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**Do U.S. and Colombian macro factors improve the forecasting
ability of unrestricted VAR models of the local term structure of
interest rate?**

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Final Work Grade report

-Final investigation report-

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Summary

This empirical study compares the ability of unrestricted Vector Autoregressive Models (VAR) to forecast the Colombian term structure of interest rates. We compare simple VARs with factor augmented VARs using Colombian and U.S. macroeconomic and financial factors. We find that including information from oil prices, Colombian credit risk, and an international proxy for risk aversion improves the out-of-sample forecasting ability of unrestricted VAR models for short-term maturities at monthly frequency. For mid and long-term maturities, the models without macroeconomic variables perform the best suggesting that mid and long-term yields already include all the meaningful information necessary to forecast them. Our findings have important implications for portfolio managers, market participants, and policy makers.

Key words

Term structure of interest rates, yield curve forecasting, VAR models, macroeconomic factors.

Resumen

Este estudio empírico compara la capacidad de los modelos Vectores auto-regresivos (VAR) sin restricciones para predecir la estructura temporal de las tasas de interés en Colombia. Se comparan modelos VAR simples con modelos VAR aumentados con factores macroeconómicos y financieros colombianos y estadounidenses. Encontramos que la inclusión de la información de los precios del petróleo, el riesgo de crédito de Colombia y un indicador internacional de la aversión al riesgo mejora la capacidad de predicción fuera de la muestra de los modelos VAR sin restricciones para vencimientos de corto plazo con frecuencia mensual. Para vencimientos de mediano y largo plazo los modelos sin variables macroeconómicas presentan mejores pronósticos sugiriendo que las curvas de rendimiento de mediano y largo plazo ya incluyen toda la información significativa para pronosticarlos. Este hallazgo tiene implicaciones importantes para los administradores de portafolios, participantes del mercado y responsables de las políticas.

Palabras clave

Estructura a plazos de las tasas de interés, pronóstico de la curva de rendimientos, modelos VAR, factores macroeconómicos.

Introduction

The term structure of interest rates or yield curve provides important information for financial markets and it is a necessary tool for managing fixed income portfolios. In addition, it gives information about inflation expectations and the future path of interest rates (Cano, Correa & Ruiz, 2010). Increases in the slope of the yield curve may signal the authorities of rising inflation expectations (Banco de la República, 2003). The yield curve is also useful in predicting future economy activity; for example, an inverted yield curve usually precedes U.S. recessions (Estrella & Trubin, 2006).

In this paper, we investigate if U.S. and Colombian macroeconomic variables help forecast the local yield curve using unrestricted Vector Autoregressive Models (VARs). Anecdotic evidence suggests that central banks, portfolio managers, and investors care about the movements of the yield curve and use information beyond what is been priced into the current yield curve to try to anticipate such movements. However, previous studies have found conflicting results regarding the ability of econometric models in forecasting the yield curve. While most studies focus on the U.S. and other developed countries, just a few focus on developing countries.

Abbritti et al. (2013) show that global shocks affect the long-term of the U.S. yield curve. Mönch (2005) shows that interest rate forecasts improve if wide common factors of macroeconomic information are included in the model, instead of using just a few variables. De Pooter et al. (2010) find that models that include macroeconomic information forecast better before and during recession.

There are no previous available studies for Colombia examining the relationship between macroeconomic factors and the forecasting of the yield curve; in this regard, it is important to perform an empirical study useful for future reference.

For Colombia, we use monthly data for the period between April 2005 and April 2015. The macro variables selected for this study correspond with those currently used by financial

analysts and market participants to explain the movements of the Colombian yield curve (see Appendix 1).

As base case models, we use unrestricted Vector Autoregressive Models, VARs. Then, we augment them with Colombian and U.S. macroeconomic exogenous variables to test if they have some predicting power for future yield curves. VAR models are widely used in yield curve studies either as the main model of interest (Ang and Piazzesi, 2002; Mönch, 2005; Abbritti et al., 2013) or as a benchmark model against which more sophisticated models are compared to (De Pooter et al., 2010).

To our knowledge, this is the very first study that investigates the ability of unrestricted VAR models augmented with U.S. and local macro factors to forecast the Colombian yield curve. Given the scope of the present work, we chose the simple unrestricted VAR models as a natural first step. More sophisticated models would be more appropriate but go well beyond of the scope of the present work. In addition, such models require a simple benchmark to compare with. We offer a deep empirical analysis of the forecasting ability of such a benchmark which could be useful for policymakers and market participants as well.

Our results indicate that including macroeconomic variables help improve short-term yield forecasts. In particular, oil prices, the EMBI Colombia Index, Corporate U.S. bonds, Option Adjusted Spreads (OAS), and the volatility Index (VIX) contain additional information to forecast the yield curve that has not yet been spanned by the current yield curve. For mid and long-term maturity yields, the models without macroeconomic variables outperformed the models that included them, suggesting that mid and long-term yields already include all meaningful information necessary to forecast them.

The following section presents the conceptual framework of the term structure of interest rates and motivates the use of macroeconomic variables in forecasting the yield curve. Section 2 includes the solution method, describes data in detail and presents models and forecasting methods selected to carry out the present study. Section 3 shows the empirical results. Section 4 concludes and presents recommendations for future research.

1. Conceptual Framework

In the previous section, the importance of improving the forecasting of the term structure of interest rates in Colombia was explained, but to achieve it a correct understanding of the term structure itself is necessary. The objective of this section is to help familiarize the reader with these topics.

1.1. Term structure of interest rates

The term structure of interest rates is “the relation between the returns of the securities with the same credit qualification, regularly risk free, but with different maturity times. The graphical representation of the term structure of the interest rates is known as the yield curve (Arango et al, 2002, p.3)”.

There are some characteristics the yield curve has. Firstly, the yields tend to increase with longer maturities, and recognize a term premium that can be based on the risk aversion, the preferences, or the liquidity wanted by the investors. Agudelo & Arango (2008) proved the liquidity premium hypothesis in the Colombian market concluding that it appeared in the fixed income markets in the Term Deposits (CDT by its acronym in Spanish) and TES. Secondly, the volatility tends to decrease over time, which means that the long-term rates are less volatile than the short ones. Besides, the yield curve has high correlation, visible in the persistence shown in the series of the curve (Diebold & Rudebusch, 2013).

As mentioned before, the yield curve offers important information for the economy agents, who based on this information price different assets and make inversion choices. Consequently, they continually follow the movements of the yield curve and try to forecast its behavior over time. If the agents base their decisions on an accurate forecasting, they could have better returns than other investors based on a poorer forecasting; and even a country would better cope with a recession if it could predict and modify its monetary policy.

According to Diebold et al. (2013), three factors are necessary to explain most of the yield variation: level, slope, and curvature. The level factor, as its name implies, is the level that

represents the yield curve. A shock in the level factor produces a parallel movement of the curve, implying thus that shocks affect the rates of different maturities by the same amount. Macroeconomic variables strongly affect the level factor.

The slope factor shows how steep the yield curve is. If the short-term interest rates are much lower than the long-term ones, the curve would be steep. This may alert the authorities of an increase on the expectation of a future inflation (Banco de la República, 2003). A shock to the slope factor could make the short interest rates change in a higher amount than the long-term ones, thus making the curve less steep.

Meanwhile, the curvature shows how concave the curve is; a shock to the curvature factor affects mostly the mid-term interest rates rather than short or long-term ones, which causes the yield curve to be more hump-shaped (Wu, 2003). The movements on the yield curve factors generally do not happen in an isolated way, since they are correlated (Julio et al, 2002).

1.2. Term structure of interest rates and macroeconomic factors

Some studies have used macroeconomic factors in the forecasting of the term structure of interest rates based on the link between finance and macroeconomics, as was explicit during the 2008 economic crisis. Today's countries do not act as independent entities, they are interconnected since they trade goods, money, services, and knowledge to the point that a shock to one's economy can threaten the stability of several others, or even the whole world's economic stability. On this account, the previous studies included not only local variables but foreign variables as well.

Ang and Piazzesi (2002) work with a model where inflation and economic growth factors are the macroeconomic variables selected to regress with bond yields as joint dynamics. The findings reveal that macroeconomic factors improve the forecasting of the model compared with models that just consider unobservable factors, which is true especially in the short and middle end of the yield curve, although unobservable factors are good at predicting the

movements in the long end of the curve. Another significant finding is that, unlike the level factor, both the slope and curvature of the yield curve can be affected by macroeconomic factors.

Abbritti et al. (2013) used the yield curves of seven different countries to study the effects of global factors on the dynamics of interest rates. The model included, along with the traditional determinants of the yield curve, some other factors such as global level, global slope and global curvature. The results led to the conclusion that global factors are very good at explaining long-term dynamics of the yield curves as they help to account for almost the 80% of the variation; on the other hand, local factors are useful in making clear short-term dynamics.

From another point of view, Mönch (2005) used the common component of a large number of macroeconomic variables as explanatory variables, as well as the short-term interest rates based on the belief that authorities responsible for the monetary policy will consider these variables. In other words, it is expected that the monetary policy be done based on a “data rich environment”. As a result, this model outperforms those not including macroeconomic variables as well as those including only individual macroeconomic variables, which has been proved for all maturities and horizons, providing a good in-sample fit of the yield curve.

De Pooter et al. (2010) did something similar to forecast the term structure of interest rates in the U.S. They extracted factors from series of individual macroeconomic variables and ran the models with and without the factors, concluding that both models can have a good predictive power depending on the moment. Models without macroeconomic factors performed well in low volatility times, while models that included the factors worked well during and near to recession periods.

