tradable goods increases and there is a decrease of employment in the tradable sectors other than mining and energy, and concluding that the total employment rate is not significantly altered.

According to Arguello, Jimenez, Torres and Gasca (2014), low qualification of labor in the non-tradable production sector implies that an increase in wages and employment in this sector could encourage some not qualified informal workers to offer their workforce in the non-tradable sector. Therefore, an appropriate modeling strategy that incorporates labor market adjustment against mining and energy shocks, consists in distinguishing between formal and informal employment instead an aggregate employment rate, and considering the existence of an unemployment rate.

In this regard, Jaramillo, Obando and Guataquí (2015) developed a closed economy with flexible prices that include a formal and an informal sector. The authors assume the existence of efficiency wages in the formal sector of the economy in order to incorporate unemployment in the model. Thus, a productivity aggregate shock takes into account the dynamic adjustment of the unemployment rate, the formal employment rate and the informal employment rate, which leads to a dynamic determination of the total employment rate in the economy.

In order to holistically understand the labor market adjustment facing a Dutch disease phenomenon in small open economies arises the need to merge the macroeconomic modeling strategies conducted by Ojeda, Parra and Vargas (2014) with Jaramillo, Obando and Guataquí (2015). Since these two
modeling strategies are based on certain modifications on the conventional model of Real Business Cycle (RBC), the creation of a dynamic macroeconomic model based on characteristics of both economies is feasible.

3. Dutch disease and the Colombian macroeconomic context

Between 2004 and 2014, Colombia experienced an increase in mining and oil production in response to high commodity international prices and abundant foreign capital inflows represented as Foreign Direct Investment (FDI). Therefore, the mining and energy sector had become a sector that had significantly increased its share of GDP (López, Montes, Garavito and Collazos, 2013).

Figure 1: International oil prices (BRENT)

As a sign of the rise of natural resources worldwide, figure 1 shows that since 2004 the international oil price had been steadily rising, witnessing a fall in September 2008 (because of the 2008 crisis originated in the United States), but it quickly recovered from March 2009. Despite of the positive expectations in emerging economies that this price rise would continue indefinitely or at least stabilize at levels close to USD 120, from April 2011 fluctuations started to show a negative trend. From July 2014 onwards, the price fell dramatically to USD 47.76 in January 2015. This has discouraged production and exports of crude oil in emerging market economies and generated speculation about reversals of foreign capital in local mining and energy activities, i.e. a FDI drop in this sector.

Source: Federal Reserve Economic Data (FRED).
Moreover, total FDI in Colombia began to increase from 2003, then fell during the 2008 crisis. Following Cano (2010), total FDI fell from USD 10,593 million in 2008 to USD 6,923 million in 2009, which is equivalent to a decrease of 35%. However, the fast recovery of international oil prices prevented the FDI directed to the Colombian mining and energy sector from falling during the crisis (graphic 2). As noted by Cano (2010), mining and energy FDI rose from USD 5,530 million in 2008 to USD 6,819 million in 2009, which is equivalent to an increase of 23%. This phenomenon positively compensated outflows of foreign capital in sectors such as manufacturing, contributing to avoiding a strong decline in total FDI due to the financial crisis.

Once the financial crisis was attenuated and the international oil price recovered its growth momentum, then total, energy, and mining FDI regrew. However, given the recent fall in international oil prices the annual growth of total energy-mining and manufacturing FDI in the third quarter of 2014 decreased to levels of -24.6%, -25.6% and -53.8%, respectively.
The rise of massive inflows of foreign capital to the country, represented in the form of FDI, can be seen reflected in the behavior of the Nominal Exchange Rate, which is equivalent to the “Representative Exchange Rate (TRM)”. In figure 3, it can be seen the TRM revaluation pressures from 2003, the year in which the flows boom of FDI in the country had begun.

However, the 24.6% declining on the total FDI flows in the third quarter of 2014, caused in part by the dramatic decline the crude oil international price, has influenced the TRM to return to similar nominal devaluation levels experienced during the 2008 crisis, period in which a similar reversal of foreign capital occurred. On the other hand, greater flows of FDI in the country aimed to finance explorations and heavy cargo, coupled with high oil prices since 2004 to July 2014, necessarily involved an increase in productivity in the oil sector, which has led to a considerable increase in local oil production during the boom years (Figure 4).

As noted by Lopez, Montes, Garavito and Collazos (2013), the National Hydrocarbons Agency (NHA) revealed that crude oil production in Colombia was in an average of approximately 540,000 barrels per day (540.000 bpd) between 2004 and 2007. Subsequently, from 2008 production increased to 587 bpd, then reached 900 bpd in 2011. However, data from oil production until June 2014 published by the NHA, shows that daily production average from January to June has remained in 981 bpd.
Fluctuations in total and mining-energy FDI flows, plus changes in international commodity prices, can be seen also reflected in the trade balance behavior (figure 5). According to Lopez, et al. (2013), in 2011 70% of the Colombian exports was determined by commodity sales, whose prices were determined abroad. In addition, 49.7% of total exports was explained by oil and its derivatives sales.

In that regard, Cano (2010) noted that the mining and energy sector generated about 40% of the current account in the balance of payments of the country in 2010, while determined the 60% of total exports in that year. Therefore, the mining and energy sector had become the main source of foreign exchange, prices and production, and during the FDI boom this activity generated a surplus exchanging balance and an increasing rate in exports. Despite the good dynamics of the sector,
graphic 5 shows that the surplus in its own trade balance is no longer growing since 2011, and has begun to decline at a faster rate since May 2014, due to the current negative international situation of commodities.

Figure 6: Inflation Rate: Tradable and Non- Tradable Goods in Colombia

![Inflation Rate Graph]

Source: Central Bank of Colombia (Banrep).

Figure 7: Real Exchange Rate Index

![Real Exchange Rate Graph]

Source: Central Bank of Colombia (Banrep).

The higher crude oil production since 2004, has generated a significant increase in government revenues and the GDP in general. As noted by Cano (2010) and Lopez, et al. (2013), the mining and energy activities contribute about 40% of the total fiscal revenue, while GDP is fed by growing exports in this sector. According to Anguello, Jimenez, Torres and Gasca (2014), the Colombian
economy grew 7% in 2007 then grew an average of 1.5% between 2008 and 2009, and then grew at a rate of 4.5% in 2011.

This means that the mining and energy boom has greatly contributed to the increased in public and private spending in recent years, which has caused an increase in aggregate demand. This demand positive shock could be related to the increasing level of non-tradable goods prices, in relation to the behavior of tradable goods prices (figure 6). The positive difference between the inflation rates for non-tradable goods, with respect to the inflation of tradable goods, has probably contributed to the real exchange rate trend towards appreciation during the period of the mining and energy boom (Figure 7). The common trend towards nominal and real appreciation has been accompanied by a significant decline in employment and output in the manufacturing tradable sector, plus a significant employment and product expansion in non-tradable sectors such as building and services, setting in this way the so called Dutch disease. This phenomenon has contributed to foreign capital inflow falls (lower FDI) in important tradable sectors of the economy such as manufacturing and to increasing deficits on its trade balance, as shown in figures 2 and 5. As evidence of this, figure 8 shows that since 2009 the Colombian Industrial Production Index (IPI) has shown growth rates close to zero, which verifies the presence of the natural resources curse.

