MULTIFACTOR SPREAD MODELS FOR CAT BONDS IN THE PRIMARY AND SECONDARY MARKET

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Abstract

As a result of the reinsurance industry seeking for additional capital capacity in the financial markets, a new class of financial instruments for trading insurance related assets has emerged. This class is known as Insurance Linked securities (ILS), being Cat bonds the most successful class of ILS so far, reaching an outstanding trading volume of US$7 billion just after 10 years since its public appearance. Their success derives from its innovative structure, which is attractive to the sponsors as an alternative to reinsurance protection against catastrophic losses, and to the investors as a high yield asset, uncorrelated with other financial securities.

This research seeks to address the need for market players to fully understand the dynamics of Cat Bonds prices in the primary and secondary market, and, to provide a reliable valuation tool for making sound investment decisions. We propose multifactor spread models in which several variables are included as determinants for the Cat Bond's spread. Our results are robust, and have a general applicability in both for the P&C and Life market.

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Part I

INTRODUCTION

Over the last decades, there has been an increasing trend in converting assets into financial instruments, which can become liquid and easily tradable in capital markets. That process is called securitization, which according to Weber (2011) "is the process of removing assets, liabilities or cash flows from the corporate balance sheet and transferring them to third parties through the creation of tradable securities".

The insurance industry has found a growing attractiveness in securitizing its assets with the purpose of raising capital and transferring part of their risk into the capital markets. Those financial instruments originated specifically from the insurance industry are known as Insurance linked securities (ILS). Currently there are numerous classifications of ILS, given the constant creation of new securities related to the coverage of insurance risks. In this thesis we follow Cummins (2008), who distinguishes 5 different types of ILS: Cat Bonds, Side-Cars, Catastrophic Equity Puts, Catastrophe Risk Swaps, and Industry Loss Warranties.

The use of ILS was initially triggered by the devastation caused by Hurricane Andrew in 1992, the costliest hurricane up to that moment, and the third in the United States history after Katrina (2005) and Ike (2008). According to the Munich Re, insured losses amounted to US$17 billions, caused by 62 deaths and 125,000 homes destroyed in Florida.

Hurricanes, floods, earthquakes, tsunamis, and other related natural disasters are classified as catastrophic events, and since their economic impact is so high, the industry was forced to search for alternative mechanisms to fund them. It was then, when the financial markets seemed as the perfect option to which risk could be transferred and shared with investors willing to buy those securities.

It is the high cost of claims derived from the aggregation of many losses, the most important factor for securities with catastrophic coverage to be among the most used

ILS nowadays. Catastrophic events are characterized by being severed and infrequent, forcing insurers and reinsurers to seek adequate protection, in order to avoid operative volatility, assure solvency and release capacity to underwrite other risks. Such protection is often found on Cat Bonds, which according to Cummins (2008) have been the most successful ILS so far.

Cat Bonds are defined as "fully collateralized instruments that pay off on the occurrence of a defined catastrophic event" (Cummins 2008). It has been developed as a regular bond, aiming to provide the issuer with financial coverage in case a specified catastrophic event occurs, causing high losses to the company. Their success derives from its innovative structure, being attractive to the sponsors as an alternative to reinsurance protection against catastrophic losses, and to investors as a high yield asset, uncorrelated to other financial securities

It becomes then of upmost relevance for all market players to fully understand the dynamics of Cat Bonds both in the primary and secondary market, as well as to have a reliable valuation tool for making sound investment decisions. This thesis aims to address the former need, by developing multifactor spread models, working with several relevant variables that have a direct effect over the spread.

1.1   Definition

CAT Bonds are collateralized securities, with contingent payments upon the occurrence of a defined catastrophic event. According to Cummins (2008), the first CAT Bond ever traded in the capital market was issued by the Reinsurance Hannover Re in 1994. Ever since, the market has set a revolutionary upsurge in the use of these instruments, moving from volumes of US$542 in 1994 to a record high of US$7.33 billion in 2007 (A.M. Best 2012).

Cat Bonds are structured by Investments banks, and are issued by large insurance or reinsurance companies or by governments (the sponsor), who seek an alternative method for covering the high value of their undertaken risks. Their role usually gets

materialized once a catastrophic event occurs, in which case the high cost of claims could lead the company to insolvency.

1.2   Triggering types

Unlike the corporate and government bonds, Cat Bonds do not have an underlying asset. They are priced and structured according to the events covered, which will determine the cash flow of the investor in case of occurrence. Those events are known as triggering events, and according to Cummins (2008), Cat Bonds are classified under the following 3 triggering categories:

1. **Indemnity Trigger:** the triggering event is the actual loss incurred by the sponsoring company, after the occurrence of a specific catastrophe, in a specific location.

2. **Index Trigger:** The triggering event is linked to the performance of an Index, not directly related to the sponsor. There are 3 major types of indices used as Cat Bonds triggers

   (a) Industry Loss Indices: When a certain level on a chosen catastrophe index is exceeded, the payoff is triggered.

   (b) Modeled Loss Indices: works as the industry index, but the values are derive from calculations replicating catastrophic scenarios, instead of real events.

   (c) Parametric Indices: The trigger here is represented by a physical condition related to the catastrophic event, such as the wind speed in a hurricane, or the magnitude of an earthquake.

3. **Hybrid Triggers:** a mix of several trigger events in one bond.

Usually indemnity triggers have a better match with the losses incurred by the issuing company (sponsor), as the trigger itself is the real loss incurred by the company, reducing the risk of under or over coverage.

On the other hand, "Index triggers tend to be favored by investors because they minimize the problem of moral hazard; i.e., they maximize the transparency of the transaction." (Cummins 2008) The estimation of losses for the indemnity trigger could have misleading information for investors, charging higher losses than incurred, therefore, an index publicly known and calculated by a third party, significantly increases transparency.

"Indices also have the advantage of being measurable more quickly after the event than indemnity triggers, so that the sponsor receives payment under the bond more quickly" (Cummins 2008). The disadvantage lies on the risk of mismatching the funds received with the real capital needed for paying the catastrophe incurred, faced by the sponsor.

1.3 Structure

Cat Bonds are structured by financial entities, under the usual characteristics of a regular Bond, however, the probabilistic scenario given by the triggering event, and the insurance purpose of the bond change the way transactions are conducted.

The steps in issuing a Cat Bond are:

1. A Special Purpose Vehicle (SPV) is established by the sponsor. "SPVs are entities created for a specific, limited and normally temporary purpose. They are limited companies or partnerships to which the debt of another company is transferred. By transferring its debt off its balance sheet into an SPV a company is able to isolate itself from any risk that the debt might pose. SPVs are often used in the securitization of loans or other instruments."(Thomson Reuters 2013)

   In the case of Cat Bonds, the SPV created is known as a *Single Purpose Reinsurer* (SPR), which would be the entity receiving all the income from the Cat Bond sales and responsible in return of providing insurance to the sponsor who seeks protection against a catastrophic event.