All the previous studies were carried out in the U.S. and other developed countries; there are few available for developing countries and there is none for Colombia. Thus, the present study constitutes a base on this field in this country.

1.3. Forecasting methods

There are plenty of models used to forecast the yield curve; some are more complex although not necessarily more accurate. Shinobu et al. (2010) shows that simple models (such as Nelson-Siegel's), can better predict interest rates. Brooks (2008) also claims that simple models like VAR are better than traditional structural models, which perform poorly during the out-of-sample periods. McCandless et al. (2001) also maintain that most of the times VAR models can make better predictions than complex models of simultaneous equations.

2. Solution Method

To calculate the forecasting of the interest rates in Colombia it is necessary to get the proper data, know the method, and have the right software with the right functionalities.

2.1. Data

For the period between April 2005 and April 2015, we used months 3rd and 6th, and years 1, 2, 3, 5, 7, 8, 9, 10 and 15 of the Colombian yield curve. Data were obtained from www.bloomberg.com and were measured in percentage points, as it is a rate. We labeled these variables as M3, M6, Y1-Y15, respectively.

The Colombian macroeconomic variables used are the following: 1) the exchange rate (TRM, by its Spanish acronym) or the value of the U.S. dollar in terms of the Colombian peso. TRM was measured in Colombian pesos and was taken from the official web page of the Superintendencia Financiera de Colombia, the governmental entity that supervises the country's financial system. 2) The Colombian inflation rate was taken from Banco de la República de Colombia official web page and is measured as an index set equal to 100 in December 2008. 3) EMBI-Colombia is an indicator of the country's investment risk measured in basis points; it was taken from www.ambito.com, an Argentine financial newspaper.

The U.S. variables included are:

- 1) The Effective Federal Funds Rate (FFR) is the rate at which financial institutions trade federal funds in the U.S. measured in percentage points;
- 2) Crude Oil Prices is the international price of the West Texas Intermediate (WTI) measured in U.S. dollars;
- 3) BofA Merrill Lynch U.S. Corporate Master Option-Adjusted Spread (OAS) is a spread between an index of corporate bonds and a spot rate Treasury curve measured in percentage points;
- 4) CBOE Volatility Index (VIX) shows market expectations of near-term volatility; it is measured as an index;
- 5) Consumer Price Index for All Urban Consumers (CPI-U.S.) shows the changes in the prices of goods and services; it is measured as an index with 100 as a base;
- 6) 10-Year Treasury Constant Maturity Rate, 2-Year Treasury Constant Maturity Rate, and 3-Month Treasury Constant Maturity Rate are measured in percentage points and are used to calculate the factors of the U.S. yield curve: level, slope and curvature. The level factor corresponds to the 10Y yield curve, the slope to the difference between 10Y and 3M yields curve, and the curvature is 2 times the 2Y minus the sum of 3M plus 10Y yield curves.

All these data were taken from the Federal Reserve Bank of Saint Louis. The EMBI + Global is also taken from the financial newspaper www.ambito.com and is an index measured in basis points to show investing risks in emerging markets bonds.

Table 1 reports the summary statistics for all Colombian and U.S. macroeconomic variables along with the Colombian yields for the sampling period on a monthly basis. Columns contain the statistics and rows the maturity term of the Colombian yield curve and the macroeconomic variables.

Table 1. Summary statistics of variables

maturity/ variable	Number of observations	mean	standard deviation	percentile 5	percentile 25	percentile 50	percentile 75	percentile 95
M3	120	-0,017	0,395	-0,634	-0,185	0,020	0,217	0,478
M6	120	-0,018	0,396	-0,701	-0,189	0,040	0,180	0,484
Y1	120	-0,020	0,394	-0,634	-0,199	-0,022	0,211	0,501
Y2	120	-0,032	0,465	-0,807	-0,262	-0,051	0,213	0,771
Y4	120	-0,043	0,561	-0,948	-0,279	-0,079	0,262	0,939
Y5	120	-0,048	0,572	-0,879	-0,318	-0,091	0,250	0,975
Y7	120	-0,055	0,578	-0,896	-0,341	-0,054	0,296	0,905
Y8	120	-0,053	0,560	-0,880	-0,310	-0,029	0,247	0,973
Y9	120	-0,051	0,559	-0,942	-0,329	-0,034	0,215	0,831
Y10	120	-0,048	0,553	-1,024	-0,332	-0,028	0,202	0,837
Y15	120	-0,039	0,552	-0,980	-0,310	-0,031	0,261	0,928
levelcol	120	-0,048	0,553	-1,024	-0,332	-0,028	0,202	0,837
slopecol	120	-0,030	0,518	-0,958	-0,233	-0,026	0,217	0,823
curvaturecol	120	0,001	0,492	-0,785	-0,308	-0,038	0,244	0,914
Oilprices	121	81,862	19,761	49,640	66,250	81,640	97,100	107,980
OAS	121	1,889	1,170	0,900	1,120	1,540	2,150	4,870
FFR	121	1,516	2,039	0,060	0,090	0,160	3,220	5,310
VIX	121	20,217	9,217	11,570	13,750	17,470	23,540	40,000
usdlevel	121	3,234	1,039	1,680	2,350	3,210	4,020	4,860
usdslope	121	1,873	1,167	-0,290	0,860	2,120	2,690	3,380
usdcurvature	121	-1,165	0,792	-2,200	-1,850	-1,350	-0,620	0,210
TRM	121	2043,981	2,290	1773,240	1857,980	1965,320	2259,720	2426,000
EMBIcol	121	210,488	86,800	132,000	157,000	193,000	229,000	407,000
EMBI	121	312,661	106,251	178,000	251,000	296,000	348,000	529,000
U.S.CPI	121	218,841	12,803	198,100	209,190	217,605	231,165	236,950
Inflation col	121	102,228	11,033	83,757	91,980	103,813	111,687	117,489

Source: Compiled by the author.

This table provides the summary statistics for all the variables under the columns. Colombia yield curve maturities, Colombia and U.S. yield curve factors, and Colombian and U.S. macroeconomic factors are in the rows. Statistics include Number of Observations, Mean, Standard Deviation and Percentile 5, 25, 50, 75 and 95. Yield information for Colombian and U.S. yield curve and factors (level, slope and curvature) of the yield curve are measured in percentage points, as well as OAS and FFR. EMBI and EMBICol are measured in basis points. The variables VIX, U.S.CPI and Inflation-col are measured as an index with a base of 100. Oil prices are expressed in U.S. dollars and TRM are in Colombian pesos.

2.2. VAR models

Vector Autoregressive Models, VAR, are multivariate models based in two or more equations where some variables are dependent variables in an equation and explanatory variables in another. In other words, “the structure is that each variable is a linear function of past lags of itself and past lags of the other variables (Department of Statistics Online Programs, 2015).” It is called vector because it considers two or more variables; it is autoregressive because it depends on the lag of the variables.

One of the advantages of VAR models is that it is not necessary to choose between exogenous or endogenous variables, a major problem in structural models, because this differentiation can be subjective (McCandless et al., 2001). VAR models treat all the variables as endogenous.

Other advantage of VAR models (Brooks, 2008) is that it can capture more features of the data due to its flexibility compared with Autoregressive models, AR, and can be easily estimated because of its ability to use the Ordinary Least Squares method, OLS, used to estimate the unknown parameters in linear regressions.

All variables included in VAR models should be stationary since they are based on time series analysis and it is important to carry out joint significance tests. It is also necessary to select the same lag level or number of past periods to be included in all the variables; otherwise, the model would be restricted rendering the task difficult because the selection of a high level of lags could mean losing levels of freedom leading to inaccurate estimates. To do this one useful tool is to compare the information criterion. To prove the significance of all jointly lags the F-test is used.

2.3. VAR models with macro factors

Ang et al. (2002) used a VAR model to describe the joint dynamics of bond prices yields and macroeconomic factors with a two-step estimation procedure, using the OLS method for both of them. In the first part, the macro dynamics and coefficient of the macro factors are estimated. In the second, the remaining parameters of the model are estimated keeping the previous estimated parameters fixed. The model is estimated in several iterative rounds.

Then to estimate the model and determine the effect of each macro factor on the model itself, an Impulse Response (IR) analysis is used, thus giving weight to each yield of maturity in the term structure model. A Variance Decomposition analysis is also used to show the relative contribution of each macro and latent factor on the forecasting. At the end, to prove the predicting power of the model it is important to carry out an out-of-sample forecasting and

to compare the prediction with the real observable data, using the Root Mean Square Error, RMSE.

Abbritti et al. (2013) and Mönch (2005) used a factor augmented VAR known as FAVAR, an alternative model where the standard VAR is augmented with estimated factors.

Based on the previous studies and due to the simplicity and applicability of the VAR model, we chose it to carry out the forecasting of the term structure of interest rates in Colombia and to compare it later with the forecasting augmented with the macroeconomic factors.

2.4. Software and model selection

The software chosen to calculate the models is Stata, which allows performing estimation procedures as linear regression. The function used is the Vector Autoregressive (VAR) model, which allows the comparison of two or more equations that depend on the lag of the variables. By using this software and the monthly data of yields for different maturities, it is possible to forecast the unrestricted VAR models of the term structure of interest rates in Colombia. Regarding the Colombian and U.S. macro factors it is possible to identify the ones that are relevant for the term structure in Colombia and then include them in the new forecasting of the term structure.

To get the most accurate results it is necessary to select the proper models. For this empirical study, more than one hundred models were ran and statistical properties were tested for each: a) The optimal lag number (including number of past periods); b) the presence of autocorrelation; c) the normality assumption; d) the stability of the model; and e) the individual and joint significance of the coefficient of all variables. According to the results of the previous statistical properties, the best models were selected to carry out this study. The data used for the selection are not included in the paper due to space limitations but are available by the author.