Figures 9 and 10 show the GDP evolution discriminated by economic activity in quarterly frequency. The data are seasonally adjusted. A steady slowdown in manufacturing GDP can be noticed from the third quarter of 2007 until nowadays. The opposite is observed in the behavior of non-tradable GDP, while the mining and energy GDP always evidenced an increasing trend, although it begins a slowdown from the fourth quarter of 2013, a period that coincided with the slowdown in the rise of natural resources boom. In the same direction, the share of these sectors of total GDP is presented in figure 10. The shares of the mining and energy sector and non-tradable sectors increased during the natural resources boom to the detriment of the manufacturing sector share of GDP. The preceding analysis of the behavior of macroeconomic aggregates suggests that in Colombia the last mining and energy boom generated symptoms of Dutch disease. To confirm this fact, some empirical studies in Colombia related to the effects of mining and energy boom, conclude that the Dutch disease phenomenon actually occurred. In this regard, Clavijo, Vera, and Fandiño (2012) estimated a VEC model between 1965 and 2012; finding that a gradual process of deindustrialization has occurred which has been a result of the mining and energy exports dynamics, trade liberalization, and the real exchange rate. The authors find that these factors negatively affect the share of manufacturing on total GDP. Meanwhile, Goda and Torres (2015) analyze the possibility that the mining and energy boom in Colombia, together with the greater inflows of foreign capital to finance its expansion, may have caused a Dutch disease context. The authors find, through estimating an Error-Correction model with distributed lags that FDI directed at financing the mining and energy sector and other sectors generated an appreciation of the real exchange rate that cannot be attributed to productivity gains of the domestic economy only, but also to higher exports, mainly in the mining and energy sector. Additionally, they found that these effects on the real exchange rate had a negative and significant effect on the economy's sectoral composition as the GDP ratio between tradable and non-tradable decreased, accompanied by a fall in the share of manufacturing of total GDP. Therefore, there is evidence that the mining and energy boom generated a Dutch disease phenomenon, which was reinforced by capital inflows to this sector and other sectors.

In terms of labor market adjustment, figure 12 shows the number of employed by economic activity. The data has been updated to December 2014. Since the mining and energy sector is highly intensive in the use of capital and technology, the labor-capital ratio is 6.61 for the coal subsector, and 5.29 for the oil subsector, these being the highest values of the Colombian economy (Arguello, 2011). Additionally, Cano (2010) notes that the energy and mining sector directly provides around 220,000 jobs and Lopez, et al. (2013) points out that the sector generates less than 1% of total national employment. For these reasons, the mining and energy sector boom does not seem to have substantially altered the behavior of employment in the sector.
By contrast, the increasing non-tradable output share of GDP seems to have resulted in an increase in the number of employed, since figure 12 shows that industries such as building, trade, hotels and restaurants, transport, storage and communications, financial intermediation, real estate activities, renting and business, have increased their employment rates during the mining and energy boom.

Finally, the slowdown in manufacturing activity has resulted in a widespread lower level of labor for the entire industry, especially since September 2011. However, since July 2014 has shown a slight recovery in the number of employed in the sector.

Another important aspect to consider for the labor market adjustment in the context of the Dutch disease phenomenon, is the evolution of formal and informal employment. In this respect, Argeló, Jiménez, Torres and Gasca (2014) use a Computable General Equilibrium (CGE) model in order to assess the impact of mining and energy boom in some variables of interest in the Colombian economy.

Since a large part of the non-tradable sector is intensive in using unskilled labor, it is possible that some fraction of non-tradable not-qualified sector absorbs workers from the informal economy. In this sense, figure 11 shows that the Informal employment / Formal employment relationship is falling considerably, especially during the mining and energy boom. In this regard, data from the Great Integrated Household Survey (GEIH) show that the percentage ratio between the number of informal employed and the number of formal employed (measured as the natural logarithm of the ratio of these two variables) reached a maximum 9.8% in March 2009 and after that began to fall, reaching 3.1% in January 2014, in a context where the unemployment rate exhibited a clearly negative trend.

In conclusion, we can say that Colombia has presented the typical symptoms and consequences of the Dutch disease. However, once international oil prices returned to levels prior to the mining and energy boom and FDI decreased for the natural resources and manufacturing sectors, it seems that despite the boom have disappeared and the real exchange rate returning to its initial equilibrium levels (real depreciation process), the manufacturing sector shows no signs of recovery. Finally, the aggregate adjustment of the Colombian labor market has resulted in lower informal employment in relation to the greater formal employment, both accompanied by lower unemployment.

Figure 8: Industry Production Index (IPI)

Source: Central Bank of Colombia (Banrep).

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Figure 9: Gross Domestic Product (GDP) per Economic Activity

Source: National Administrative Department of Statistics (DANE).

Figure 10: Contribution of each Sector to the Whole GDP

Source: National Administrative Department of Statistics (DANE).
Figure 11: Informal / Formal Employment Ratio

Source: National Administrative Department of Statistics (DANE).
4. The Model

The model is a decentralized small open economy consisting of three types of agents: households, firms and government. Closely following Alexopoulos (2004) and Jaramillo, et al. (2015), there is a continuum of households of unit mass, whose members are classified into formal workers, informal workers and the unemployed. Meanwhile, the productive sector is divided between an informal sector and a formal sector, the latter being divided into three subsectors: non-tradable, tradable manufacturing and tradable mining and energy, as shown in Ojeda, et al. (2014). In addition to this, the government collects taxes from formal firms and public spending is unproductive, in the sense that it takes place in the form of lump-sum transfers to households.

4.1. Households

Representative household members can be classified into formal workers, informal workers and the unemployed. Household preferences are the sum of each of its member’s preferences, which means they each have a particular budget constraint, while each specific utility function is separable in each of its arguments. However, the formal firms’ profits are equally directed to the restriction of each of
the household members, and each member receives lump-sum transfers from the government, which means that in the absence of income labor, household members receive the same income and have the same level of consumption.

In order to obtain unemployment positive values in equilibrium, it follows Alexopoulos's (2004) approach, in which the effort of the formal workers is imperfectly observable by the formal sector firms, and unlike the traditional approach of Shapiro and Stiglitz's (1984) efficiency wages where detected shirker workers are dismissed, in this scheme the shirkers' punishment is that the firms pay them just a percentage $s$ of total the salary agreed in the contract. Specifically, the formal firms employ workers by offering a contract to the end of a period, which specifies the fixed number of hours an employee must work $h$, the level of effort required $e_f$, and an advance $s \in (0, 1)$ of hourly nominal wage $W_f$, but the firms retain the remaining $(1 - s)W_f$ until the final contract. Shirker workers are detected with an exogenous probability $\delta$, and formal firms will not pay the remaining $(1 - s)W_f$ to the shirker workers that had been detected at the end of the period.

According to Alexopoulos (2004), this way of incorporating efficiency wages is fundamentally plausible for two reasons. The first one has a theoretical advantage because, on the one hand, setting efficiency wages in Shapiro-Stiglitz implies that if a shirker worker is detected he/she will be immediately fired. Therefore, if the unemployment rate falls, it reduces the punishment for shirkers because if the employee is dismissed, the length of unemployment will be lower. As a result the workers’ efforts will be less, which induces the firms to increase the wage in order to prevent effort contraction, which implies procyclical wages. In contrast to the Shapiro-Stiglitz’s approach, the punishment on Alexopoulos (2004) is not to pay the remaining fraction (constant) of the total salary specified in the contract to the worker, implying procyclical wages in a scenario in which the punishment is independent of the unemployment rate.

The second advantage of adopting the Alexopoulos’ (2004) approach, is that there is empirical evidence that suggests i) most firms punish the shirkers detected directly in the salary but these are not fired, and ii) workers who make an effort are not likely to receive bonuses or salary increases (Agell and Lundborg, 1995; Hall, 1993; Weiss, 1990; and Malcolmson, 1998).

The difference in the modeling strategy of this paper in relation to Alexopoulos, is that in this case if members seeking to be employed in the formal sector do not achieve that goal, then they have the option of self-employment in the informal sector of the economy (as self-employed workers), or they can choose to stay unemployed, as shown in Jaramillo et al. (2015).

At any moment in time, each household member decides their level of consumption, effort and labor supply. In principle, all household workers are offered in the formal sector of the economy. However, since not all the labor supply in the formal sector is absorbed, members who fail to join it must decide between going either to the informal sector or staying unemployed. In this regard, an important difference of this modeling strategy in comparison to the one adopted by Alexopoulos and Jaramillo, et al., is that there is no insurance for the unemployed members, whose finality is to generate voluntary unemployment in the case of full insurance, and involuntary unemployment when insurance is incomplete. As Alexopoulos argues, what generates higher fluctuations in the employment rate

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1. Later in the formal firms' subsection, the mechanism through which procyclical wages are generated adopting Alexopoulos punishment strategy will become clearer.