   Sponsors do not issue themselves the bond, because in that way they eliminate

the credit risk associated to the transaction, since that money will be hold in a separate account from the assets of the company, so in case they go in bankruptcy, the money would be safe from credit default risk. Other reason for having the issue through a SPR, is because those vehicles are usually created on tax haven jurisdictions, representing financial advantages.

2. The sponsoring company enters into a reinsurance agreement with the SPR, who commits in making a payment once the trigger event defined is set. The sponsor in return pays an initial premium as a cost for that protection.

3. The SPR issues the CAT Bond, with the characteristics specified by the sponsoring company (trigger event, rate, and expiration) according to the protection needed. The structure, issuance and sell of the Cat Bond are made with the assistance of an investment bank.

4. The proceeds from the sale of the Cat Bond are deposited on a segregated collateral account (trust account), in order to invest those funds and generate a return. Those investments are usually made on instruments with risk free rates. The coupon payment made to the investor is derived from two sources: the investment income generated from all the funds collected from the Bond sale, and the premium paid by the sponsor to the SPR.

5. If no trigger event occurs, the SPR pays periodically to the bond holders the coupons as accorded, and at the end returns the principal and final coupon to the investors.

6. If a triggering event does happen, the funds collected from the sale are transferred from the SPR to the sponsor, in order to pay for the claims arising from the catastrophic event, just as stated on the reinsurance contract between those 2 entities.

According to Cummins (2008), "In most Cat bonds, the principal is fully at risk, i.e., if the contingent event is sufficiently large, the investors could lose the entire principal in the SPR. In return for the option, the insurer pays a premium to the investors."

Figure 1: Structure of a Cat Bond Transaction
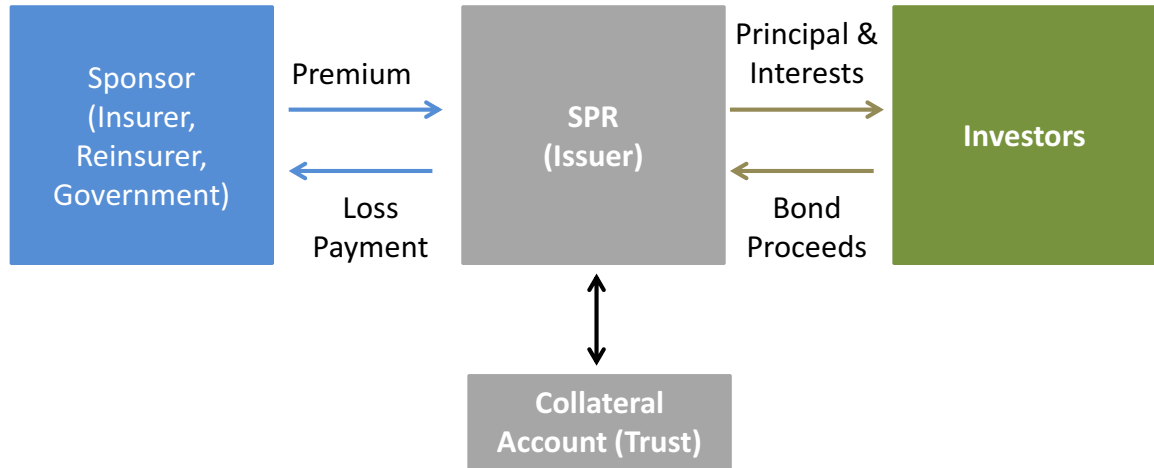
"The bonds are attractive to investors because catastrophic events have low correlations with returns from securities markets and hence are valuable for diversification purposes."(Cummins 2008)

Because of the novelty, and complexity of these financial securities, all current investors are institutional. According to a report issued by RMS (2012), Fund Managers are the largest investors for Cat Bonds, buying approximately 70% of all the bonds issued.

1.4   Risks

The sponsor of a Cat Bond faces, among other, the two following risks:

- **Basis Risk:** In Cat Bonds, is reflected in the possibility that the Bond does not fully cover the losses suffered by the sponsor, leading them to be under covered.

- **Credit Risk:** In Cat Bonds, credit risk does not exist, as the money collected from the issuance is put safely in an independent account from the sponsor (Trust Account), and hence there is no reason why the transfer of money to investor would be denied.

1.5 Cat Bonds Vs Reinsurance

Insurance companies buy protection against the risks they assumed (policies issued to clients), through a contract signed with a reinsurance company, who compromises on paying part of those risks in case of losses, in return for a fee or participation on the premiums received by the insurer.

Insurers chose to share their risk with reinsurers, because of their larger capacity in terms of equity and assets to support the liabilities arising from those policies covered. However, not all reinsurers manage to achieve the level of capital needed for assuming large risks from insurers. According to RMS (2012), the reinsurance industry has become very concentrated, with the top 5 worldwide reinsurers (Munich Re, Swiss Re, Berkshire Hathaway, Hannover Re and Lloyd's) accounting for 50% of the worldwide premiums written.

Reinsurers work by the *law of large numbers*, which affirms that after many trials, the outcome tends to converge to the expected value. Meaning that after a big number of risks insured, losses tend to stabilize on normal levels. That's why reinsurers tend to be diversified geographically and by products, reducing therefore the probability of large deviated losses.

Reinsurance business is growing at a fast pace, given the increasing value of insured risks. People now are more aware of all uncertainties, fostering companies, governments and individuals to increase the purchase of insurance policies. From all the different types of risks insured, those covering catastrophic events are the most expensive in losses and therefore premiums (fee paid by client). A catastrophe in insurance is defined as an event affecting several independent risks.

However, as convenient as reinsurance seems, there is a constant concern about the volatility in the premium rates. Reinsurance is priced based on the amount of losses incurred in the prior year, meaning that after a year of high losses the price of the premium charge by reinsurers will increase significantly. And that increase is passed latter on to the clients through their policies rates.

Cummins and Trainar (2009) explain the above, by affirming that "The traditional and still prevalent model of risk diversification and risk transfer in the insurance industry is the risk warehouse; i.e., insurers and reinsurers served a risk absorption or risk-warehousing function in the economy. Traditional reinsurers provide risk diversification and risk management products but typically do not pass the risks inherent in these instruments along to the capital markets but rather hold them on balance sheet."

Therefore, as previously mentioned the insurance industry is turning into the financial markets for transferring part of their risks and seek further capacities. Nowadays with the boom in ILS, and more specifically the high success of Cat Bonds, the question of whether those financial instruments or the traditional reinsurance is the best protection alternative, has arisen.

Let's suppose that an insurer is exploring the differences between buying reinsurance and issuing a CAT bond. He will find the following differences (Tower Watson 2012):

1. **Type of Collateral:** In reinsurance is the contract signed, backed by a legal paper. In Cat Bonds is the cash paid by the investor deposited on a collateral account.