3. Results and its analysis

The first model built includes all the yield maturities to try to forecast the next steps using their previous values. The maturities of the yield curve included are: 3, 6 months, 1, 2, 4, 5, 7, 8, 9, 10 and 15 years (the notation used is 3M, 6M, 1Y, 2Y, 4Y, 5Y, 7Y, 8Y, 9Y, 10Y and 15Y, respectively). The model is estimated for three different periods, all of which begin in April 2005 but finish in different months, August and December 2013, and April 2014. Then, after each of these periods, we carry out out-of-sample forecasts for each of the following twelve months (1 year).

Throughout this period, the models displayed a significant coefficient of the variable 1Y for all the maturities, except for the third where the coefficient of 1Y is not significant for short-term maturities (3M, 6M, 1Y). This outcome is obtained both when the model includes two lags (information of the two past periods) as when has only one. When the model has one lag, the coefficient of the variable 8Y is significant for the maturities from 4Y to 9Y. This finding is important because it suggests that these are the variables to be monitored in the market and they will give important information for the other maturities.

Therefore, the-out-of sample forecasting is conducted 12 steps/months ahead for three different periods: from September 2013 to August 2014, from January 2014 to December 2014, and from May 2014 to April 2015. The objective is to compare the forecasting power of the model in different moments and not just in one specific period, because this may lead to a wrong inference.

To compare the forecasts, two criteria were used: the Root Mean Squared Error, RMSE, and the Mean Absolute Deviation, MAD, where the lowest values indicate the best forecast. Table 2 displays the main results for models estimated without macroeconomic and financial factors. The columns show the results for each of the three above-mentioned periods for VAR models with one and two lags (number of past periods included) which were selected as optimal, according to the statistical test. The rows show the values of RMSE and MAD criteria for each model. Boldface numbers indicate the lower RMSE and MAD. The

forecasting is better for the short-term maturities (M3, M6, and Y1) and the forecasting power starts decreasing from 2Y; the higher criteria values are obtained at long-term maturities.

Table 2. Forecast criteria comparison for yield curves

Criteria	yield maturity	Sep/13 - Aug/14		Jan/14 - Dec/14		May/14 - Apr/15	
		2 lags	1 lag	2 lags	1 lag	2 lags	1 lag
RMSE	M3	0,321	0,318	0,323	0,315	0,202	0,255
	M6	0,285	0,282	0,300	0,282	0,200	0,237
	Y1	0,250	0,244	0,259	0,245	0,229	0,234
	Y2	0,280	0,273	0,313	0,327	0,376	0,341
	Y4	0,278	0,277	0,359	0,366	0,417	0,395
	Y5	0,281	0,269	0,355	0,361	0,420	0,391
	Y7	0,354	0,326	0,408	0,375	0,390	0,365
	Y8	0,362	0,347	0,438	0,398	0,392	0,371
	Y9	0,303	0,307	0,415	0,359	0,362	0,344
	Y10	0,334	0,345	0,394	0,341	0,352	0,318
	Y15	0,420	0,428	0,456	0,414	0,382	0,352
MAD	M3	0,250	0,250	0,260	0,249	0,181	0,209
	M6	0,228	0,226	0,256	0,234	0,169	0,210
	Y1	0,184	0,182	0,222	0,200	0,202	0,193
	Y2	0,242	0,228	0,264	0,285	0,324	0,279
	Y4	0,232	0,234	0,306	0,311	0,349	0,315
	Y5	0,208	0,211	0,287	0,297	0,358	0,312
	Y7	0,267	0,256	0,327	0,306	0,328	0,291
	Y8	0,277	0,287	0,359	0,327	0,319	0,294
	Y9	0,273	0,283	0,363	0,322	0,297	0,281
	Y10	0,312	0,319	0,351	0,308	0,290	0,264
	Y15	0,343	0,343	0,390	0,335	0,304	0,279

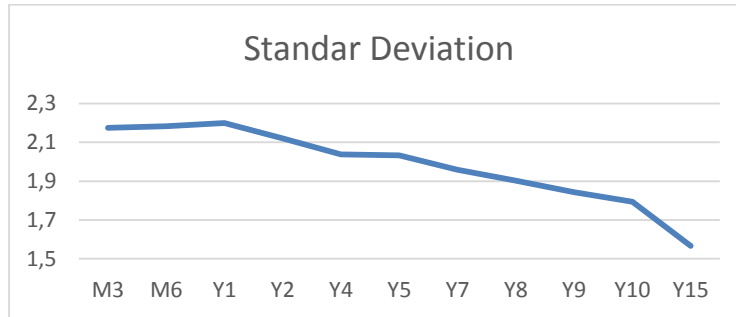
Source: Compiled by the author.

The forecasting for 12 months ahead (out-of-sample) for 3 different periods (Sep/13 - Aug/14, Jan/14 - Dec/14, May/14 - Apr/15). The criteria to compare the best forecasting are calculated: Root Mean Squared Error, RMSE, and the Mean Absolute Deviation, MAD, where the lowest values indicate the best forecast. For each period a model with one lag and another with two lags (past periods) is calculated. The columns show the results for each period for VAR models with one and two lags (number of past periods included); the rows present the values of RMSE and MAD criteria for each model. Boldface numbers indicate the lower RMSE and MAD.

This finding is especially important considering that the standard deviation is higher at short maturities and decreases at long maturities, as shown in Graph 1, also reported in previous empirical studies (Ang et al., 2002; Diebold et al., 2013). Those studies also suggest that

yield curves present high autocorrelation (Pooter et al., 2010) and do not present a normal distribution, which is coherent with the findings in the models of the present empirical study.

Graph 1. Standard deviation of Colombian yield curve.



Source: Compiled by the author.

The graph shows the standard deviation for the historical monthly data from April 2005 to April 2015 for the different maturities of the Colombian yield curve, which are at the bottom row.

In general, the model only considers one lag period and presents lower criteria values along the three periods forecasted, which suggests that it is better to work with only one lag instead of two. Throughout the three periods considered, it was better to use two lags for some maturities. But each maturity is different in each period: over the first period (Sep/13 - Aug/14) the criteria were lower in the model with two lags for long-term maturities 9Y, 10Y and 15Y, but over the second period (Jan/14 - Dec/14) it was better for mid-term maturities 2Y, 4Y and 5Y. On the other hand, the third period (May/14 - Apr/15) suggests that the two lags model outperformed that of the one lag model for short-term maturities M3, M6 and Y1. These results might suggest that the relationship between the number of lags and the forecasting quality is not stable.

It is also important to build models with the main factors of the yield curve (level, slope and curvature), as we did for each of the maturities of the yield curve. The models for the three periods show the most significant contemporaneous movements. For the slope variable, the

past values of level, slope and curvature are important; for the level variable, the past values of level and slope (also curvature over the first period) are important; and for the curvature variable the past values of level and curvature (except level over the first period) are important. This suggests that is important to monitor the three factors of the yield curve.

Then the 12-step ahead forecasting for the three periods is estimated and the RMSE and MAD criteria are calculated to compare the results. In this case, only a two-lag model is used because the model with one lag is not optimal, according to the statistical test. Table 3 shows the results. The columns show the results for each of the three periods for VAR models; the rows present the values of RMSE and MAD criteria for each model. Boldface numbers indicate the lower RMSE and MAD. The model better predicts the level factor over the two first periods and the slope factor over the third. The relationship between the forecasting power of the factors and the quality is not stable either.

Table 3. Forecast criteria comparison for yield curve factors.

Criteria	yield factor	Sep/13 - Aug/14	Jan/14 - Dec/14	May/14 - Apr/15
RMSE	levelcol	0,324	0,329	0,345
	slopecol	0,403	0,410	0,296
	curvaturecol	0,398	0,418	0,456
DMA	levelcol	0,300	0,294	0,302
	slopecol	0,331	0,347	0,230
	curvaturecol	0,316	0,330	0,399

Source: Compiled by the author.

The level factor corresponds to the 10Y, the slope to the difference between 10Y and 3M, and the curvature is 2 times the 2Y minus the sum of 3M plus 10Y. The forecasting for 12-months ahead (out-of-sample) for three different periods (Sep/13 - Aug/14, Jan/14 - Dec/14, May/14 - Apr/15) is made and then the criteria to compare the best forecasting are calculated: Root Mean Squared Error, RMSE, and the Mean Absolute Deviation, MAD, where the lowest values indicate the best forecast. Only models with two lags are calculated. The columns show the results for each of the three periods for VAR models; the rows present the values of RMSE and MAD criteria for each model. Boldface numbers indicate the lower RMSE and MAD.

The next step is to identify the most appropriate macroeconomic factors to forecast the yield curve. The starting point is to pick the variables to be included in the model (it would be interesting to select those used by the economy agents to explain the movements of the yield curve).

Researchers commonly use oil prices to explain the movement of the yield curve; for instance, how a decrease of the prices increases the rates of the yield curve (Córdoba et al, 2015, p.1; Reyes et al, 2015a, p.27, 2015b, p.27). Some reports even link the movement of the oil prices to the movement of other macroeconomic variables, like the exchange rate devaluation, and the increase in the EMBICol as impacting the rates of the yield curve, especially the long-term maturities (Espinosa, Riveros & Otero, 2015, p.15-16).

Inflation and the expectation of its behavior, the increase of its causes and on the rates, especially in the short-term maturities are among other widely used variables (Pardo, Ramos & Salcedo, 2015a, p.10).