2. The optimization problem of formal entrepreneurs consists of setting wages (optimal contract) that maximize the discounted expected benefits subject to that formal worker utility is strictly greater than the unemployed and informal workers' utility, and greater or equal to the expected value of utility of shirkers. This means that workers who fail to become formal employees, will have to compare the utility of self-employment in the informal sector with the utility of being unemployed. In this situation, the decision will depend on the income that is perceived in the informal sector, because in principle, the benefit of formal firms and lump-sum transfers make the incomes of informal and unemployed totally equal.

3. Unemployment insurance feeds on the earnings of the employed members in the case of Alexopoulos (2004), and on income of formal and informal members in the case of Jaramillo, et al. (2015). When the unemployment insurance is complete, it is guaranteed at all times that the income of the unemployed members is equal to that of the employed.
and lower volatility of wages, is essentially the rigidity that the imperfectly unobervable effort by firms creates, and not so much the existence of these unemployment funds. Instead, the advantage brought by the non-existence of this unemployment insurance in this scheme, is that once the formal employment opportunities are saturated, household members can decide between self-employment in the informal sector, or stay unemployed depending on informal income, because unlike Alexopoulos and Jaramillo, et al., this model allows for different levels of consumption for each household member.

All household members are Ricardoian, meaning they can borrow (buy bonds) with agents around the world at an exogenous risk-free interest rate. In addition, each household member can decide her/his participation in firms’ assets in the next period. Finally, all household members receive fixed lump-sum transfers by the government. Home preferences at any point in time $t$ are represented by a utility function, which is separable in consumption, leisure and effort for each member, and takes the following form:

$$ U(C^f_t, C^s_t, C^i_t, C^u_t, N^f_t, N^s_t, N^i_t, N^u_t, e^f_t, e^i_t) = $$

$$ N^f_t \ln C^f_t + dN^s_t \ln C^s_t + N^i_t \ln C^i_t + N^u_t \ln C^u_t + N^f_t \theta \ln [T - he^f_t - \varepsilon] + N^i_t \theta \ln [T - he^i_t - \varepsilon] + N^u_t \theta \ln T $$

$$ + N^f_t \theta \ln T $$  \hspace{1cm} (1)

Where $C^f_t$ is the consumption index for formal workers (who do not shirk, or at least are not detected), $C^s_t$ is the consumption index for formal shirker workers, $C^i_t$ is the consumption index for informal workers, $C^u_t$ the consumption index for unemployed workers, $N^f_t$ the number of non-shirker formal members in the household, $N^s_t$ the number of formal shirker workers, $N^i_t$ the number of informal members, $N^u_t$ the number of unemployed members, $e^f_t$ the amount of effort of formal non-shirking workers and $e^i_t$ the amount of effort made by informal workers in their productive activities.

The consumption index for each member $C^j_t, j = f, s, i, u,$ is a consumption compound of tradable goods, $C^j_{T,t},$ and non-tradable goods, $C^j_{N,t},$ such that:

$$ C^j_t = \left[ \gamma_c^{1/\rho_c} C^j_{N,t}^{(\rho_c - 1)/\rho_c} + (1 - \gamma_c)^{1/\rho_c} C^j_{T,t}^{(\rho_c - 1)/\rho_c} \right]^{\rho_c/(\rho_c - 1)} $$  \hspace{1cm} (2)

Where $\gamma_c$ and $\rho_c$ are the consumption non-tradable share of total consumption, and the elasticity of substitution of non-tradable goods for tradable goods respectively. Similarly to the previous case, consumption of tradable goods is an index composed of local manufactured goods, $C^j_{M,t},$ and imported goods, $C^j_{F,t},$

$$ C^j_{T,t} = \left[ \gamma_m^{1/\rho_m} C^j_{M,t}^{(\rho_m - 1)/\rho_m} + (1 - \gamma_m)^{1/\rho_m} C^j_{F,t}^{(\rho_m - 1)/\rho_m} \right]^{\rho_m/(\rho_m - 1)} $$  \hspace{1cm} (3)

Where $\gamma_m$ and $\rho_m$ are the local consumption share of manufactured goods of the consumption of tradables, and the elasticity of substitution of local manufactured goods for imported goods respectively. An assumption here is that the production of mining and energy assets are fully exported, which implies that the consumption of this kind of goods is not included within the domestic consumption of tradable goods.
Thus, households maximize their discounted utility flows over time:

$$E_0 \sum_{t=0}^{\infty} \left[ \exp \sum_{\tau=0}^{t-1} \beta_{\tau} \right] U(C_{t}^{f}, C_{t}^{s}, C_{t}^{u}, N_{t}^{f}, N_{t}^{s}, N_{t}^{u}, e_{t}^{f}, e_{t}^{s})$$  \hspace{1cm} (4)

Where the discount factor is endogenous, and takes the form:

$$\beta_{t} = \left\{ 1 + N_{t}^{f} \ln C_{t}^{f} + \frac{dN_{t}^{f}}{N_{t}^{f}} \ln C_{t}^{f} + \frac{N_{t}^{j}}{N_{t}^{f}} \ln C_{t}^{f} + \frac{N_{t}^{s}}{N_{t}^{f}} \ln C_{t}^{f} + \frac{N_{t}^{k}}{N_{t}^{f}} \ln [T - he_{t}^{f} - \varepsilon] + N_{t}^{s} \theta \ln T \right\}^{-k}$$  \hspace{1cm} (5)

Since this model is a small open economy, Schmitt-Grohe and Uribe (2003) demonstrate that this endogenous discount factor ensures that the model has a stable dynamic solution. Each member in the household has his/her own budget constraint. In that order, the budget set of formal workers is:

$$P_{t}C_{t}^{f} + V_{M,t}X_{M,t+1} + V_{E,t}X_{E,t+1} =$$

$$(V_{M,t} + d_{M,t})X_{M,t} + (V_{E,t} + d_{E,t})X_{E,t} + W_{t}^{f} h - B_{t} + \frac{1}{1 + r} B_{t+1} + T_{r,t}$$  \hspace{1cm} (6)

Where $P_{t}$ is the Consumer Price Index, $X_{M,t}$ and $X_{E,t}$ are the shares of formal members (purchased in $t$) of the manufacturing and mining and energy firms’ benefits, respectively. The prices of these shares are respectively $V_{M,t}$ and $V_{E,t}$, while the corresponding dividends are respectively $d_{M,t}$ and $d_{E,t}$. The formal member’s wage is $W_{t}^{f}$, and he/she buys bonds abroad in the current period, $B_{t}$, and then receives returns in the next period, at a risk-free exogenous rate international interest, $r$, and $T_{r,t}$, are lump-sum transfers from government to households.