2. **Payment Flow:** The reinsurance coverage is paid through a premium set on the contract. For Cat bonds, the payment takes place in the form of coupon payments.

3. **Premium Adjustment:** For reinsurance, the change in the premium is set at the beginning of the period, and is expressed in money according to the losses incurred previously. On the other hand, CAT bonds are typically bought at par and have a predetermined coupon spreads, varying across time in the secondary market.

4. **Fees & Expenses:** Reinsurance may imply a fee charged by a broker. CAT bonds represent a financial instrument that must be carefully designed by professionals and issued by an investment bank, for which expenses for the sponsor are significantly higher.

However, CAT bonds and reinsurance must not be seen as substitutes, but as Cummins and Trainar (2009) say, both instruments are rather complements, although for certain

types of risk such as catastrophic, they do become substitutes. "Thus, given the effi-
ciencies of risk warehouses in handling numerous, relatively small, independent risks,
we do not expect that securitization will replace reinsurance. However, for larger, more
correlated risks, securitization begins to compete with reinsurance, and securitization
may be the only solution for the largest, most catastrophic risks." (Cummins 2008) For
example, an insurance company may buy reinsurance to cover the lower bound of its
risks, while at the same time issuing a Cat bond to cover the higher layers in which
catastrophic losses fall.

Now, given the mentioned advantages of Cat Bonds, one wonders why the use of this
instrument is not higher than traditional reinsurance. The explanation arises from two
main reasons: "first, the costs of cat bond issuance are significantly higher than for
a traditional reinsurance contract, and are not economically viable for small principal
amounts. Second, the number of investors willing to buy cat bonds is still limited,
mostly due to lack of familiarity with catastrophe risk."(RMS 2012)

1.6 Historical volumes traded

According to A.M. Best (2012), Cat Bonds first appeared publicly on 1997, and ever
since there has been a marked increase in the volume issued, going from US$643 million
in 1997 to a peak of US$7 billion in 2007. Consequently those assets are considered
to be "one of the more recent financial derivatives to be traded on the world markets"
(Baryshnikov et al 2001). Their innovative nature, and historical high yields have led
investors to turn their attention into an asset that was originally developed for insur-
ance and reinsurance companies only.

The following graph (Guy Carpenter 2013) shows the volumes of Cat Bonds issued on
a quarterly basis from 1997 to 2013. There is a clear dominance in the volumes issued
on quarters 2 and 4, as they represent the middle and end of a year, being the most
common time for companies to renew their annual protection contracts, or in this case,
seek for alternative protection.

Figure 2: Risk Capital Issued by Quarter 1997-2013
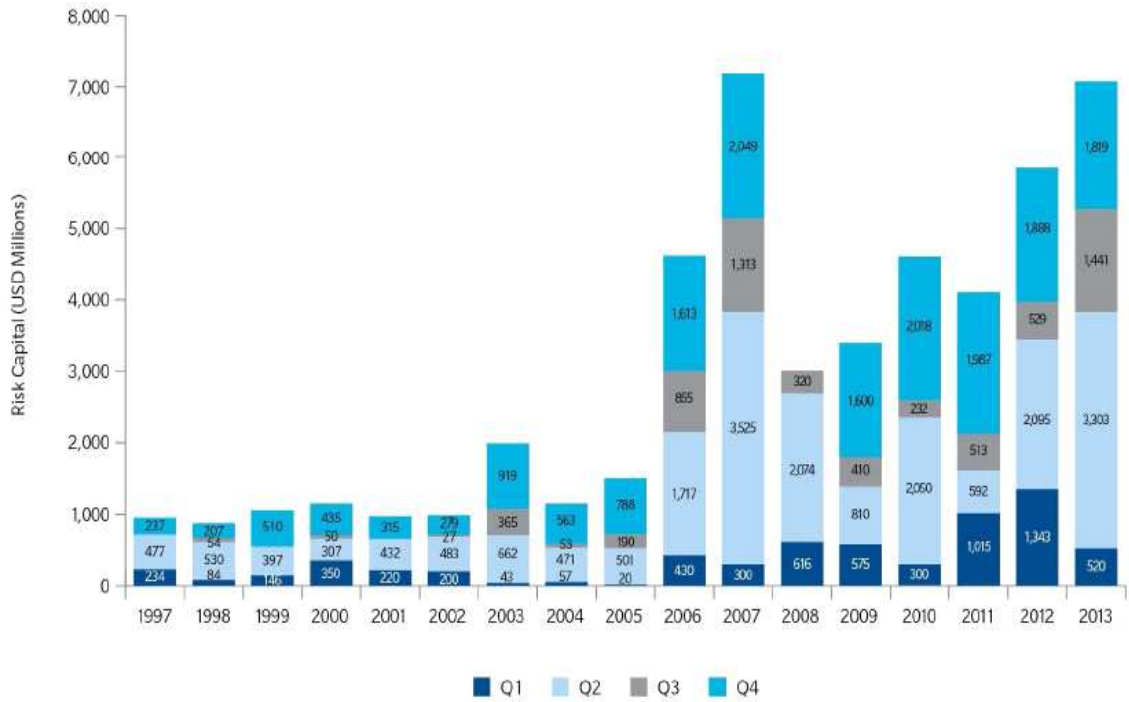
The issuance of Cat Bonds reached its highest peak on 2007, with a near US$7 billion, followed by US$6.8 billion in 2013. "Capacity is expanding because sophistication and attention to transaction mechanics is increasing, not decreasing."(Guy Carpenter 2013)

Cat Bonds usually have maturities from 1 to 3 years, for which at a specific year there are several bonds trading from different maturities, reflecting the risk capital outstanding in the market, i.e, the face value of all the bonds outstanding.