Among other commonly used variables, we have:

- 1) The Exchange Rate (Espinosa, Riveros, Otero & Torres, 2015a, p.9);
- 2) The Federal Fund Rate for all Maturities (Espinosa, Riveros, Otero & Torres, 2015b, p.13);
- 3) The EMBI + Global and the EMBICol, specially for the slope (Investigaciones Grupo Bancolombia, 2014c, p.12);
- 4) The American Treasury Bonds Rate (Pardo & Ramos, 2014b, p. 9, 14, 18); and
- 5) The VIX (Castañeda & García Rojas, 2014, p.1).

Considering the explanatory power given to these macroeconomic variables, we decided the new model would include oil prices, Option Adjusted Spread (OAS), Federal Funds Rate (FFR), Volatility Index (VIX), level, slope and curvature of the U.S. yield curve, Exchange Rate (TRM, Spanish acronym), EMBI Colombia (EMBICol), EMBI + Global, U.S. inflation rates (U.S. CPI) and Colombia inflation rate.

The augmented model is calculated for the three same periods used in the model with only yield curves, but now including the macroeconomic variables as exogenous in two different ways: a model including only one exogenous variable (one model each) and another including all the macroeconomic variables at the same time.

Table 4 shows that some contemporaneous movements are significant because of the coefficient of the variables in each model, and at the same time varying those that are significant according to the maturity term. The rows present the periods where the out-of-sample forecast is done, and the columns the variables that are significant in each model according to its coefficient. The first column shows the variables significant for all the maturities; the second, only those significant for short-term maturities; the third, those significant for mid and long-term maturities; and the last shows the variables that are not significant at any maturity.

Table 4. Significance of macroeconomic variables according to the yield curve maturity.

Period	Maturity significance			
	For all maturities	Short - term	Mid and long-term	None
Sep/13 - Aug/14	Oilprices VIX	OAS USDlevel USDslope	USDcurvature TRM inflationcol	FFR EMBIcol U.S. CPI EMBI
Jan/14 - Dec/14	Oilprices VIX	OAS USDlevel USDslope	TRM EMBI inflationcol	FFR USDcurvature EMBIcol U.S. CPI
May/14 - Apr/15	Oilprices VIX	OAS USDlevel USDslope	TRM EMBI inflationcol	FFR USDcurvature EMBIcol U.S. CPI

Source: Compiled by the author.

In the first column, the table shows the macroeconomic variables significant for all the maturities; over the second, those significant only for short-term maturities; over the third, those significant for mid and long-term maturities; and those that are not significant for any maturity are in the last column. The rows present the results for each period. The results for the same three periods used in the model including only yield curves are reported.

Variables of oil prices and VIX are significant for all the maturities. OAS, USD level and slope are significant only for short-term maturities; TRM , EMBI and Colombia inflation rate are significant for long-term maturities (the second is not for the model of the first period). The variables FFR, EMBICol, U.S. CPI, and U.S. curvature are not significant at any maturity term (the last only in the model of the first period).

These results suggest that some of the variables used by the economy agents are not important for the yield curve and monitoring should not be necessary. Other variables should only be monitored according to the maturity term in which they are inserted. Just two of the variables shown impact all the maturity terms of the yield curve.

Considering the previous results, new models are thus calculated: one model including all yield curve maturities and only the macroeconomic variables significant at any maturity; other for short-term yield curve maturities (M3, M6, Y1, and Y2) with macroeconomic variables significant at all maturities. In addition, a model for those only significant at short-term maturities, another for mid and long-term yield curve maturities (Y2, Y4, Y5, Y7, Y8, Y9, Y10, Y15) with macroeconomic variables significant at all maturities. One more for those only significant at mid and long-term maturities, and a last one only for long-term yield curve maturities (Y8, Y9, Y10, Y15) with macroeconomic variables significant at all maturities plus a model only significant at long-term maturities.

With all models built, the 12-steps ahead forecasting is carried out for the three periods considered. Table 5 contains the three models for each maturity displaying the lowest RMSE and MAD criteria. The rows show the criteria for each maturity and the columns the three best models for every period of the three evaluated.

Table 5. Forecast criteria comparison for yield curves and macroeconomic variables.

Models with lower criteria									
Criteria	Sep/13 - Aug/14			Jan/14 - Dec/14			May/14 - Apr/15		
RMSE M3	EMBICol	OAS	Oil prices	only significant variables	OAS	only significant variables and significant maturities	EMBICol	VIX	OAS
RMSE M6	EMBICol	OAS	VIX	only significant variables	OAS	only significant variables and significant maturities	EMBICol	VIX	IPC USA
RMSE Y1	EMBICol	VIX	Oil prices	OAS	only significant variables	EMBICol	VIX	no exogenous variables	USD curvature
RMSE Y2	no exogenous variables	EMBICol	VIX	OAS	mid and long-term model	VIX	only significant variables	mid and long-term model	VIX
RMSE Y4	no exogenous variables	USD curvature	short-term model	OAS	USD slope	IPC USA	mid and long-term model	VIX	no exogenous variables

RMSE Y5	no exogenous variables	USD curvature	TRM	U.S. CPI	inflation col	USD slope	mid and long-term model I	no exogenous variables	VIX
RMSE Y7	no exogenous variables	USD curvature	TRM	no exogenous variables	inflation col	IPC USA	mid and long-term model	no exogenous variables	VIX
RMSE Y8	no exogenous variables	USD curvature	TRM	long-term model	no exogenous variables	OAS	long-term model	mid and long-term model	no exogenous variables
RMSE Y9	USD curvature	TRM	no exogenous variables	long-term model	no exogenous variables		long-term model	mid and long-term model	no exogenous variables
RMSE Y10	USD curvature	TRM	no exogenous variables	long-term model	no exogenous variables	mid and long-term model	mid and long-term model	long-term model	no exogenous variables
RMSE Y15	USD level	FFR	USD slope	long-term model	no exogenous variables	only significant variables and significant maturities	long-term model	mid and long-term model	no exogenous variables
DMA M3	EMBIcol	TRM	Oil prices	OAS	only significant variables	only significant variables and significant maturities	EMBIcol	OAS	VIX
DMA M6	EMBIcol	OAS	VIX	only significant variables	OAS	only significant variables and significant maturities	EMBIcol	OAS	VIX
DMA Y1	TRM	EMBIcol	Oil prices	OAS	only significant variables	EMBIcol	no exogenous variables	VIX	USD level
DMA Y2	mid and long-term model	short-term model	no exogenous variables	all variables	OAS	mid and long-term model	no exogenous variables	short-term model	USD level
DMA Y4	USD level	USD curvature	inflation col	OAS	mid and long-term model	Oil prices	no exogenous variables	mid and long-term model	VIX
DMA Y5	USD curvature	U.S. CPI	TRM	OAS	Oil prices	VIX	no exogenous variables	mid and long-term model	Oil prices
DMA Y7	USD curvature	U.S. CPI	no exogenous variables	no exogenous variables	OAS	mid and long-term model	mid and long-term model	no exogenous variables	VIX
DMA Y8	USD curvature	U.S. CPI	Oil prices	long-term model	mid and long-term model	no exogenous variables	long-term model	mid and long-term model	no exogenous variables
DMA Y9	USD curvature	U.S. CPI	Oil prices	long-term model	mid and long-term model	only significant variables and significant maturities	mid and long-term model	long-term model	no exogenous variables
DMA Y10	USD curvature	U.S. CPI	Oil prices	long-term model	mid and long-term model	no exogenous variables	mid and long-term model	long-term model	no exogenous variables
DMA Y15	long-term model	USD curvature	TRM	long-term model	mid and long-term model	no exogenous variables	no exogenous variables	long-term model	IPC USA

Source: Compiled by the author.

Forecasting for 12 months ahead (out-of-sample) for 3 different periods (Sep/13 - Aug/14, Jan/14 - Dec/14, May/14 - Apr/15) is made for each model including macroeconomic variables. A model with each single variable; a model with all variables; a model only with significant variables; a model with significant variables only for short term maturities (M3, M6, Y1, Y2); a model with significant variables for mid and long-term maturities (Y2, Y4, Y5, Y7, Y8, Y9, Y10, Y15); a model with only significant variables for long-term maturities (Y8, Y9, Y10, Y15). Then the criteria to compare the best forecasting are calculated: Root Mean Squared Error,

RMSE, and the Mean Absolute Deviation, MAD, where the lowest values indicate the best forecast. The models with the three lowest RMSE and the three MAD are shown in the table for each maturity in every period. Rows show the criteria for each maturity and columns the periods. Appendix 2 contains the complete criteria for all the models in every period.

Over the first period (Sep/13 - Aug/14), the models with lower RMSE and MAD included as exogenous variables: the EMBICol for short-term maturities, oil prices for short and long-term maturities, TRM, U.S. curvature, and the model without exogenous variables (with one lag) for mid and long-term maturities.

Over the second period (Jan/14 - Dec/14), the lowest criteria values included as exogenous variables:

- OAS for short and mid-term maturities;
- The model including the variables with significant coefficients for mid and long-term maturities, plus the macro variables with significant coefficients for all maturities to forecast mid and long maturities yield curves;
- The model with macro variables with significant coefficients for long-term maturities and those with significant coefficients for all terms to forecast long maturity yield curves; and
- The model without exogenous variables (with 1 lag) for mid and long-term maturities.

Over the third period, the results were very similar to the second, differing only in the variables with significant coefficients for short and mid-term maturities, now being VIX instead of OAS.

The results were not the same for all the three periods forecasted. For short-terms, it always worked best a model including some macroeconomic variables rather than none. Nevertheless, the macroeconomic variable with significant coefficients was different for each period. Thus, over the first period the variables were oil prices and EMBICol; over the second, OAS; and over the third, VIX. This behavior suggests that the model is not stable, even though all these variables present low values in all the models. Monitoring all four of them might be important to forecast the short maturities yield curves (M3, M6, Y1, Y2).