Similarly, budget constraints for formal shirker members, informal and unemployed, can be expressed respectively as follows:

$$P_{t}C_{t}^{s} + V_{M,t}X_{M,t+1} + V_{E,t}X_{E,t+1} =$$

$$(V_{M,t} + d_{M,t})X_{M,t} + (V_{E,t} + d_{E,t})X_{E,t} + SW_{t}^{f} h - B_{t} + \frac{1}{1 + r} B_{t+1} + T_{r,t}$$  \hspace{1cm} (7)

$$P_{t}C_{t}^{u} + V_{M,t}X_{M,t+1} + V_{E,t}X_{E,t+1} =$$

$$(V_{M,t} + d_{M,t})X_{M,t} + (V_{E,t} + d_{E,t})X_{E,t} + Y_{t}^{i} h - F_{t}^{i} + \frac{1}{1 + r} B_{t+1} + T_{r,t}$$  \hspace{1cm} (8)

$$P_{t}C_{t}^{u} + V_{M,t}X_{M,t+1} + V_{E,t}X_{E,t+1} =$$

$$(V_{M,t} + d_{M,t})X_{M,t} + (V_{E,t} + d_{E,t})X_{E,t} - B_{t} + \frac{1}{1 + r} B_{t+1} + T_{r,t}$$  \hspace{1cm} (9)
Note that in the budget constraint of informal members informal wage is not specified and in the restriction of the unemployed ones no labor income is specified. Since informal workers are self-employed in activities on their own account, their salary is equivalent to the income that they generate($Y_i^i$). The total number of members in the household is normalized to 1, such that at all times the following physical constraint is always satisfied:

$$N_i^u + N_i^f + N_i^s + N_i^f = 1 \quad (10)$$

Thus, the household problem will consist of maximizing their utility function, subject to the budget constraints of each one of its members. The solution to the problem of maximizing the utility constitutes the First Order Conditions (F.O.C.) with respect to $C_t^f, C_t^s, C_t^u, N_t^f, e_t^f, e_t^s, X_{M,t+1}, X_{E,t+1}, B_{t+1}$. The F.O.C. with respect to consumption levels imply the corresponding (decreasing) marginal utilities of consumption for each member:

$$C_t^f : \lambda_{1t} = \frac{1}{P_t C_t^f}$$

$$C_t^s : \lambda_{2t} = \frac{1}{P_t C_t^s}$$

$$C_t^u : \lambda_{3t} = \frac{1}{P_t C_t^u}$$

The F.O.C with respect to $X_{M,t+1}, X_{E,t+1}, B_{t+1}$ give the following optimal expressions, respectively:

$$X_{M,t+1} : V_{M,t} = \beta_t E_t \left\{ \frac{P_t}{P_{t+1}} \left[ \frac{N_{t+1}^f}{C_{t+1}^f} + \frac{N_{t+1}^i}{C_{t+1}^i} + \frac{N_{t+1}^u}{C_{t+1}^u} \right] \right\} (V_{M,t+1} + d_{M,t+1}) \quad (11)$$

$$X_{E,t+1} : V_{E,t} = \beta_t E_t \left\{ \frac{P_t}{P_{t+1}} \left[ \frac{N_{t+1}^f}{C_{t+1}^f} + \frac{N_{t+1}^i}{C_{t+1}^i} + \frac{N_{t+1}^u}{C_{t+1}^u} \right] \right\} (V_{E,t+1} + d_{E,t+1}) \quad (12)$$

$$B_{t+1} : \beta_t (1+r) E_t \left\{ \frac{P_t}{P_{t+1}} \left[ \frac{N_{t+1}^f}{C_{t+1}^f} + \frac{N_{t+1}^i}{C_{t+1}^i} + \frac{N_{t+1}^u}{C_{t+1}^u} \right] \right\} = 1 \quad (13)$$

Therefore, decisions of foreign indebtedness and participation in the firms’ profits will determine the intertemporal consumption decisions of each household member, generating three Euler equations for each one.
With regard to the participation of household members in the labor market, the existence of efficiency wages in the formal sector makes individuals seek to be always employed in it, because the utility of working in the formal sector is strictly greater than the utility from working in the informal sector and being unemployed. Once vacancies in the formal sector are filled, the decision to work in the informal sector or be unemployed depends on whether the income in the informal sector is higher than the subsistence income of being unemployed. This can be seen in the production function of informal workers, which is a linear function of informal effort, the number of informal workers and a fixed number of hours worked:

\[ Y_t^i = \rho c_t^i N_t^i h \]  

(14)

Solving for the number of informal workers in equation (14), we have:

\[ N_t^i = \frac{Y_t^i}{\rho c_t^i h} \]

However, since the adjustment on the informal labor market is also given through the extensive margin, there are a fixed number of working hours, but with different levels of effort (informal), which directly affects the value of the informal production. Therefore, the level of informal effort is a decision variable for informal members:

\[ e_t^i \sim \frac{\theta}{T - h e_t^i - \varepsilon} C_t^i = \rho \]  

(15)

Where \( \rho > 0 \). This condition indicates that informal individual effort is inversely related to their level of consumption. Finally, F.O.C. with respect to the consumption compositions (inframarginal problem) between tradable and non-tradable goods for each household member, \( j = f, s, i, u \), can be expressed in the form:

\[ \frac{C_{N,t}^j}{C_{T,t}^j} = \frac{\gamma_c}{(1 - \gamma_c)} \left( \frac{P_{N,t}}{P_t} \right)^{-\rho_c} \]  

(16)

\[ C_{T,t}^j = (1 - \gamma_c) C_t^f \left( \frac{P_{T,t}}{P_t} \right)^{-\rho_c} \]  

(17)

\[ C_{N,t}^j = \gamma_c C_t^f \left( \frac{P_{N,t}}{P_t} \right)^{-\rho_c} \]  

(18)

This procedure makes the consumer price index \( P_t \) be a compound between the tradable goods price index \( P_{T,t} \) and non-tradable goods price index \( P_{N,t} \):

\[ P_t = \left[ \gamma_c P_{N,t}^{1-\rho_c} + (1 - \gamma_c) P_{T,t}^{1-\rho_c} \right]^{1/1-\rho_c} \]  

(19)

\(^4\)The intramarginal consumption problem is to minimize the total expenditure on consumption of tradable and non-tradable goods \( P_t C_t^j = P_{T,t} C_{T,t}^j + P_{N,t} C_{N,t}^j \) subject to the total consumption CES index equation described by equation (2), the decision variables being consumption of tradable and non-tradable goods. Similarly, the next intertemporal problem consists in minimizing total expenditure on tradable imported and local manufactured goods \( P_{T,t} C_{T,t}^j = P_{T,t} C_{T,t}^j + C_{M,t}^j \) subject to the tradable consumption CES index described by equation (3), the decision variables being consumption of imported tradable and local manufactured tradable goods.
Similarly happens with the intratemporal optimality conditions of consumption of tradable goods, which is a compound of the consumption of local manufactured goods, and imported goods:

\[
\frac{C_{M,t}^{j}}{C_{F,t}^{j}} = \frac{\gamma_m}{(1 - \gamma_m)} \left( \frac{1}{P_{F,t}} \right)^{-\rho_m}
\]

(20)

\[
P_{T,t} = \left[ \gamma_m + (1 - \gamma_m)P_{F,t}^{1-\rho_m} \right]^{1/(1-\rho_m)}
\]

(21)

\[
C_{M,t}^{j} = \gamma_m C_{T,t}^{j} \left( \frac{1}{P_{F,t}} \right)^{-\rho_m}
\]

(22)

\[
C_{F,t}^{j} = (1 - \gamma_m) C_{T,t}^{j} \left( \frac{P_{F,t}}{P_{T,t}} \right)^{-\rho_m}
\]

(23)

Since the model is abstracted from nominal considerations and focuses only on real issues, the determination of price levels will be relative to the locally manufactured goods price index, which is chosen as the numerary. The fact that the price of local manufactured goods does not change (equal to unity), can be interpreted as the fact that actually the price of tradable goods is determined by international movements of supply and demand, and therefore, a small economy takes this price as given. Considering tradable goods prices as exogenous is usual in the literature that addresses the (real) phenomenon of Dutch disease in general equilibrium scenarios (Corden and Neary, 1982; Beverelli, et al., 2011; Ojeda, et al, 2014; among others).

4.2. Firms

The whole economy productive sector is divided into two subsectors: a formal sector and an informal one. The informal sector does not require physical capital stock, so its production function only needs informal workforce, as shown in equation (14).

The formal sector is in turn divided into three subsectors: non-tradable goods (N), manufactured tradable goods (M), and mining and energy tradable goods (E). The firms’ modeling strategy, follows closely Acosta, Lartey and Mandelman (2009), Ojeda, Parra and Vargas (2014), and Jaranaillo Obando y Guataqui (2015). There is only a formal wage that covers the three formal subsectors, which implies free mobility of labor between formal activities. Additionally, there is a physical constraint on the total number of formal workers:

\[
N_{N,t}^{f} + N_{M,t}^{f} + N_{E,t}^{f} = N_{f}^{f}
\]

(24)

Where \(N_{N,t}^{f}\) is the number of workers in the non-tradable subsector, \(N_{M,t}^{f}\) is the number of workers in the manufacturing tradable subsector, and \(N_{E,t}^{f}\) the number of workers in the mining and energy subsector.