Figure 3: Risk Capital Outstanding 1997-2013
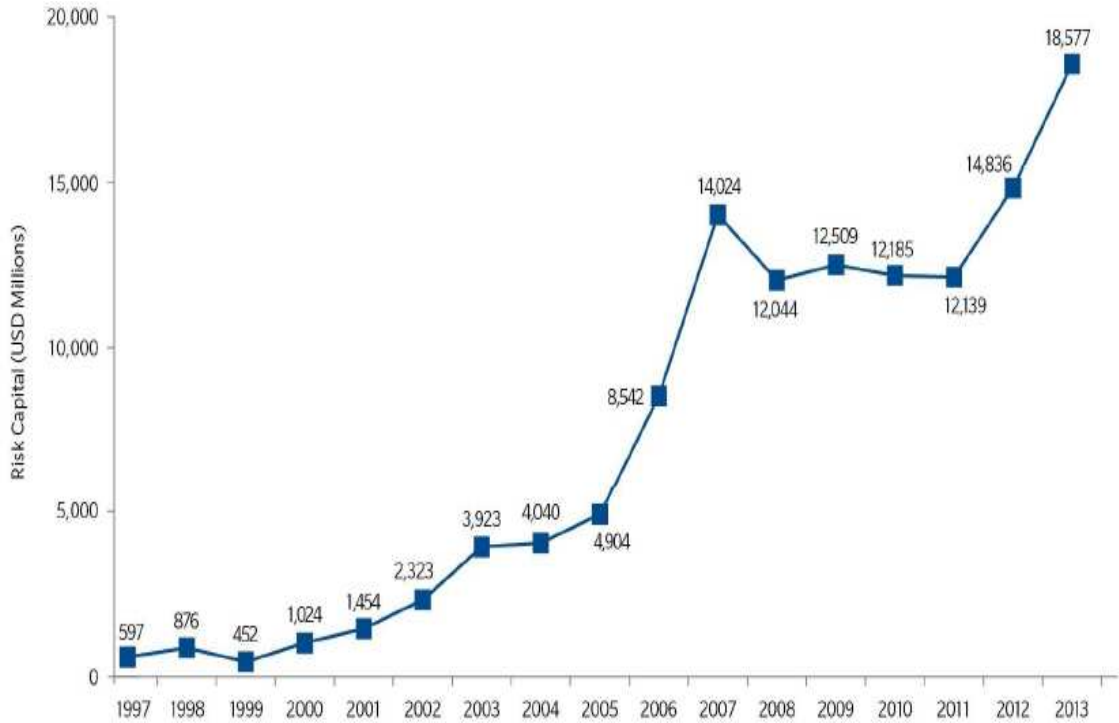
Source: Guy Carpenter

Risk capital outstanding "increased during the 2013, reaching an all-time high water mark of US$18.57 billion – up from USD14.83 billion at year-end 2012, representing a net increase of 25 percent (US$ 3.7 billion)." (Guy Carpenter 2013)

Currently the amount of notional exposure that trades in the catastrophe reinsurance market each year is approximately US$220 billion, while the total issuance typically averages around US$4 billion per annum[1].

1.7   Spreads and Returns

Historical returns of Cat Bonds have been higher than those of most corporate bonds, given the fact that they represent a higher risk for the investor. Once a trigger event oc-

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[1]Nephila Capital Limited. 2013

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curs, investors lose fully or partially their investment, offering therefore a higher spread for the risk undertaken. The most used index for assessing the Cat Bonds performance in the market, is the Swiss Re Cat Bond Total Return Index (ticker: SCATTRR), which is an index developed by the Reinsurance company Swiss Re, tracking the total rate of return for all outstanding dollar denominated cat bonds, priced by them.

It is usually used as a benchmark, and when compared to other indices as the S&P500, shows a sign on how well Cat Bonds are doing, against equities and other financial instruments. "This index, as well as others in the Swiss Re Cat Bond series is based on secondary market data and tracks the price, coupon and total rate of return for cat bonds since 2002." (RMS 2012)

The historical performance of that index, benchmarked with others indices in the market, is shown in figure 4.

Figure 4: Performance of the Swiss Re Cat Bond Total Return Index compared to other asset classes.

Source: RMS (2012)

The return on each index depends on the assets tracked, which comprises a certain level of risk. "Obtaining a financial rating is a critical step in issuing a CAT bond because buyers use ratings to compare yields on Cat Bonds with other corporate securities. Consequently, almost all bonds are issued with financial ratings." (Cummins 2008) Most Cat Bonds are rated below investment grade, because the criterion here is the risk of losing the investment rather than the financial creditworthiness of the sponsor. "Because Cat Bonds are fully collateralized, Cat Bond ratings tend to be determined by the probability that the bond principal will be hit by a triggering event. Thus, the bond ratings merely indicate the layer of catastrophic-risk coverage that is being provided by the bonds." (Cummins 2008)

Historical returns of Cat Bonds have been higher than those of most corporate bonds, given the fact that they represent a higher risk for the investor. Once a trigger event occurs, investors lose fully or partially their investment, offering therefore a higher spread for the risk undertaken.

Table 1: Comparison of historical returns and volatility (2002-2012)

Index	Historical Annual Returns	Volatility
Swiss Re Cat Bond Total Return Index (SCATTRR)	7.98%	2.79%
Dow Jones Credit Suisse Hedge Fund Index	6.38%	5.91%
S&P 500 Index	1.06%	16.24%
Dow Jones Corporate Bond Index	1.19%	6.70%
Private Equity Total Return Index	-2.26%	30.23%

Source: RMS (2012)

According to RMS (2012), the historical (2002-2012) annual return of the Swiss Re Cat Bond is significantly higher (7.89%) than other indices like the Dow Jones (6.38%), S&P500 (1.06%) and the Dow Jones Corporate Bond Index (1.19%).

Part II

PRELIMINARY REVIEW OF CAT BONDS VALUATION

MODELS

In the 90's decade when Cat bonds started actively trading, a spur of authors with different approaches for valuing this type of assets appeared. Following the classification proposed by Galeoti et al (2013), we identify 3 categories of Cat Bonds models: the Bond pricing, the indifference pricing and the premium calculation model.

2.1 Bond Pricing

The base for these types of models is the interpretation of a Cat Bond as a portfolio with a variable interest bond and an option whose exercise will depend on a catastrophic event.

One of those models was first developed by Cummins and Geman (1995), in which they explore both the pricing of catastrophe Insurance Futures and Call Spreads. Other authors have been also Loubergé et al (1999) who propose a valuation method for a bond already trading in a secondary market. They analyze Cat bonds as financial portfolios combining a straight bond and catastrophe options. "Using option pricing theory and simulation analysis in a stochastic interest rate environment, we show that investors attracted by the potential for diversification benefits should not overlook the optional features when including these securities in an asset portfolio." (Loubergé et al, 1999)

An illustrative model developed under a bond pricing approach, is that of Vendenov et al (2006), in which they explore the design of a catastrophe bond for protecting agricultural risks. The authors propose a Cat bond structured as a zero-coupon bond that is initially sold at a discount. An investor's return is then the difference between the purchase price and the face value. The sponsor reserves the make payment of part or all of the face value if a predetermined triggering event occurs.

Let us assume a zero-coupon Cat bond is issued at time 0 with the face value F and time to maturity T. The payoff VT of the bond at maturity is conditional on realization of a certain index L relative to the predetermined trigger value D so that:

$$V_T = \begin{cases} A \times F \ if \ L > D \ (Bond \ is \ triggered), & (1) \\ F \ if \ L \leq D \ (otherwise) \end{cases}$$

where

$$0 < A < 1$$

is the proportion of the face value repaid to investors.