Oil prices are related to the economic cycle. When oil supply is high but demand is not enough because of a recession, prices go down and the rates of the yield curve increase because risk increases too, possibly affecting all maturities. EMBICol, for instance, shows how risky is to invest in Colombia, causing an increase to this variable, thus affecting the rates of the yield curve in a higher grade. OAS shows the behavior of the bonds market in the United States, which is related to its economy and that at the same time it can affect the Colombian economy and the rates of yield curves. When the CBOE Volatility Index (VIX) increases, the market could get nervous and push an increase on the rates of yield curves because they seem to be riskier.

Over the first period, the best models for mid and long-term maturities included TRM, U.S. curvature, oil prices, and the model without macroeconomic variables. Over the second and third periods, the best models included the mid and long-term yield curves plus the macroeconomic variables with significant coefficients for all maturities, as well as the variables with significant coefficients for mid and long-term maturities. And also the model that included long-term yield curves plus the macroeconomic variables with significant coefficients for all maturities, besides the variables with significant coefficients for mid and long-term maturities, and the model without macroeconomic variables, as well.

The only consistent model throughout the three periods was that without macroeconomic variables, probably meaning that the yield curves include all the significant information to forecast their future values. In other words, the yield curves have absorbed the movements of the factors of the economy and their addition to the models will not be necessary. For some periods, it might be relevant to include macro variables, especially those with significant coefficients for all maturity terms (oil prices and VIX), and those with significant coefficients for mid and long-term maturities, (TRM, EMBI, Colombian inflation).

TRM represents the exchange rate between dollars and Colombian pesos; an increase of the first currency leads to the devaluation of the second, probably due to economic crisis or nervousness in global markets. As Colombia is a risky place to invest, these rates would

increase. EMBI global tracks the external debt instruments in the emerging markets; this index also provides information of investment risks associated with a country, like Colombia. Thus, when the risks are high it determines an increase in the rates of yield curves.

A country's inflation and the rates of yield curves are strongly related; that is why the Central Bank has to increase the interest rates to control inflation, affecting thus the economy because borrowing money becomes more expensive making investments and consumption to decrease.

A study carried out in the United States (Mönch, 2005), points out that to explain or forecast yield curves it is better to include macroeconomic variables. Abbritti et al. (2013) also supported this idea with a study carried out in seven developed countries. Ang et al. (2002) agreed, but suggested that macroeconomic variables are more significant for short and mid-term maturities of yield curve than for long-term maturities. The present research obtained similar results, demonstrating that models with macroeconomic variables performed better for short maturities.

Shinobu et al. (2010) conducted a study that brought about different results between the U.S. and Brazil markets. Whereas for the U.S. markets the results above stated are similar, there is evidence that including financial variables in the Brazilian market improve the forecasting power of a model, which is not the case when macroeconomic variables are included.

In the present study, the model without macroeconomic factors is the best to forecast mid and long-term maturities. This fact is in accordance with Shinobu's suggestion that the results might well be affected by the different sample periods. This fact could be relevant considering that Brazil and Colombia are not developed countries and have much more in common between them than with the U.S. or some European countries.

De Pooter et al. (2010) indicate that the predictive power of the models vary significantly over time, being especially good around recession periods and decreasing at low-volatility periods, where models without macro factors outperformed those that included them. It would be interesting to test if the different results obtained along the three periods considered

in the present study, would be somehow affected by the presence of recession or low volatility periods, as suggested by De Pooter et al.

Forecasting was also done for the 12-steps ahead of the yield curve factors for the same three periods. Table 6 contains the three models with lowest RMSE and MAD criteria for each factor of the yield curve. The rows have the criteria for each factor, and the columns have the three best models for every period of the three evaluated.

Table 6. Forecast criteria comparison for yield curves factors and macroeconomic variables.

Models with lower criteria									
Criteria	Sep/13 - Aug/14			Jan/14 - Dec/14			May/14 - Apr/15		
RMSE levelcol	USD slope	U.S. CPI	FFR	U.S. CPI	OAS	VIX	USD curvature	EMBIcol	VIX
RMSE slopecol	VIX	EMBIcol	OAS	TRM	VIX	EMBIcol	USD curvature	VIX	EMBIcol
RMSE curvaturecol	all macro variables (2 lags)	FFR	USD level	USD level	FFR	TRM	all macro variables (1 lag)	only significant variables	USD curvature
DMA levelcol	U.S. CPI	USD slope	Oil prices	U.S. CPI	OAS	VIX	USD curvature	VIX	TRM
DMA slopecol	EMBIcol	EMBI	USD slope	TRM	VIX	EMBIcol	EMBIcol	USD curvature	EMBI
DMA curvaturecol	all macro variables (2 lags)	TRM	no exogenous variables	USD level	USD slope	TRM	all macro variables (1 lag)	only significant variables	inflation col

Source: Compiled by the author.

The forecasting for 12-months-ahead (out-of-sample) for 3 different periods (Sep/13 - Aug/14, Jan/14 - Dec/14, May/14 - Apr/15) is made for each model that includes macroeconomic variables: model with each single variable, model with all variables, model only with significant variables, model with only level and slope factors plus the significant variables. Then the criteria are calculated to compare the best forecasting: Root Mean Squared Error, RMSE, and the Mean Absolute Deviation, MAD, where the lowest values indicate the best forecast. The models with the three lowest RMSE and the three MAD are shown in the table for each maturity in every period. The rows have the criteria for each factor and the columns have the periods. Appendix 2 contains the complete criteria for all the models in every period.

Over the first period, the models with lower criteria to forecast the level factor were those that included the USD slope as a macroeconomic variable and the U.S. CPI, the model with EMBICol for the USD slope, and the model with all macroeconomic variables at once for curvature.

Over the second period, the best models to forecast the USD level were those that included the OAS, VIX and the U.S. CPI, to forecast the USD slope those including VIX, TRM and EMBICol, and for USD curvature models including level and TRM .

Over the last period, the models with lowest criteria to forecast the USD level were those that included the VIX and USD curvature, for USD slope those with the curvature and EMBICol, and for curvature the model with all the macroeconomic variables and the model with only the macroeconomic variables with significant coefficients.

All the results suggest that it is better to include macroeconomic variables in the forecasting of the factors, but the macroeconomic variable that has significant coefficient is not the same for each factor, showing that the model is both unstable and changes the macro variable with significant coefficient from one period to another.

Throughout two of the periods, the model with the U.S.CPI variable had significant coefficient for the level factor as was also the case of the model with the VIX. Solely over one of the periods, the model with USD slope, OAS, and USD curvature had significant coefficient. Oil prices, FFR, USD level, TRM , EMBICol, EMBI and inflation-col were variables with no significant coefficient at any period, suggesting that it might be not necessary to monitor them by the economy agents to forecast their level.

The model with EMBICol for the slope factor had significant coefficient throughout all of the periods. During one of the periods, the model with VIX, TRM, and the model with USD curvature had significant coefficient. Oil prices, OAS, FFR, USD level, USD slope, EMBI, U.S. CPI and inflation-col were variables with no significant coefficient at any period, suggesting that it might be not necessary to monitor them to determine their slope.

On the other hand, the models with significant coefficient for the curvature factor during two periods were those that included all the macro variables; over one period, one of the best models used level and TRM, and other included only the variables with significant coefficient. Oil prices, OAS, FFR, VIX, USD level, USD slope, USD curvature, EMBICol,

EMBI, U.S.CPI, and inflation-col were variables with no significant coefficient at any period, suggesting that it might be not necessary to monitor them to determine their curvature.

Ang and Piazzesi (2002) showed that macroeconomic factors are important to forecast the level and slope factors, which is in accord with the findings on the present study. Nevertheless, it was also reported that macro factors were not important to forecast the curvature factor, which differs from this study. One of the possible reasons for this dissent could be the fact that their study was carried out in the U.S. market, while this was made in Colombia's market.

4. Conclusions and recommendations

As mentioned before, it is important for economy agents to be aware of the behavior of the yield curve. It is also relevant to know if some of the macroeconomic variables could improve the forecasting of the models that include only the past values of the different yield curve maturities.

In this paper, we estimated unrestricted VAR models without macroeconomic variables using Colombian yield curves of different maturities, as well as the factors of the yield curve: level, slope and curvature. After those models were compared with models augmented with macroeconomic factors as exogenous variables in different combinations depending on the significance of its coefficients, the results vary according to the maturity terms.

In general, for short-term rates the macroeconomic variables increase the forecasting power of the model. The variable with significant coefficient that should be monitored is not stable across the periods. Nevertheless, it might be important to monitor oil prices, EMBICol, OAS and VIX that could provide relevant information.

For mid and long-term maturities, the results suggest that it is not necessary to include macroeconomic variables in the model because the yield curves include all meaningful information of the market. Even though, it might be relevant to monitor oil prices, VIX,

TRM, EMBI and Colombian inflation due to the fact that the forecasting improves on some periods .

It has been found that there are some variables that despite their current use to explain the movements on the yield curve, showed no significant coefficient at any maturity. This suggests that FFR, USD level, USD slope, USD curvature and U.S.CPI should not be monitored.

For the factors of the yield curve it is better to include macroeconomic variables, although the models are not stable from one period to another either. For the level factor CPI, VIX, OAS, USD slope, and USD curvature might be relevant. EMBICol, VIX, TRM and USD curvature might give relevant information for the slope factor. All macro variables and the TRM variable by itself could be important for the curvature factor.

These findings have important implications for portfolio managers, market participants, and policy makers because they should focus only on the relevant macroeconomic variables, instead of the wide number of variables that they are currently monitoring.