Formal firms hire workers through a one-period contract, where \(sW_{t}h\) is the guaranteed compensation for all workers and \((1 - s)W_{t}h\) is the payment guaranteed to workers who are not detected shirking. As in Alesina (2004), it is assumed that \(s \in (0, 1)\) is exogenously fixed and firms set \(W_{t}\) in order to maximize the effort of formal workers (which is not observable by the firms) and minimize laziness (which is detected with exogenous probability \(d > 0\)), which implies setting efficiency wages by formal entrepreneurs.
Since the formal firms face uncertainty about whether their workers make laziness in their respective jobs, each formal firm will have to design an optimal contract that maximizes the utility of their workers, maximizes their own dividends, and minimizes asymmetric information between employee and employer by offering a salary to ensure that workers do not shirk. Thus, the formal firm maximizes its expected benefits, subject to the restriction of individual rationality (IR) and the incentive compatibility constraint (IC):

\[ \text{Max formal profits} \]

s.a.

\[ u(C^f_t, c^f_t) > \{ u(C^e_t, c^e_t), u(C^n_t, 0) \} \quad \text{(IR)} \]

\[ u(C^f_t, c^f_t) \geq du(C^n_t, 0) + (1 - d)u(C^f_t, 0) \quad \text{(IC)} \]

Where \( u(\cdot, \cdot) \) is the utility of the representative household member, and \( d \) is the exogenous probability of a lazy worker getting detected. The restriction of individual rationality ensures that household members prefer working in the formal sector while the restriction of incentive compatibility ensures that formal workers are indifferent between shirking and carrying out the effort required by the contract. In equilibrium, this incentive constraint (IC) is satisfied with equality, which defines the link between formal efforts and formal wages:

\[ c^f_t = \frac{T}{h} \left[ 1 - \left( \frac{C^n_t}{C^f_t} \right)^{d/h} \right] - \frac{\varepsilon}{h} \quad \text{(25)} \]

Manipulating the algebra of formal members’ budget constraints shows that the formal wage is an increasing function of the gap between non-shirker consumption and formal shirker consumption:

\[ W^f_t = \frac{p(C^f_t - C^n_t)}{1 - \rho h}. \]

This result combined with equation (25) implies that formal workers’ effort responds positively to an increase in formal wage. Then, by dividing equation (6) by equation (7) and making some algebraic modifications, the following is obtained:

\[ \frac{C^f_t}{C^n_t} = \frac{\text{profits} + W^f_t h - B_t + \frac{1}{1+\tau} B_{t+1} + Tr_t}{\text{profits} + sW^f_t h - B_t + \frac{1}{1+\tau} B_{t+1} + Tr_t} \quad \text{(26)} \]

Where \( \text{profits} = (V_{M,t} + d_{M,t})X_{M,t} + (V_{E,t} + d_{E,t})X_{E,t} - V_{M,t}X_{M,t+1} - V_{E,t}X_{E,t+1} \). Equation (26) indicates that the ratio \( \frac{C^f_t}{C^n_t} \) diminishes when \( \text{profits} \) are rising and/or when the transfers from government to households increases. Finally, as a result of equation (26) there are no shirker workers in equilibrium, so \( N^n_t = 0 \).

4.2.1. Formal Non-Tradable Sector

For simplicity, non-tradable firms do not require physical capital in the production process, which implies that labor is the only input factor. Therefore, the production function takes the following form:
\[ Y_{N,t}^f = \exp(A_{N,t}^f) N_{N,t}^f h e_t^f \]  

(27) \[ A_{N,t}^f = \rho_N A_{N,t-1}^f + \epsilon_t^N \]  

(28) Where \( A_{N,t}^f \) is the autoregressive exogenous stochastic productivity process and \( \epsilon_t^N \) is a white-noise term. Each non-tradable firm pays an output tax to the government, \( \tau_N \). The F.O.C. that ensure the maximization of profits of non-tradable firms, regarding labor and wages, are respectively:

\[ (1 - \tau_N) P_{N,t} \exp(A_{N,t}^f) = \frac{W_t^f}{e_t^f} \]  

(29) \[ (1 - \tau_N) P_{N,t} \exp(A_{N,t}^f) = \frac{1}{e_t^{f+1}(W_t^f)} \]  

(30) Thus, marginal productivity of labor equals the wage divided by the level of effort. Moreover, optimality condition with respect to wages implies that the marginal productivity of labor is equal to unity divided by the derivative of effort with respect to wages. Combining the last two equations, the following is guaranteed:

\[ \frac{W_t^f}{e_t^f} e_t^{f+1}(W_t^f) = 1 \]  

(31) This expression, known as the Solow Condition, is the unitary elasticity of effort to wages, which says that in this situation the non-tradable firm sets a wage (of efficiency), such that an increase in this wage by 1% encourages workers to increase their level of effort by 1%, which solves the problem of asymmetric information. Differentiating equation effort (25) with respect to the formal wage, and replacing the resulting equation (31), we obtain:

\[ e_t^f = \frac{T}{h} \frac{d}{\partial} \left( \frac{C_t^f}{C_t^f} \right)^{-(\hat{\eta})-1} \left( \frac{C_t^f - sC_t^f}{(C_t^f)^2} \right) h W_t^f \]  

(32) Therefore, the link between wages and the level of effort can be interpreted as follows. By equation (26), an increase in formal entrepreneurs benefits and/or an increase in government transfers, makes the gap between formal consumption and shirker formal consumption \( \frac{C_t^f}{C_t^f} \) decrease, which reduces the punishment for lazy workers. With the objective of preventing workers from reducing their efforts, firms respond by increasing wages. This means that when facing a positive productivity shock, firm benefits and public transfers rise, implying a wage increase in order to avoid shirking behaviour in formal workers. This wage raise makes formal employment fall because of an increase in formal labor costs, but the increase in total productivity raises formal marginal labor productivity, which in turn raises labor demanded quantity at the new higher formal wage. The increased labor productivity then positively compensates for the negative effect on employment which involves a negatively sloped labor demand. Therefore, formal wages and employment are procyclical, formal employment being more volatile than formal wages.

Since there is a single salary through formal subsectors, the Solow Condition must also be satisfied for manufacturing and mining-energy firms.
4.2.2. Formal tradable manufacturing sector

Manufacturing firms require factor labor and capital stock to carry out their production processes. The equation of capital motion is:

\[ K_{M,t+1} = I_{M,t} + (1 - \delta_m)K_{M,t} \]  \hspace{1cm} (33)

Where \( K_{M,t} \) is the capital stock of the manufacturing firm, \( I_{M,t} \) is investment and \( \delta_m \) the capital depreciation rate. In addition, the model has costs of adjusting capital to its optimum level, which are proportional to the capital stock:

\[ \frac{\phi_m}{2} \left( \frac{I_{M,t}}{K_{M,t}} - \delta_m \right)^2 K_{M,t} \]  \hspace{1cm} (34)

Additionally, capital goods that manufacturing firms demand \( I_{M,t} \) to the price \( P_{I,M,t} \) is a CES compound by local investment \( I_{H,t} \) and foreign (imported) investment \( I_{F,t} \), such that:

\[ I_{M,t} = \left[ \gamma_i \left( \frac{I_{H,t}^{1/\rho_i} + (1 - \gamma_i)I_{F,t}^{1/\rho_i}}{P_{I,M,t}} \right)^{\rho_i} \right]^{\rho_i/(\rho_i - 1)} \]  \hspace{1cm} (35)