Their model supposes that Cat bond prices are based on the discounted expected payoff of the bond over different possible scenarios. Such an approach assumes that the triggering index underlying every Cat bond is characterized by a stationary distribution. With the former assumptions, the authors proceeded to develop their model under a two-steps approach: First, estimating the distribution of the index underlying the Cat bond, and thus the probabilities of triggering the bond. And second, incorporating the estimated probabilities and the required rate of return into the bond's price.

They use a Kernel density distribution, which is a nonparametric technique, in order to derive the **probability distribution of the triggering index**, form historical data. Authors affirm they chose Kernel density estimation over a parametric estimation, because of its properties of accommodating to the complexities of the data.

Kernel density estimation constructs the probability distribution of a random variable 'x' as a sum of specially selected functions or kernels, under the following form:

$$f_h(x) = \frac{1}{n} \sum_{i=1}^{n} K_h(x - x_i) \quad (2)$$

where $f(x)$ is the kernel density function; $x, ..., xn$ are observations (realizations) of the random variable, and $K(u)$ is the kernel. H is a smoothing parameter called bandwidth: $K(h) = \frac{1}{h} K(\frac{x}{H})$

Replacing H in equation (2), we have:

$$f(x) = \frac{1}{nH} \sum_{i=1}^{n} K_h \frac{(x - x_i)}{H}$$

For the distribution of the percentage deviations of the triggering variable from its long-term average, the authors used the Epanechnikov kernel, which is one of the standard kernel functions. The functional form for the Kernel is written as:

$$K(u) = \begin{cases} \frac{3}{4}(1 - u^2) \ |u|<1, \\ 0 \ otherwise \end{cases}$$

Following the concept of a regular bond, authors assumed that a Cat bond is valued by taking the discounted expectation of its possible payoffs under the derived distribution of realized yield losses (deviation of triggering variable) and the required rate of return on investment. Therefore, the formula for pricing a Cat bond with time to maturity T is as follows:

$$V = E_{\vartheta\eta} \left[V_T exp \left(-\int_0^T r(t)dt \right) \right]$$

where VT is the payoff of a Cat bond in (1), $r(t)$ is the appropriate interest rate used to discount future cash flows, and $E_{\vartheta\eta}$ indicates expectations with respect to two state variables. The state variable ϑ, encompasses the term structure of interest rates, is independent of the state variable η, which reflects the catastrophe risk per se. Under this assumption, the Cat bond price becomes:

$$V = E_{\eta} V_T \times E_{\vartheta} exp \left(-\int_0^T r(t)dt \right) \quad (3)$$

where now $E_{\eta} V_T$ is the expected payoff of the Cat bond.

From equation 3 we can observe the separation between the risk of default due to occurrence of a catastrophic event, and the risk of default due to all other factors.

From the former analysis, the authors obtain an analytical pricing formula for a conventional zero coupon bond under the assumption of a constant interest rate $(t) \equiv r$

. Then, the solution $B(0,T)$ is found by discounting the face value of the bond at the appropriate discount rate for the time period T.

We can express the right term of equation 3 as:

$$E_{\vartheta} exp\left(-\int_0^T rdt\right) = B(0,T) = exp(-rT) \quad (4)$$

Now, using equation 1, the expected payoff of the Cat Bond can be written as:

$$E_{\eta}V_T = F \times Pr\{L \le D\} + A \times F \times Pr\{L > D\} \quad (5)$$

where $PrL \le D$ is the probability of the realized yield loss less than or equal to the trigger level D, and $PrL > D$ is the probability of the opposite event.

Finally, combining (4) and (5) to express (3), we reach the general pricing formula for Cat Bonds:

$$V = B(0,T) \times [F \times Pr\{L \le D\} + A \times F \times Pr\{L > D\}]$$

The above expression shows how the price of Cat Bonds can be represented as the product of the price of a conventional zero-coupon bond and the expected payoff from the Cat bond. Such pricing model assumes that the financial market is liquid and there are no arbitrage opportunities. The price of a specific Cat bond depends on the parameters of the bond (face value F, trigger level D, and proportion A of face value repaid in case of a catastrophic event) as well as the selected discount rate and the probability distribution of the triggering index.

However, one of the restrictions with this model developed by Vendenov et al (2006), is the fact that authors rely on the efficient market hypothesis, which implies that the risk of default can be incorporated into the bond price through an appropriate rate of return used to discount future cash flows. However, in financial markets, such discount rate is influenced by other factors such as investors' perceptions which are difficult to be accurately quantified.

2.2 Indifference Pricing

These kind of models are based on the concept of a Utility function, in which the indifference price is that for an agent with the same expected utility level between exercising a financial transaction and not. It resembles the concept of the reservation price, which is the highest price a buyer is willing to pay for a good or service; or; the smallest price at which a seller is willing to sell a good or service. An advantage of these models is that they remain valid for incomplete markets.

Young (2004) proposed such an approach for valuing Cat Bonds. Later, Egami and Young (2008) modified it in order to consider structured Cat Bonds.

2.3 Premium Calculation

While the other pricing models take the premiums which are paid by the sponsor, for granted, these approaches define as "price" of a Cat Bond the determination of the premium which has to be set. According to Galeoti et al (2013), "in this approach the price which is also referred to as spread consists of the expected value of loss plus a load for risk margin and expenses."

Usually, in reinsurance pricing, the risk is divided into several layers if the protection contracted is non-proportional. Now, let us assume we have a non-negative random loss variable X. Also, the layer of first loss is 'a' and the one of last loss is 'h'. Then, the loss function is given by:

$$X_{(a,a+h]} = \begin{cases} 0 & if \ x \leq a \\ X - a & if \ a < X \leq a + h \\ h & if \ a + h < X \end{cases} \quad (1)$$

The loss variable X has a cumulative distribution function

$$F_X(x) = P\left(X \leq x\right)$$

Accordingly,

$$S_x(x) = 1 - F_x(x) = P(X > x)$$

denotes the so-called loss exceedance probability curve. This is an important index, since we are interested in the loss which is occurring above a determined level. Now, let us assume that there also exists the density function,

$$f_x(x) \ and, \ thus, \ s_x(x) = S'_x(x)$$

Now, the authors state the loss exceedance probability curve under the following equation:

$$S_{x(a,a+h]}(y) = \begin{cases} S_x(a+y) = P\left[X > a+y\right], \ if \ 0 \le y \le h \\ 0 \quad if \ h \le y \end{cases} \tag{2}$$

The above means that if the loss is between 0 and h, then the loss exceedence function will be given by the probability of the risk (X) exceeding the first loss (a) plus the actual loss (y).