For future research, these models can be run over a wide number of periods to establish the stability of the models. They could also be further tested, if the results vary during recession or low volatility periods.

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Appendix 1

“En los últimos meses el mercado de deuda pública ha incorporado una mayor expectativa de inflación de corto plazo, lo anterior se reflejó en aumentos en los TES de corto plazo (...) En medio de la caída en las volatilidades y los niveles de aversión al riesgo a nivel global, la curva de rendimientos se desplazó hacia abajo.” (Pardo, Ramos & Salcedo, 2015, p.10)

“Luego de una importante publicación de referencias macroeconómicas relevantes, la reaparición de Grecia como factor de riesgo y una caída en los precios del crudo, la deuda soberana de Estados Unidos concluyó la jornada en medio de valorizaciones. En este sentido, los tesoros de 10 años mostraron una apreciación del orden de 8 pbs básicos cerrando con una tasa de negociación de 2,1390%.” (Reyes et al, 2015a, p.27)

“En medio de la especulación ante un posible acuerdo de Grecia con sus acreedores y una nueva caída en la cotización del petróleo, la deuda soberana estadounidense concluyó la jornada en medio de valorizaciones.” (Reyes et al, 2015b, p.27)

“En línea con la recuperación de la moneda colombiana, mencionada previamente, el mercado de deuda pública local evidenció un comportamiento favorable en la semana en curso. Concretamente, los TES TF que más se valorizaron fueron aquellos con vencimiento en May-22 (-6 pbs) y Sep-30 (-5,9 pbs). Por tanto, los mercados siguen mostrando un apetito considerable por los títulos emitidos por el gobierno colombiano, pese a que los datos recientes de actividad real han sorprendido negativamente. En efecto, factores como el deterioro en la confianza empresarial y la ampliación en el déficit comercial no parecen preocupar considerablemente a los mercados.” (Espinosa, Riveros, Otero & Torres, 2015a, p.9)

“Por otro lado, el mercado de TES presentó valorizaciones a lo largo de toda la curva en el último mes. En efecto, la postergación en las expectativas de alzas de la tasa de los fondos federales implica que el apetito por riesgo por parte de los inversionistas tanto internacionales como locales siga siendo alto (...) realizamos un análisis de regresión simple. Los resultados de este muestran que hay una relación estadísticamente significativa entre el número de meses en el que los agentes prevén que se presente el primer incremento de tasas en EE.UU y el comportamiento del USDCOP y la rentabilidad de los TES Jul-24. Adicionalmente, la correlación entre el índice de expectativa y el USDCOP es de -74,5%, y para la tasa de los TES Jul-24 es de -69,7%. En conclusión, encontramos que los mercados en Colombia continúan reaccionando de forma significativa al desempeño de la economía de EE.UU y sus decisiones de política monetaria.” (Espinosa, Riveros, Otero & Torres, 2015b, p.13)

“El precio del petróleo ha sido el principal determinante de la evolución del mercado financiero local. En efecto, en los últimos 30 días las cotizaciones tanto de la referencia Brent como la del WTI han caído 25%. Como respuesta a ello, en el mismo período la moneda local registró una devaluación de 7,1% frente al dólar. Entre tanto, la prima de riesgo país medida a través del spread del índice EMBI Colombia se ha ampliado más de 45 pbs hasta 221 pps. Por su parte, la curva de deuda pública en tasa fija se amplió de forma considerable, con las referencias de mayor duración presentando alzas de tasas superiores a 50 pbs. Los movimientos antes descritos ponen de manifiesto que el impacto del choque del precio del petróleo sobre la economía colombiana puede ser significativo (...) prevemos que en los próximos meses la correlación entre el mercado local y el precio del petróleo se mantenga alta.” (Espinosa, Riveros & Otero, 2015, p.15-16)

“Es razonable suponer que la dirección del mercado local va a continuar altamente relacionada con el movimiento en los precios del petróleo. No

obstante, es importante advertir que, a diferencia de lo ocurrido con otras economías emergentes productoras de crudo como Rusia, la prima de riesgo país de Colombia ha permanecido relativamente estable en esta coyuntura.” (Espinosa, Riveros, Otero & Beltrán, 2014, p.14)

“La curva de deuda pública de títulos denominados en tasa fija experimentó en septiembre un empinamiento, con aumentos en las rentabilidades de las referencias con vencimiento en Abr-2028 (31 pbs), Jul-2024 (28 pbs) y Jul-24 (28 bps). Esto se ha dado en un contexto en el que las primas de riesgo latinoamericanas se han ampliado, puesto que el spread del EMBI Global de JP Morgan se incrementó en 55 pbs hasta 424, en tanto que el de Colombia lo hizo en 22 pbs hasta 168 pbs.” (Investigaciones Grupo Bancolombia, 2014c, p.12)

“El buen desempeño de la deuda pública ha sido el resultado de varios factores. Entre los globales está que los TES siguen siendo un activo atractivo en términos relativos, no solo por las lecturas positivas de la economía en este año, sino también porque las rentabilidades de la renta fija en los mercados desarrollados continúan cayendo, gracias al impulso reciente que el BCE ha dado a los mercados. Como consecuencia de ello, la demanda por deuda local por parte de agentes internacionales ha seguido firme. En el frente local, los TES se han favorecido por el fin esperado del ciclo alcista en las tasas de interés locales después de la última reunión del Banco de la República, así como por la finalización del programa de subastas de TES para el presente año.” (Investigaciones Grupo Bancolombia, 2014b, p.12)

“En el caso de la deuda pública, la ampliación de las curvas que se presentó en el último mes responde a las noticias en el frente fiscal. En particular, la publicación del Marco Fiscal de Mediano Plazo para 204 incluyó la revisión del déficit del Gobierno Nacional Central de 2,3% a 2,4% del PIB para el presente año, así como el aumento de \$1 billón en las colocaciones de TES por subasta.

Lo anterior ha generado preocupación en los agentes locales, lo que a su turno puede llevar a que el potencial de valorizaciones de estos activos en el corto plazo sea limitado.” (Investigaciones Grupo Bancolombia, 2014a, p.11)

“El EMBI+ Global también presentó un incremento durante desde nuestro último informe, aumentando 61 pbs hasta ubicarse en 402 pbs. No obstante, el nivel máximo observado fue de 117 pbs, superior al registrado el 3 de febrero (406 pbs) en medio de la crisis de emergentes. En línea con el comportamiento del EMBI+ y los CDS a 5 años, los Libor z-spreads¹ de los bonos globales colombianos también aumentaron en el último mes. Los z-spreads de los bonos globales colombianos también aumentaron desde el 24 de octubre (fecha de cierre de nuestro último informe), registrando un cambio promedio de 47 pbs. (...) En medio de la fuerte tendencia al alza en las distintas primas de riesgo y de la fuerte reducción en las tasas de los Bonos del Tesoro en EEUU (según lo detallamos en la sección anterior), las tasas de los bonos globales colombianos registraron un aumento de 30 pbs en promedio desde el 24 de octubre. (...) El comportamiento del mercado de deuda continuó influenciado por los acontecimientos de política monetaria en EEUU y Europa, y por el deterioro en las expectativas de inflación tras la caída en los precios del crudo a nivel internacional. (...) Durante el último mes se observó un incremento importante en las principales medidas de riesgo de Colombia. Este comportamiento se asoció con el deterioro en las expectativas de inflación global y con el impacto de la caída de los precios del petróleo sobre las perspectivas de las economías altamente dependientes de este producto. Así mismo, ante la fuerte reducción observada en los Bonos del Tesoro, las tasas de los bonos globales colombianos presentaron aumentos de 30 pbs en promedio.” (Pardo & Ramos, 2014b, p. 9, 14, 18)

“Una caída del crecimiento del PIB a niveles del 2.8% permitiría al Gobierno aumentar su déficit fiscal en hasta 5.5 billones de pesos, lo cual podría tener un

impacto significativo al alza sobre las tasas de los TES. (...) En medio de la fuerte reducción en las tasas de los Bonos del Tesoro de EEUU, y de la volatilidad observada en primas de riesgo, las tasas de los bonos globales colombianos registraron un descenso promedio de 2 pbs desde el 6 de febrero (...) En los últimos dos meses, el mercado de deuda pública incorporó una mayor expectativa de inflación de corto plazo tras los datos de enero y febrero. Lo anterior se reflejó en aumentos en los TES de corto plazo y disminuciones en las tasas de los TES UVR de corto plazo. (...) Ante la fuerte reducción observada en las tasas de los Bonos del Tesoro luego de la reunión de marzo de la Fed, la cual sembró dudas sobre un inicio anticipado de la normalización monetaria, las tasas de los bonos globales colombianos presentaron descensos de 2 pbs en promedio.” (Pardo & Ramos, 2015, p. 4, 7, 11, 17)

“En medio de los persistentes bajos niveles observados en las distintas primas de riesgo, que reflejan una disminución en la aversión hacia bonos de economías emergentes, las tasas de los bonos globales colombianos continuaron a la baja, descendiendo 18 pbs en promedio en junio. Lo anterior se observó a pesar de la publicación de positivos datos del mercado laboral en EEUU que generaron incrementos en las tasas de los Tesoros a 10 años. (...) En junio, el mercado de deuda pública continuó ajustándose a las nuevas condiciones monetarias tras los dos aumentos en la tasa de intervención desde 3.5% a 3.75% a finales de mayo, y posteriormente a 4.0% en la tercera semana de junio, situación que se reflejó especialmente en el desplazamiento de la curva de rendimientos, donde se observó un aumento en tasas de negociación de los TES de corto, mediano y largo plazo. (...) Durante el último mes del año, el riesgo asociado a los países emergentes ha disminuido continuamente, y se ubica en algunos casos como los CDS a 5 años en niveles cercanos a los mínimos históricos. Esta reducción en la percepción de riesgo, específicamente en la deuda de Colombia, se evidencia en los bajos niveles del EMBI del país (niveles evidenciados antes del anuncio de

la Fed en mayo de 2013). Sin embargo, creemos que la continua caída en todas las primas de riesgo de Colombia reducen la probabilidad de que los bonos globales continúen valorizándose y, a su vez, incrementan el riesgo asociado con la sensibilidad de las tasas ante cambios en las perspectivas de riesgo.” (Pardo & Ramos, 2014a, p. 9, 13, 19)