It is assumed that a final manufactured good (whose price is equal to the unity) can be immediately converted into a manufacturing investment good. Therefore, the price of local manufacturing investment \( I_{M,t} \) is equal to the unity. The solution to the manufacturing firm intratemporal problem consists of choosing the optimally demanded quantity of local and foreign investment goods for each period, to the price that will minimize their spending on the total investment quantity purchased. Then it involves the following first order conditions:

\[ \frac{I_{H,t}}{I_{F,t}} = \frac{\gamma_i}{(1 - \gamma_i)} \left( \frac{1}{P_{I,t}} \right)^{-\rho_i} \]  \hspace{1cm} (36)

\[ P_{I,M,t} = \left[ \gamma_i + (1 - \gamma_i)P_{I,F,t}^{1-\rho_i} \right]^{1/(1-\rho_i)} \]  \hspace{1cm} (37)

\[ I_{H,t} = \gamma_i I_{M,t} \left[ \frac{1}{P_{I,M,t}} \right]^{-\rho_i} \]  \hspace{1cm} (38)

\[ I_{F,t} = (1 - \gamma_i)I_{M,t} \left[ \frac{P_{I,F,t}}{P_{I,M,t}} \right]^{-\rho_i} \]  \hspace{1cm} (39)

Manufacturing firms have a Cobb-Douglas production function with constant returns to scale:

\[ Y_{M,t}^f = \exp(A_{M,t}^f)K_{M,t}^{\alpha_m}[N_{M,t}^f]^{1-\alpha_m} \]  \hspace{1cm} (40)

\[ A_{M,t}^f = \rho_M A_{M,t-1}^f + \epsilon_t^M \]  \hspace{1cm} (41)

Where \( A_{M,t}^f \) is the exogenous autoregressive stochastic productivity process and \( \epsilon_t^M \) is a white-noise term. Each manufacturing firm pays a tax on their income to the government, \( \tau_M \). Manufacturing firms maximize the present value of dividends, such that:
\[ E_t \sum_{s=t}^{\infty} \left[ \exp(\beta_{s-t}) \right] \frac{\lambda_t \phi_{s} P_{s}}{\lambda_{c,s}} d_{M,s} \]  

\[ d_{M,s} = (1-\tau\eta)f_I Y^f_{M,s} - P_{M,t} \left( I_{M,s} + \frac{\phi_d}{2} \left( \frac{I_{M,s}}{K_{M,s} - \delta} \right)^2 \right) - W^f_s h N^f_{M,s} \]  

Where \( \lambda_{c,s} = \frac{N^f_{s}}{C^i_t} + \frac{N^r_{s}}{C^i_t} + \frac{N^m_{s}}{C^i_t} \). It is the marginal utility of consumption of all household members. The way to discount the future expected flow of dividends in equation (46), has been used in works such as Mendoza (1991), Ojeda, et al., (2014) and Schmitt-Grohe and Uribe (2015). This strategy of intertemporal discounting can be interpreted as the value assigned by households to contingent payments of consumption goods in period \( s \geq t \). This maximization process subject to capital stock motion equation, incentive compatibility constraint (IC) and individual rationality (IR) constraint, generates the following optimality conditions regarding \( K_{M,t+1}, I_{M,t}, N^f_{M,t}, W^f_t \) respectively:

\[ E_t \exp(\beta_{t+1}) \frac{\lambda_{c,t+1}}{\lambda_{c,t}} \frac{P_t}{P_{t+1}} \left[ P_{M,t+1} \left( \phi_m \left( \frac{I_{M,t+1}}{K_{M,t+1} - \delta} \right) - \frac{\phi_m}{2} \left( \frac{I_{M,t+1}}{K_{M,t+1} - \delta} \right)^2 \right) \right] + \alpha_m \frac{(1-\tau_{m})Y^f_{M,t+1}}{K_{M,t+1}} + \lambda_{IM,t+1}(1-\delta_{m}) = \lambda_{IM,t} \]  

\[ P_{M,t} \left[ 1 + \phi_m \left( \frac{I_{M,t}}{K_{M,t} - \delta} \right) \right] = \lambda_{IM,t} \]  

\[ (1-\tau_m)\exp(a_m) (1-\alpha_m) K_{M,t}^{\alpha_m} \left[ N^f_{M,t} h c^f_t \right]^{-\alpha_m} = \frac{W^f_t}{e^f_t} \]  

\[ (1-\tau_m)\exp(a_m) (1-\alpha_m) K_{M,t}^{\alpha_m} \left[ N^f_{M,t} h c^f_t \right]^{-\alpha_m} = \frac{1}{e^f_t} (W^f_t) \]  

Equation (44) is the optimality condition with respect to the capital stock, which says that the shadow price of a unit of capital in the present is equal to the discounted value of the future shadow price per unit capital net of depreciation, plus the discounted value of the expected marginal productivity of future capital, net of investment adjustment costs. Meanwhile, the optimality condition with respect to investment is equal to the ratio of the shadow price of capital relative to the price of the investment good \( \frac{\lambda_{IM,t}}{P_{M,t}} \) which is equal to the Tobin’s \( q \): \( q_t = 1 + \phi_m \left( \frac{I_{M,t}}{K_{M,t} - \delta} \right) \). As there are no investment adjustment costs in steady state, then \( q = 1 \) and \( \lambda_{IM,t} = P_{M,t} \). Finally, the optimality conditions regarding labor input and wages involves again the Solow Condition of efficiency wages, discussed in the profits maximizing problem of non-tradable firms:

\[ \frac{W^f_t}{e^f_t} c^f_t (W^f_t) = 1 \]
4.2.3. Formal Mining and Energy Sector

Unlike the case of manufacturing firms, investment in the mining and energy sector is all imported. Similarly to the manufacturing sector, in the mining and energy firms the capital motion equation, installation costs of capital, and production function can be respectively expressed as follows:

\[ K_{E,t+1} = I_{E,t} + (1 - \delta_E)K_{E,t} \]  

(48)

\[ \frac{\phi_E}{2} \left( \frac{I_{E,t}}{K_{E,t}} - \delta_E \right)^2 K_{E,t} \]  

(49)

\[ Y_{E,t}^f = \exp(A_{E,t}^f)K_{E,t}^{\alpha_E} \left[ N_{E,t}^{f}he_{t}^f \right]^{1-\alpha_E} \]  

(50)

\[ A_{E,t}^f = \rho_E A_{E,t-1}^f + \epsilon_t^E \]  

(51)

Where \( K_{E,t} \) is the stock of mining and energy capital, \( \delta_E \) is the depreciation rate, \( A_{E,t}^f \) is the exogenous autoregressive stochastic productivity process, \( \epsilon_t^E \) is a white-noise term and the production technology is Cobb-Douglas type with constant returns to scale.

As in the case of manufacturing firms, each mining and energy firm pays a tax on their income to the government, \( \tau_E \), and maximizes the present value of its dividends, such that:

\[ E_t \sum_{s=t}^{\infty} \left[ \exp^\beta_s \right] \left[ \frac{\lambda_{c,t} P_t}{\lambda_{c,t} P_s} \right] d_{E,s} \]  

(52)

\[ d_{E,s} = (1 - \tau_E)Y_{E,s}^f - P_{F,t} \left( I_{E,s} + \frac{\phi_E}{2} \left( \frac{I_{E,s}}{K_{E,s}} - \delta_E \right)^2 K_{E,s} \right) - W_s^f hN_{E,s}^f \]  

(53)

This maximization process subject to the equation of motion of capital stock generates the following optimality conditions concerning \( K_{E,t+1}, I_{E,t}, N_{E,t}, W_t^f \), respectively:

\[ E_t \exp(\beta_{t+1}) \left[ \frac{\lambda_{c,t+1}}{\lambda_{c,t}} \frac{P_{t}}{P_{t+1}} \right] \left[ P_{F,t+1} \phi_\varepsilon \left( \frac{I_{E,t+1}}{K_{E,t+1}} - \delta_\varepsilon \right) \right] = \lambda_{IE,t} \]  

(54)

\[ P_{F,t} \left[ 1 + \phi_\varepsilon \left( \frac{I_{E,t}}{K_{E,t}} - \delta_\varepsilon \right) \right] = \lambda_{IE,t} \]  