Since $S_{x(a,a+h]}(y)$ and also $S_x(x)$ are continuous on the interval $(a, a + h]$ the expected value of the random variable X, is given by:

$$E\left[X_{(a,a+h]}\right] = \int_0^\infty y \cdot s_{x(a,a+h]}(y)dy$$

$$= \int_a^{a+h} x \cdot s_x(x)dx$$

Using partial integration

$$E[X_{(a,a+h])}] = xS_x(x)|_a^{a+h} - aa + hS_x(x)dx$$

$$E[X_{(a,a+h]}] = [(a + h)S_x - as_x(a)] - \int_a^{a+h} S_x(x)dx$$

By definition of the limits in equation 2:

$$E[X_{(a,a+h]}] = [-as_x(a)] - \int_a^{a+h} s_x(x)dx$$

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Replacing $s_x = f_x(x)$, and following the limits definitions on equation 1, we now have:

$$E[X_{(a,a+h]}] = \int_a^{a+h} s_x(x)dx$$

The above expression is referred to in the literature as the net premium. Now eliminating the layer, for having a general form:

$$E[X_{(a,a+h]}] = \int_0^\infty s(x)dx$$

Since the net premium is a lower bound for a premium, a risk load is needed. The expected loss (EL) of a layer is also referred to as the spread or price of a layer in case of risk neutrality. According to Galeoti et al (2013), if the market is complete then the risk neutral price of a layer is the EL of this layer. Since, usually, there is no market completeness, a risk premium is demanded.

Empirically it can be stated that the observed market price for layer always includes a risk load in addition to the expected loss:

$$EL^* = EL + \lambda$$

In literature there are various possibilities in order to account for a risk load (λ). According to Bantwal and Kunreuter (1999) there arises a Cat Bond premium puzzle. They found that the spreads are too high to be explained only by investor risk aversion. They suggested considering also ambiguity aversion, myopic loss aversion and fixed costs of education as impact factors for the high spreads.

Other authors following these types of model are Berge (2005) who accomplished a multivariate regression analysis in order to determine factors explaining the risk loads of Cat Bonds, in addition to the expected loss. Mainly he found that risks which arise out of market imperfections have an impact on the level of the risk load.

Major and Kreps (2003) found that the relationship between the spread and the determined expected loss is best explained through a log-linear function. Furthermore, Lane and Mahul (2008) assume a linear relationship between those two factors.

Conclusion:

The Bond Pricing approach was not chosen for the development of our model, since it posses several challenges, being the most relevant the inclusion of catastrophe risk. Usually a portfolio which replicates this payment structure cannot be found. Thus, the market for Cat Instruments is incomplete and no unique equivalent martingale measure exists. The former causes that a unique price cannot be derived. Completeness of the market is important because in an incomplete market there are a multitude of possible prices for an asset corresponding to different risk-neutral measures.

Apart from that, the modeling of the catastrophic event remains a challenge. Furthermore, the cash flows of the Cat Bond are considered to depend only on catastrophe risk variables, which is not necessarily true.

On the other hand, the indifference pricing models are do not fit under the structure of the Cat Bonds market. Since the market for Cat Bonds lacks transparency for those privately placed bonds, and every contract is individually designed, it is almost impossible to verify the use of the pricing models by empirical analysis. Therefore, the construction of a utility function is challenging and lacks an adequate information input.

Therefore, we chose to follow a premium calculation approach, since it focuses on determining the spread paid to investors, which is ultimately the variable were risk and return converges. Moreover it has a comprehensive vision, in which assumes the spread depends not only on the expected loss but on a risk premium, which is open to authors to define how it is measured.

Such risk premium, as is shown later in our research, depends on several factors which should not be overlook, since these types of securities have complex pricing dynamics, and investors make decisions based upon different elements they take into account, and translate after in their expected coupon rate.

Part III

CAT BONDS' SPREAD IN THE PRIMARY MARKET –

CASE STUDY AND RESULTS

3.1 Background

The recent boom of Cat Bonds has created an increasing need for understanding their pricing dynamics in the primary market. Thus, on the last two decades a stream of authors have surged, developing and proposing numerous valuation approaches. The most common valuation approaches developed are based on a simple linear model, which basically consists in expressing the spread as a multiple of the excepted loss attached to the Cat Bond. Further insights have replaced expected loss with other volatility metrics or risk measures, associated to the bond.

A relevant example in the literature is the linear model proposed by Bodoff and Gan (2009), in which the spread is a function of the expected loss, multiplied by a risk factor that depends on the perils covered by Cat Bond. Nevertheless, the lack of other relevant factors included in the calculation of the spread, resulted on a model with a low consistency for different periods of time.

We took all public registers of the Cat Bonds issued between 1998 and 2008 and replicated Bodoff and Gan (2009) methodology, to finally conclude about the relevance of their results across different periods of time. We replicated the coefficients for every peril defined by the authors, and assessed them in different periods, concluding about their robustness according to the Wald test results.

Table 2: Robustness evaluated to the linear model proposed by Bodoff and Gan (2009)

	Bodoff and Gan (2009) 1998-2008	1997-2013	Bodoff and Gan (2009) 2006-2007	2011-2012
Additional Constant $_{peak}$	1.280 [0.000]	0.023* [0.000]	2.310 [0.000]	0.018* [0.298]
Additional Constant $_{diversifying}$	-1.090 [0.002]	-0.002* [0.761]	-1.660 [0.001]	-0.006* [0.849]
Loss Multiplier $_{EQ}$	1.600 [0.000]	1.403 [0.000]	1.870 [0.000]	0.595 [0.583]
Loss Multiplier $_{Wind}$	2.290 [0.000]	1.169* [0.000]	2.310 [0.000]	1.065 [0.116]

*Significantly different (Wald Test) against Bodoff and Gan (2009). α=0.05
[ ]: p-value

Table 2 shows the results of 4 variables assessed by Bodoff and Gan (2009) for the periods of 1998-2008 (complete sample) and the 2006-2007 subsample. The latter was made in order to reflect the changes in variables when the reinsurance market undergoes a hard cycle, which happens after large catastrophes occurred, being the case in 2006 after hurricane Katrina.

We have replicated those coefficients for other periods of time, being 1997-2013 and the hard market cycle of 2011-2012, for comparative purposes. We found that for the complete sample, the only coefficient equal to the one founded by Bodoff and Gan, belong to the Loss Multiplier of Earthquake, since Cat Bonds with earthquake coverage have proved historically to be more stable both in their expected loss and in their spread, with a ratio of Spread over expected loss between 1.4 and 19. Wind covered Cat Bonds on the other hand, exhibit a more volatile historical spread over expected loss ratio ranging between 1.2 and 71.

For the subsample comprising hard market cycles (2006-2007 Bodoff and Gan versus 2011-2012), Loss Multipliers for Earthquake and Wind are equal; this is explained by the direct impact those periods have on Cat Bonds spreads. However, for all periods the Additional Constants acting as the intercept in their lineal model are significantly

different, affecting their output of the estimated spread.

Results suggest that a linear simple model, in which a Cat Bond price relying solely on the expected loss or any other risk measure, is inaccurate in predicting the spread dynamics across different periods of time. Hence, it becomes quite important to develop a multifactor pricing model that incorporates other determinants in the spread, and with a robust fit that remains unchallenged across any period of time.