“La deuda pública local presentó un desempeño positivo en el último mes, con una caída promedio de 51,6 puntos básicos en la curva de TES Tasa Fija, especialmente los TES TF Oct 2018, que bajaron en 65 pb. Lo anterior ocurre como repuesta a los bajos precios del petróleo.” (Córdoba, Angie Lorena, 2015, p.1)

“La deuda pública local mostró un desempeño positivo en el último mes, con una caída promedio de 30 puntos básicos en la curva de TES tasa fija y 23 puntos básicos en la curva TES UVR. El movimiento se explicó por la caída en los niveles de aversión al riesgo a nivel global evidenciados en el mes de Septiembre e inicios de Octubre; los bonos del tesoro a 10 años alcanzaron un mínimo de 1.86% para luego presentar tendencia al alza en los rendimientos hasta 2.38%, mientras el VIX alcanzó un máximo de 31 para estabilizarse finalmente alrededor de 14. El mencionado incremento en la aversión obedeció a renovadas preocupaciones sobre el crecimiento global (desaceleración en China y Europa).” (Castañeda & García Rojas, 2014, p.1)

Appendix 2

yield curves	M3 M8 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M8 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10	M3 M6 Y1 Y2 Y4 Y5 Y5 Y7 Y8 Y9 Y10
EXOG			Oilprices	OAS	FRR	VIX	USDlevel	USDslope	USDcurvature	TBM	EMBIcol	EMBI	U.S. CPI	inflationcol	all variables	only significant variables	significant short term	significant medium and long term	significant long term	only significant variables														
RMSE M3	0,321	0,318	0,303	0,298	0,351	0,304	0,350	0,340	0,325	0,311	0,295	0,327	0,329	0,352	0,431	0,346	0,320			0,353														
RMSE M6	0,285	0,282	0,267	0,261	0,318	0,265	0,317	0,305	0,288	0,274	0,258	0,292	0,297	0,320	0,399	0,320	0,285			0,325														
RMSE Y1	0,250	0,244	0,238	0,240	0,282	0,236	0,282	0,268	0,249	0,244	0,236	0,261	0,264	0,289	0,400	0,318	0,252			0,322														
RMSE Y2	0,280	0,273	0,289	0,292	0,299	0,291	0,300	0,289	0,275	0,279	0,297	0,302	0,285	0,299	0,465	0,418	0,275	0,279		0,413														
RMSE Y4	0,278	0,277	0,302	0,291	0,291	0,286	0,291	0,282	0,277	0,278	0,298	0,302	0,281	0,289	0,527	0,539		0,330																
RMSE Y5	0,281	0,269	0,302	0,289	0,294	0,284	0,293	0,286	0,279	0,280	0,293	0,303	0,282	0,288	0,503	0,528		0,353																
RMSE Y7	0,354	0,326	0,373	0,359	0,365	0,356	0,361	0,359	0,350	0,352	0,366	0,377	0,356	0,358	0,544	0,589		0,433																
RMSE Y8	0,362	0,347	0,378	0,366	0,372	0,365	0,370	0,365	0,356	0,361	0,373	0,384	0,361	0,364	0,518	0,554		0,420	0,399	0,549														
RMSE Y9	0,303	0,307	0,327	0,310	0,311	0,307	0,310	0,304	0,299	0,302	0,319	0,322	0,306	0,306	0,533	0,543		0,379	0,353	0,522														
RMSE Y10	0,334	0,345	0,369	0,343	0,339	0,338	0,337	0,335	0,332	0,333	0,351	0,351	0,343	0,339	0,611	0,631		0,452	0,421	0,609														
RMSE Y15	0,420	0,428	0,461	0,434	0,416	0,429	0,415	0,418	0,423	0,420	0,446	0,438	0,439	0,426	0,693	0,686		0,524	0,487	0,651														
DMA M3	0,250	0,250	0,247	0,247	0,282	0,250	0,279	0,273	0,253	0,238	0,240	0,260	0,257	0,283	0,379	0,287	0,255			0,297														
DMA M6	0,228	0,226	0,213	0,210	0,260	0,212	0,257	0,250	0,231	0,217	0,205	0,238	0,239	0,262	0,337	0,259	0,226			0,268														
DMA Y1	0,184	0,182	0,182	0,195	0,232	0,184	0,229	0,215	0,183	0,177	0,182	0,205	0,208	0,240	0,326	0,261	0,188			0,267														
DMA Y2	0,242	0,228	0,237	0,241	0,256	0,242	0,251	0,253	0,238	0,241	0,252	0,266	0,245	0,254	0,370	0,313	0,225	0,216		0,313														
DMA Y4	0,232	0,234	0,255	0,251	0,232	0,244	0,229	0,231	0,231	0,232	0,260	0,255	0,233	0,231	0,436	0,438		0,241																
DMA Y5	0,208	0,211	0,226	0,215	0,222	0,215	0,215	0,213	0,202	0,207	0,226	0,228	0,206	0,213	0,423	0,433		0,261																
DMA Y7	0,267	0,256	0,265	0,264	0,282	0,268	0,277	0,273	0,253	0,265	0,269	0,282	0,253	0,264	0,479	0,487		0,335																
DMA Y8	0,277	0,287	0,266	0,280	0,291	0,282	0,287	0,281	0,262	0,275	0,287	0,291	0,266	0,275	0,437	0,448		0,319	0,309	0,449														
DMA Y9	0,273	0,283	0,268	0,277	0,279	0,277	0,278	0,274	0,261	0,272	0,284	0,285	0,266	0,274	0,466	0,452		0,310	0,293	0,440														
DMA Y10	0,312	0,319	0,308	0,315	0,319	0,315	0,318	0,314	0,300	0,312	0,321	0,323	0,304	0,313	0,540	0,527		0,366	0,351	0,517														
DMA Y15	0,343	0,343	0,367	0,352	0,347	0,349	0,346	0,345	0,341	0,343	0,368	0,364	0,360	0,352	0,584	0,547		0,360	0,336	0,527														
RMSE M3	0,323	0,315	0,288	0,255	0,355	0,283	0,359	0,340	0,328	0,316	0,270	0,323	0,340	0,365	0,371	0,244	0,372			0,259														
RMSE M6	0,300	0,282	0,264	0,228	0,333	0,257	0,338	0,317	0,305	0,295	0,246	0,302	0,321	0,345	0,349	0,220	0,338			0,234														
RMSE Y1	0,259	0,245	0,234	0,192	0,294	0,221	0,303	0,275	0,261	0,258	0,214	0,268	0,286	0,312	0,306	0,209	0,324			0,222														
RMSE Y2	0,313	0,327	0,315	0,280	0,336	0,298	0,342	0,321	0,315	0,316	0,301	0,337	0,330	0,345	0,318	0,313	4,647	0,295		0,306														
RMSE Y4	0,359	0,366	0,389	0,349	0,367	0,360	0,375	0,358	0,358	0,360	0,371	0,389	0,358	0,365	0,412	0,420		0,368																
RMSE Y5	0,355	0,361	0,396	0,354	0,358	0,360	0,365	0,351	0,354	0,358	0,369	0,380	0,348	0,350	0,446	0,459		0,385																
RMSE Y7	0,408	0,375	0,460	0,406	0,408	0,413	0,413	0,405	0,415	0,412	0,427	0,433	0,404	0,399	0,508	0,530		0,457																