(55)

\[ (1 - \tau_\varepsilon)\exp(A_{E,t}^f)(1 - \alpha_E)K_{E,t}^{\alpha_E} \left[ N_{E,t}^{f}he_{t}^f \right]^{-\alpha_E} = \frac{W_t^f}{\epsilon_t^E} \]  

(56)

\[ (1 - \tau_\varepsilon)\exp(A_{E,t}^f)(1 - \alpha_E)K_{E,t}^{\alpha_E} \left[ N_{E,t}^{f}he_{t}^f \right]^{-\alpha_E} = \frac{1}{\epsilon_t^E(\epsilon_t^E)} \]  

(57)
The dynamics of capital and investment in the case of mining and energy sector is similar to the manufacturing sector. Having described the dynamics of capital and investment in both formal sectors, it must be stated that in a positive productivity shock to the mining and energy sector; the marginal productivity of capital in this sector increases, in turn increasing the shadow capital price relative to the investment price, which increases mining and energy Tobin’s q, and thereby investment and energy capital stock increase. Additionally, if there is any labor mobility into this sector, the demand for capital curve moves up and to the right, further reinforcing the increase in the stock of mining and energy capital. Although there is no capital mobility between the formal sectors in the model, the lower level of labor in the manufacturing sector due to the Dutch disease alters the marginal rates of substitution of workers between the formal sectors, which shrinks the demand curve for manufacturing capital, resulting in lower manufacturing capital stock and manufacturing investment.

Similarly to the non-tradeable and manufacturing sectors, F.O.C. with respect to labor and real wages imply that in equilibrium the Solow condition is satisfied:

\[ \frac{W_i^f}{e_i^f} e_i^f (W_i^f) = 1 \]

Finally, the formal GDP is equal to the sum of the value of production in each of the formal sectors, as well as the total GDP is the sum of formal and informal GDP, so that:

\[ Y_t^f = Y_{M,t} + P_{N,t} Y_{N,t} + P_{E,t} Y_{E,t} \]  \hspace{1cm} (58)

\[ Y_t = Y_t^f + Y_t^i \]  \hspace{1cm} (59)

4.3. Fiscal Sector

Following Ojeda, et al. (2014), government income revenue from taxes resulting from taxing the formal sectors of the economy, i.e. the non-tradeable sector, the manufacturing sector and the mining-energy sector, can be expressed as:

\[ T_t = \tau_m Y_{M,t}^f + \tau_N P_{N,t} Y_{N,t}^f + \tau_E P_{E,t} Y_{E,t}^f \]  \hspace{1cm} (60)

It is assumed that the government is always in the balanced budget condition, meaning that taxes are equal to transfers to households at any moment of time:

\[ T_{r,t} = T_t \]  \hspace{1cm} (61)

The existence of government within the model is important, because against a positive shock to mining and energy productivity, not only will formal wages increase but tax collection, too. These two factors increase consumption and therefore generate upward pressures on aggregate demand, which makes the mechanism of Dutch disease possible. However, the key reason for not including a fiscal rule is that all economic agents are Ricardian. Since there are lump-sum transfers to households, the assumption of Ricardian agents make the optimal trajectories of consumption equivalent against a certain shock when the fiscal rule is either procyclical or countercyclical, due to consumption smoothing possibility of household members. Therefore, incorporating a fiscal rule does not alter the optimal allocations, making it irrelevant in the present scheme.
4.4. External Sector

Exports process (manufactured goods only) follows the form:

\[ X_{M,t} = e_t^* Y_{F,t} \]  \hspace{1cm} (62)

Where \( e \) is the real exchange rate, \( \nu \) is the elasticity of exports to the real exchange rate, and \( Y_{F,t} \) is the rest of the world GDP (exogenous). At the same time, the real exchange rate is equal to:

\[ e_t = \frac{P_{E,t}}{P_t} \]  \hspace{1cm} (63)

Moreover, the current account is defined as:

\[ CA_t = -\gamma(B_t + B_{G,t}) + X_{M,t} + P_{E,t}Y_{E,t} - P_{F,t}(C_{F,t}^i + C_{F,t}^w + I_{E,t} + I_{M,t}) \]  \hspace{1cm} (64)

For closing the model, formal manufactured product must be equal to consumption of domestic manufactures plus local manufacturing investment plus exports. In addition, production of non-tradable goods must be equal to the consumption of non-tradable goods:

\[ Y_{M,t}^f = C_{M,t}^f + C_{M,t}^i + C_{M,t}^w + I_{M,t} + X_{M,t} \]  \hspace{1cm} (65)

\[ Y_{F,t}^f = C_{F,t}^f + C_{F,t}^i + C_{F,t}^w \]  \hspace{1cm} (66)

4.5. General Equilibrium

In order to compare the effects derived from a positive shock to the international oil price \( (P_{E,t}) \) with respect to the results of a positive productivity shock to the commodity industry, the oil price follows an exogenous autoregressive process:

\[ A_{E,t}^p = \rho E A_{E,t-1}^p + c_t^p \]  \hspace{1cm} (67)

Where \( c_t^p \) is a white-noise term. Based on Schmitt-Grohe and Uribe (2015), the recursive competitive equilibrium in the decentralized economy consists of a set of dynamic processes:

\[ \{ P_t, C_t^f, C_t^a, C_t^i, C_t^w, F_t^f, F_t^i, F_t^w, N_t^f, N_t^i, N_t^w, V_{M,t}, V_{E,t}, d_{M,t}, d_{E,t}, W_t^f, W_t^i, W_t^w, B_t, Tr_t, \beta_t, e_t^f, e_t^i, P_{F,t}, \} \]

\[ P_{N,t}, P_{F,t}, P_{M,t}, P_{E,t}, C_{N,t}^f, C_{N,t}^a, C_{N,t}^i, C_{N,t}^w, C_{T,t}^f, C_{T,t}^a, C_{T,t}^i, C_{T,t}^w, C_{M,t}^f, C_{M,t}^a, C_{M,t}^i, C_{M,t}^w, C_{N,t}^f, C_{N,t}^a, C_{N,t}^i, C_{N,t}^w, C_{T,t}^f, C_{T,t}^a, C_{T,t}^i, C_{T,t}^w, C_{M,t}^f, C_{M,t}^a, C_{M,t}^i, C_{M,t}^w, \]

\[ C_{T,t}^w, C_{E,t}^w, C_{M,t}^w, N_{N,t}^f, N_{N,t}^a, N_{N,t}^i, N_{N,t}^w, N_{E,t}^f, N_{E,t}^a, N_{E,t}^i, N_{E,t}^w, Y_t^f, Y_t^i, Y_t^w, Y_{M,t}^f, Y_{M,t}^i, Y_{M,t}^w, Y_{E,t}^f, Y_{E,t}^i, Y_{E,t}^w, A_{N,t}^f, A_{N,t}^a, A_{N,t}^i, A_{N,t}^w, A_{M,t}^f, A_{M,t}^a, A_{M,t}^i, A_{M,t}^w, A_{E,t}^p, A_{E,t}^i, A_{E,t}^w, I_{M,t}, I_{H,t}, \]

\[ I_{F,t}, K_{M,t}, \lambda_{M,t}, \lambda_{E,t}, \lambda_{C,t}, I_{E,t}, K_{E,t}, Y_t, e_t, X_{M,t}, B_{G,t}, T_t, CA_t \}_{t=0}^{\infty} \]
Which satisfy the equations (1) - (24), (27) - (41), (43) - (51), (53) - (68), given:

\[ B_{-1}, K_{M,-1}, K_{E,-1}, A_{N,0}^{f}, A_{M,0}^{f}, A_{E,0}^{r}, A_{E,0}^{p} \]

And the stochastic processes \( \{\epsilon_t^N, \epsilon_t^M, \epsilon_t^E, \epsilon_t^p\}_{t=0}^\infty \).