3.2 Data

For the purpose of this research, a data base with the primary market information of Cat Bonds was constructed. Between 1997 and 2013, 248 new Cat Bonds were issued to the market. However, only 194 registers remained after taking out those lacking information, due to unreliable data sources (41) or for being private placements (13).

Given the lack of structured information about this type of financial assets, the use of several sources was needed. The data base was constructed from information available at Artemis, Standard & Poor's, Aon Benfield, Guy Carpenter and Lane Financial. The database has the basic description for each bond, such as sponsor, month and year of issue, perils and zones covered, triggering type, expected loss, coupon, and in some cases the credit rating given by a rating agency.

The coupon is always expressed as a floating interest rate such as Libor, Treasury yield, etc, plus a fixed spread. The floating component reflects the financial income generated by the collateral account where all proceeds from the bond sale are usually kept at a risk free rate. The spread, on the other hand, is the premium paid by the sponsor to investors. There are two types of floating rates in Cat Bonds: Libor and Treasury Funds, however, due to the high correlation among them, and the fact that a higher number of Cat Bonds are Libor denominated, we have decided to work solely over the base of Libor.

Additionally, the rating of the bond is given by a rating agency (S&P, Moody's, or

Fitch), based on an assessment regarding the following criteria[2]: The likelihood of a reduction in the principal amount of the bonds, or attachment probability. The issuer credit rating or financial strength rating of the Sponsor. And, the credit assessment on the collateral held in the trust account.

From the database information, a brief analysis was made in order to characterize variables and the relationship between them.

Figure 5: Expected Loss Vs Spread in the primary market
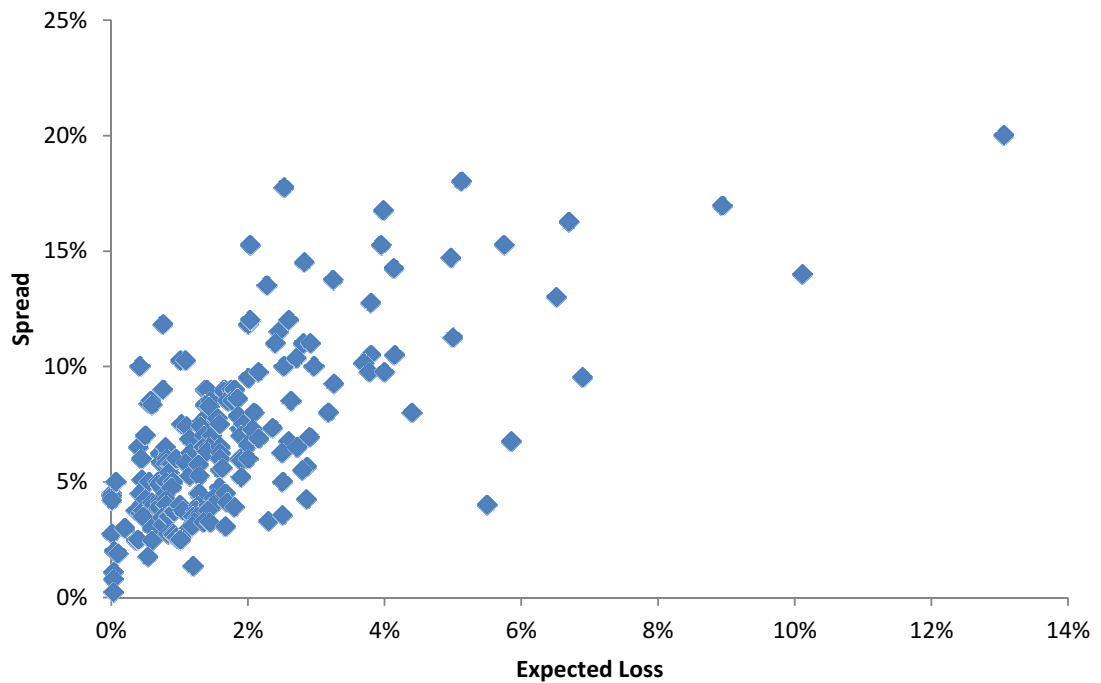
Source: Database constructed by the author.

When plotting the expected loss and spread for each bond, there is a marked concentration of Cat Bonds having an expected loss between 0% and 2%, and a respective spread between 2.5% and 7.5%. The number of bonds exceeding a 4% of expected losses decreases significantly, and only the 3 bonds (1.6%) exhibit in table 3, reported expected loss above 8%.

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[2]Standard & Poor's. Rating Natural Peril Catastrophe Bonds: Methodology and Assumptions. 2013

Table 3: Cat Bonds with the highest Expected Loss

Issuer/SPR:	Cedent / Sponsor:	Zones Covered	Perils covered:	Date issue:	Exp. Loss	Coup	Rat.
Residential Reinsurance 2013 Ltd. (Series 2013-2)	USAA	U.S	Multiperil	Dec-13	13.06%	20.00%	BB
Successor X Ltd.	Swiss Re	U.S	Multiperil	Nov-09	10.12%	14.00%	B-
Successor X Ltd. (Series 2010-1)	Swiss Re	U.S Europe Japan	Multiperil	Mar-10	8.94%	16.95%	B-

Source: Database constructed by the author.

Now, following the former analysis, a ratio to measure the times an investor is rewarded for each unit of risk taken. The ratio is compounded as the Spread over the Expected loss, showing therefore for each bond the relationship between risk and return.

Figure 6: Compensation Ratio of Spread over Expected Expected Loss
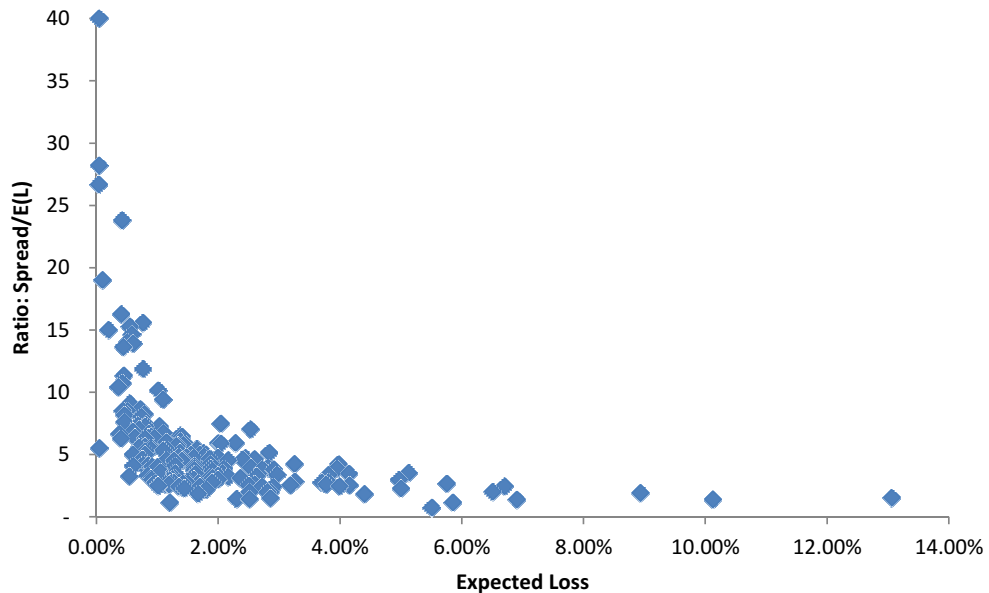
*Excluding the registers for which the ratio is higher than 100%

Source: Database constructed by the author.