RMSE Y8	0,438	0,398	0,480	0,429	0,441	0,440	0,445	0,436	0,442	0,441	0,450	0,462	0,430	0,430	0,479	0,487	0,436	0,397	0,435
RMSE Y9	0,415	0,359	0,447	0,402	0,421	0,413	0,426	0,413	0,416	0,417	0,422	0,437	0,403	0,408	0,420	0,428	0,367	0,332	0,375
RMSE Y10	0,394	0,341	0,428	0,386	0,400	0,393	0,404	0,392	0,395	0,394	0,403	0,415	0,381	0,386	0,429	0,446	0,374	0,334	0,396
RMSE Y15	0,456	0,414	0,489	0,447	0,459	0,456	0,461	0,454	0,458	0,459	0,466	0,481	0,446	0,448	0,471	0,473	0,423	0,383	0,422
DMA M3	0,260	0,249	0,224	0,202	0,291	0,224	0,299	0,274	0,265	0,255	0,216	0,262	0,273	0,304	0,337	0,203	0,324		0,212
DMA M6	0,256	0,234	0,218	0,186	0,280	0,217	0,286	0,269	0,260	0,251	0,208	0,256	0,271	0,291	0,305	0,184	0,291		0,191
DMA Y1	0,222	0,200	0,195	0,168	0,247	0,197	0,255	0,236	0,225	0,219	0,188	0,222	0,242	0,263	0,254	0,187	0,255		0,197
DMA Y2	0,264	0,285	0,247	0,226	0,273	0,246	0,272	0,267	0,265	0,265	0,249	0,272	0,271	0,278	0,224	0,262	4,599	0,243	0,243
DMA Y4	0,306	0,311	0,298	0,285	0,303	0,298	0,302	0,303	0,307	0,306	0,309	0,319	0,307	0,304	0,329	0,361	0,288		
DMA Y5	0,287	0,297	0,266	0,261	0,292	0,279	0,292	0,288	0,285	0,287	0,282	0,295	0,282	0,288	0,342	0,389	0,291		
DMA Y7	0,327	0,306	0,350	0,313	0,332	0,326	0,337	0,328	0,328	0,327	0,335	0,343	0,326	0,323	0,384	0,421	0,322		
DMA Y8	0,359	0,327	0,382	0,348	0,367	0,358	0,373	0,359	0,361	0,361	0,368	0,378	0,359	0,356	0,356	0,375	0,292	0,272	0,341
DMA Y9	0,363	0,322	0,385	0,353	0,370	0,360	0,377	0,361	0,364	0,365	0,370	0,381	0,354	0,357	0,344	0,339	0,270	0,249	0,314
DMA Y10	0,351	0,308	0,372	0,343	0,354	0,348	0,360	0,349	0,352	0,351	0,358	0,367	0,338	0,344	0,341	0,339	0,266	0,245	0,317
DMA Y15	0,390	0,335	0,410	0,383	0,392	0,389	0,393	0,388	0,390	0,393	0,398	0,407	0,379	0,382	0,385	0,387	0,326	0,310	0,360
RMSE M3	0,202	0,255	0,216	0,197	0,235	0,193	0,237	0,224	0,202	0,211	0,189	0,234	0,198	0,218	0,551	0,255	0,325		0,271
RMSE M6	0,200	0,237	0,219	0,203	0,228	0,197	0,231	0,219	0,200	0,211	0,194	0,233	0,198	0,213	0,507	0,254	0,315		0,268
RMSE Y1	0,229	0,234	0,256	0,240	0,253	0,224	0,260	0,244	0,231	0,236	0,231	0,266	0,233	0,246	0,531	0,269	0,328		0,283
RMSE Y2	0,376	0,341	0,395	0,388	0,400	0,366	0,399	0,392	0,382	0,378	0,376	0,410	0,384	0,393	0,487	0,396	0,422	0,362	0,403
RMSE Y4	0,417	0,395	0,430	0,416	0,454	0,392	0,452	0,436	0,421	0,418	0,406	0,446	0,423	0,440	0,439	0,424	0,378		
RMSE Y5	0,420	0,391	0,433	0,414	0,459	0,392	0,454	0,441	0,422	0,423	0,405	0,445	0,423	0,440	0,464	0,421	0,359		
RMSE Y7	0,390	0,365	0,409	0,384	0,426	0,372	0,422	0,407	0,383	0,393	0,376	0,411	0,380	0,397	0,450	0,385	0,322		
RMSE Y8	0,392	0,371	0,413	0,386	0,422	0,378	0,419	0,404	0,384	0,394	0,379	0,409	0,381	0,396	0,450	0,383	0,324	0,323	0,399
RMSE Y9	0,362	0,344	0,392	0,357	0,392	0,348	0,394	0,371	0,354	0,363	0,350	0,381	0,348	0,365	0,408	0,355	0,298	0,297	0,382
RMSE Y10	0,352	0,318	0,387	0,346	0,385	0,338	0,387	0,363	0,344	0,352	0,342	0,374	0,336	0,354	0,372	0,344	0,292	0,304	0,376
RMSE Y15	0,382	0,352	0,426	0,380	0,409	0,373	0,410	0,390	0,377	0,384	0,378	0,412	0,366	0,380	0,439	0,390	0,341	0,324	0,398
DMA M3	0,181	0,209	0,171	0,170	0,196	0,170	0,192	0,191	0,180	0,183	0,169	0,193	0,177	0,186	0,465	0,185	0,258		0,206
DMA M6	0,169	0,210	0,166	0,161	0,198	0,166	0,191	0,192	0,168	0,170	0,160	0,188	0,166	0,183	0,413	0,184	0,236		0,196
DMA Y1	0,202	0,193	0,210	0,211	0,205	0,196	0,200	0,208	0,204	0,206	0,205	0,219	0,205	0,203	0,422	0,228	0,231		0,239
DMA Y2	0,324	0,279	0,317	0,344	0,322	0,323	0,316	0,333	0,329	0,320	0,325	0,337	0,328	0,319	0,370	0,323	0,282	0,320	0,330
DMA Y4	0,349	0,315	0,342	0,364	0,366	0,337	0,358	0,359	0,353	0,349	0,346	0,376	0,351	0,354	0,388	0,365	0,331		
DMA Y5	0,358	0,312	0,332	0,349	0,389	0,334	0,380	0,377	0,359	0,360	0,342	0,377	0,361	0,375	0,394	0,360	0,317		
DMA Y7	0,328	0,291	0,317	0,321	0,355	0,310	0,351	0,344	0,323	0,330	0,313	0,344	0,323	0,336	0,383	0,329	0,281		
DMA Y8	0,319	0,294	0,313	0,314	0,345	0,304	0,345	0,334	0,312	0,320	0,305	0,335	0,319	0,326	0,373	0,326	0,280	0,264	0,336
DMA Y9	0,297	0,281	0,299	0,293	0,325	0,284	0,327	0,305	0,288	0,299	0,284	0,316	0,288	0,300	0,351	0,302	0,247	0,249	0,319
DMA Y10	0,290	0,264	0,311	0,287	0,318	0,278	0,319	0,299	0,281	0,291	0,278	0,309	0,273	0,291	0,334	0,287	0,242	0,249	0,315
DMA Y15	0,304	0,279	0,315	0,300	0,330	0,292	0,328	0,314	0,295	0,306	0,292	0,322	0,287	0,303	0,363	0,337	0,290	0,280	0,350

Appendix 3

variable	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	all factors	levelcol slopecol	levelcol slopecol
EXOG		Oilprices	OAS	FFR	VIX	USlevel	USDslope	USDcurvature	TRM	EMBIcol	EMBI	U.S. CPI	Inflationcol	all variables	all variables	only significant variables	only significant variables	only significant variables		
varsc	2	2	3	2	2	2	2	1	2	2	2	3	2	2	1	1	1	1		4
RMSE levelcol	0,324	0,352	0,331	0,321	0,328	0,322	0,301	0,339	0,324	0,333	0,328	0,306	0,325	0,754	0,847	0,765				0,565
RMSE slopecol	0,403	0,406	0,389	0,411	0,388	0,404	0,416	0,431	0,394	0,388	0,404	0,427	0,431	0,632	0,766	0,831				0,622
RMSE curvaturecol	0,398	0,403	0,417	0,395	0,409	0,396	0,405	0,405	0,397	0,408	0,403	0,407	0,398	0,382	0,398	0,454				
DMA levelcol	0,300	0,288	0,298	0,305	0,301	0,306	0,264	0,292	0,303	0,302	0,306	0,260	0,295	0,684	0,789	0,682				0,474
DMA slopecol	0,331	0,327	0,329	0,330	0,330	0,331	0,324	0,328	0,327	0,321	0,322	0,332	0,334	0,488	0,611	0,714				0,497
DMA curvaturecol	0,316	0,318	0,331	0,317	0,334	0,320	0,329	0,320	0,315	0,327	0,321	0,342	0,328	0,307	0,324	0,372				
RMSE levelcol	0,329	0,366	0,306	0,339	0,322	0,352	0,326	0,335	0,325	0,328	0,343	0,305	0,325	0,491	0,569	0,532	0,534			0,444
RMSE slopecol	0,410	0,417	0,416	0,416	0,396	0,413	0,419	0,418	0,394	0,399	0,408	0,426	0,422	0,598	0,756	0,653	0,653			0,536
RMSE curvaturecol	0,418	0,427	0,435	0,416	0,433	0,413	0,418	0,431	0,417	0,431	0,430	0,420	0,418	0,418	0,439	0,442				
DMA levelcol	0,294	0,296	0,267	0,305	0,286	0,316	0,292	0,292	0,291	0,288	0,304	0,262	0,289	0,406	0,496	0,452	0,456			0,369
DMA slopecol	0,347	0,354	0,354	0,349	0,337	0,349	0,350	0,341	0,329	0,337	0,340	0,338	0,347	0,514	0,657	0,547	0,546			0,398
DMA curvaturecol	0,330	0,334	0,350	0,330	0,349	0,325	0,329	0,349	0,329	0,338	0,342	0,342	0,341	0,333	0,358	0,359				
RMSE levelcol	0,345	0,382	0,340	0,363	0,328	0,375	0,345	0,308	0,340	0,334	0,351	0,344	0,346	0,473	0,429	0,397	0,395			0,378
RMSE slopecol	0,296	0,311	0,315	0,301	0,288	0,301	0,300	0,271	0,293	0,289	0,298	0,315	0,308	0,788	0,839	0,563	0,563			0,495
RMSE curvaturecol	0,456	0,465	0,493	0,443	0,487	0,438	0,452	0,432	0,457	0,472	0,458	0,441	0,434	0,478	0,427	0,430				
DMA levelcol	0,302	0,331	0,299	0,312	0,288	0,318	0,302	0,262	0,295	0,295	0,306	0,298	0,304	0,371	0,354	0,310	0,311			0,311
DMA slopecol	0,230	0,240	0,252	0,231	0,236	0,231	0,230	0,223	0,239	0,221	0,224	0,240	0,231	0,665	0,752	0,490	0,489			0,420
DMA curvaturecol	0,399	0,405	0,436	0,381	0,439	0,370	0,392	0,380	0,400	0,415	0,388	0,379	0,367	0,415	0,362	0,365				