5. Calibration

The parameters and relationships between variables in the steady state are set based on calibrations made by Ojeda, et al. (2014), standard values of previous literature, and some stylized facts of Colombian macroeconomics. In the first instance, the long-term values for the formal employment, informal employment and unemployment rates are set to coincide with their respective averaged observed values in the quarterly period 1984 - 2013, according to DANE statistics. On average, formal employment, informal employment and unemployment rates were 39.2%, 49.6% and 11.6% respectively:

\[ N^f = 0.392, N^i = 0.496, N^u = 0.116 \]

Adding the number of employed people by economic activity in each period published by DANE (period 1984-2013), that correspond to manufacturing sectors ("manufacturing industry"), Mining and Energy ("mining and quarrying") and Non-tradables ("building", "real estate activities", "local trade, restaurants and hotels"), and assuming that the sum of these three subsectors gives the total formal employment that is considered in the model, then the average employment shares of these sectors of total formal employment, are:

\[ N_{M}^{f} = 0.202, N_{E}^{f} = 0.024, N_{N}^{f} = 0.166 \]

It should be noticed that these values satisfy the constraint: \( N_{M}^{f} + N_{E}^{f} + N_{N}^{f} = N^{f} \).

The relationship between formal and informal product in steady state was endogenous to the static steady state equations system, which turned out to be \( Y^{f}/Y^{i} = 3.58 \), a very similar value to that found by Jaramillo and Gomez (2013) for 2007, which corresponds to 3.67. Total per capita GDP was 5.63, whose value is calculated by taking the GDP (at constant prices) in 2007, then it was split into 4 to obtain a quarterly measure, and finally it was divided between the economically active population (PEA) of September of that year. Given the relation \( Y^{f}/Y^{i} \), it was obtained \( Y^{f} = 4.29 \) and \( Y^{i} = 1.20 \).

Similarly, the average shares of manufacturing, mining-energy and non-tradable output levels were calculated from the GDP database discriminated by economic activity, which is available in the quarterly national accounts published by DANE. Thus in steady state:

\[ Y_{M}^{f} = 1.99, Y_{E}^{f} = 1.58, Y_{N}^{f} = 0.11 \]

In addition, the capital stock shares of output for manufacturing and mining-energy production were calculated endogenously and their values were respectively equal to \( \alpha_{m} = 0.63 \) and \( \alpha_{E} = 0.87 \), \( \alpha_{E} \)}
being very similar to that found in the fiscal rule report for Colombia (Ojeda, et al., 2015; Inter-Agency Technical Committee, 2010). Setting $s = 0.7$ the relationship between non-shirker formal consumption and shirker formal consumption was $C^f/C^s = 1.25$. This value is close to the one calculated by Jaramillo, et al. (2015) of 1.30 and estimated by Alexopoulos (2004) of 1.28 for the fast food industry in the United States. In summary, the parameter values of the model are presented in Table 1.

### 6. Impulse-Response Functions

Based on calibration and steady-state values, I proceed to solve the dynamic model by the first order approximation method. Subsequently, a positive shock to the stochastic component of productivity in 10\% is simulated, which can be interpreted as a mining and energy boom (Ojeda, et al., 2014). Additionally, the results derived from a positive shock to mining and energy productivity are compared to the results that emerge from a positive shock to the international oil price of 10\%.

#### 6.1. Positive productivity shock to mining-energy sector productivity

Figure 13 (Annex 1.1) shows the responses of the model variables to a shock to the stochastic component of the mining and energy sector productivity (10\%). In the first instance, it can be seen that the increase in energy productivity leads to an increased production in this sector, generating an increase in tax collection and in the benefits for mining and energy firms. This implies a decrease of the ratio $C^f/C^s$ and therefore the punishment for lazy workers falls and the incentive to make an effort for the formal workers then declines. To avoid falls in formal effort, formal firms decide to raise wages, initially having a negative effect on the level of employment. However, the increase in mining-energy productivity generates an up-right shift of the demand curve for labor in this sector, which more
than compensates for the employment drop in the commodity sector, as energy firm demands more workers at the new (higher) level of wages, giving a net increase in the mining-energy employment. Moreover, increased energy productivity increases the marginal productivity of capital, raising the shadow price per unit of capital relative to the investment price. Consequently, it increases Tobin’s q and thus investment and capital stock in the mining and energy sector also go up. Additionally, the free mobility of labor to this sector shifts capital demand up and to the right, further reinforcing the increase in the mining and energy capital stock. Summarizing, the mining and energy sector responses consist of increases in production, wages, employment, investment, and capital stock. The increase in formal wages, formal dividends and public transfers increase consumption levels of household members in a generalized way, causing an increase in the price level in the non-tradable subsector relative to the price level of the tradable manufacturing subsector (numerical). This generates a decrease (appreciation) in the real exchange rate, accompanied by an increase in the current account. This difference in price responses tends to push up production costs in the manufacturing sector and to increase productivity in the non-tradable sector, diminishing manufacturing employment and increasing employment in the non-tradable sector. Additionally, production increases in the non-tradable sector, and employment, product, investment and capital stock in the manufacturing sector fall.

As a result, aggregate formal employment and formal product of the economy increases, reducing informal employment and hence informal production. Declining informal incomes makes unemployed members receive a total income greater than the total income of informal members, which result in an increase in the number of unemployed and an even more pronounced fall in the number of informal workers. As a result, the unemployed increase their consumption, while informal consumption falls.

6.2. Positive productivity shock to the international commodity price

Figure 14 (Annex 1.2) shows the responses of the model variables to a positive shock (10%) to the international oil price ($P_{E,O}$). It can be seen that the results are very similar to an exogenous increase in the productivity of the mining and energy sector. This result supports López et al.’s (2013) argument that a mining and energy boom through an increase in international commodity prices, is associated with an increase of productivity in this sector. The only difference between an increase in energy productivity and an exogenous increase in the price of commodities, is that the first directly affects the level of energy production as it is present in the production function. However, the energy output response when the commodity price increases is somewhat lagged, as price of oil indirectly affects energy output through the factors’ marginal productivities.

7. Conclusions

This paper develops a Dynamic Stochastic General Equilibrium (DSGE) model in order to analyze the macroeconomic and labor market effects resulting from simulating a positive shock to the stochastic component in the productivity of the mining and energy sector. The model consists of a small open economy, which closely follows the household modeling strategy of Alexopoulos (2004) and Obando, Jaramillo and Guataqui (2015). The productive sector consists of an informal sector and a formal one, the latter being a compound of three subsectors: non-tradable goods, manufactured tradable goods and mining and energy goods (Jaramillo, et al., 2015; Ojeda, et al., 2014). Furthermore, the fiscal policy and external sector closely follow the modeling strategy by Ojeda, et al. (2014) in order to adequately simulate the transmission mechanisms that involves the mining and energy boom and the Dutch disease. Additionally, parameters and steady state values were calculated based on Jaramillo, et al. (2015), Ojeda, et al. (2014), some stylized facts of the Colombian economy, and some values of parameters from the conventional literature.
A positive transitory shock to mining-energy productivity sector generates a general increase in formal wages and tax revenues, thus increasing total consumption of household members. This generates an increase in the price of non-tradables in relation to the price of tradables, which reduces the real exchange rate (appreciation) and causes a shift of productive resources from the manufacturing tradable sector to the non-tradable one, followed by an increase in formal GDP and formal employment in the economy. This fact makes the aggregate formal sector absorb workers from the informal sector through the upward non-tradable formal subsector, reducing informal GDP. Consequently, informal members’ consumption decreases, which encourages some household members to become unemployed instead of offering their labor in the informal sector. Therefore, the final result on the labor market is a decline in the number of informal workers, where one part of the non-informal workers is in the formal sector and the remaining part is unemployed.

Additionally, the results of a positive shock to mining and energy sector productivity are compared with a shock to international commodity prices, which shows very similar qualitative and quantitative results.

References


Annexes

Annex 1.1. Positive Productivity Shock to the Mining-Energy Sector

Figure 13: Impulse-response functions due to a positive shock (10%) to the mining-energy productivity.
Annex 1.2. Positive Shock (10%) to the International Oil Price

Figure 14: Impulse-response functions due to a positive shock (10%) to the international oil price