Figure 6 shows the relationship of Expected Loss and the ratio (spread/expected loss), after excluding 4 observations considered as outliers, since their ratio was above 100,

which means that the spread is more than 100 times the expected loss. The perils of 3 of those bonds cover medical benefits claims, and the other one has a multiperil coverage for the U.S. The ratio on those bonds seems excessively high as a consequence of very small expected losses. Those 4 bonds are described in table 4.

Table 4: Cat Bonds with Spread over Expected Loss above 100

Issuer / SPV:	Cedent / Sponsor:	Zones Covered	Perils covered:	Date issue:	Exp. Loss	Coup	Ratio: Spread/ Exp. Loss
Combine Re Ltd. (Series 2012-1)	Swiss Re America	U.S	Multiperil	Mar-12	0.01%	4.50%	450
Vitality Re II Ltd.	Aetna Life Insurance Company	U.S	Mortality	Apr-11	0.01%	4.40%	440
Vitality Re III Ltd. (Series 2012-1)	Aetna Life Insurance Company	U.S	Mortality	Jan-12	0.01%	4.20%	420
Vitality Re IV Ltd. (Series 2013-1)	Aetna Life Insurance Company	U.S	Mortality	Jan-13	0.01%	2.75%	275

Source: Database constructed by the author.

Finally, it becomes relevant assessing the above characteristics according to the type of trigger defined for each bond. The trigger types can be divided on Indemnity, Index (Industry loss, Modelled Loss or Parametric) and Hybrid.

Figure 7: Cat Bonds' characterization according to Trigger Type
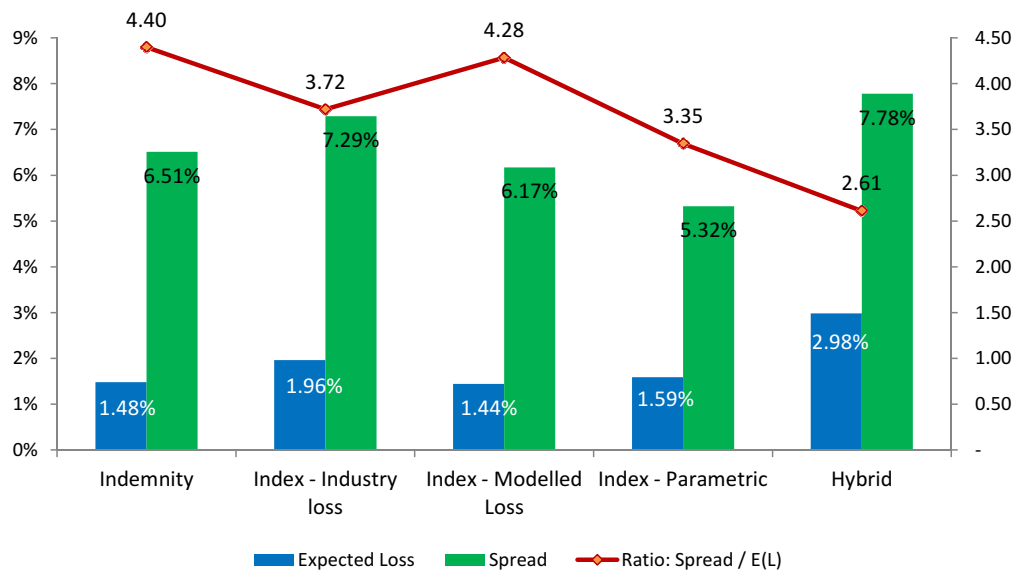
Source: Database constructed by the author.

The higher expected loss and spreads, are both given on average, by those Cat Bonds with a Hybrid Trigger, which is natural since the probabilities of having the bond set by the trigger are higher.

Now, the lowest average expected loss and spreads are given by those with a Modelled Loss trigger. The above confirms the clear relationship existent between the expected loss of a bond and its spread.

The highest ratio Spread over Expected Loss, is generated by those Cat bonds with an indemnity trigger, and on average is as high as 4.4 times. On the other hand, the lowest ratio is found for the hybrid trigger bonds, and is as low as 2.6 times.

3.3 Explanatory Variables

There has been an extensive research and several proposed models about the role of the Expected Loss on a Cat Bond's price. Nonetheless, as Braun (2012) states, there are other significant variables affecting the Cat Bonds' spread, and which are not yet fully understood. Therefore, our initial model is based on the inclusion of several internal and external variables as significant for explaining the spread. Those variables are the following:

1. **Expected Loss:** Empirical evidence in proving this factor as relevant for the spread has already been provided in several studies (Bodoff and Gan 2009, Lane and Mahul, 2008; Dieckmann, 2009; Galeotti et al., 2013). The expected loss is proved to be one of the most relevant factors in a Cat Bond price, since it represents the risk undertaken by investors, for which in return they expect compensation, translated on the spread.

    The probability of expected loss is calculated based on risk metrics prepared by modeling agencies, which simulate different catastrophic events worldwide in order to evaluate their impacts over a Cat Bond, according to the perils and zones covered, and trigger type.

2. **Zones covered:** As explored by Bodoff and Gan (2009), the zones covered by a Cat Bond comprise different levels of risk, depending to their probabilities of incurring into losses. Therefore, it becomes relevant to assess whether the type of zones is a determinant factor in the pricing of such instruments. USA, Europe, Mexico, Japan, and Multizone, follows an initial classification proposed by Braun (2012) and complemented by Bodoff and Gan (2009).

3. **Perils Covered:** Bodoff and Gan (2009) focus their research exclusively on those Cat Bonds with single peril coverage. However, due to the increasing number of multi-peril issuances and the fact that those types of Cat Bonds represent a higher risk for investors, we have included them in our classification. The types of perils used for the purpose of this paper are Earthquake, Wind, Mortality, Multi-peril, and Others.

Furthermore, most authors have focused only on those Cat Bonds covering Property and Casualty (P&C) perils for their valuation analysis. However, Life Cat Bonds with mortality coverage have been in the market since 2003, when the Swiss Re first issued one of those. Therefore, we consider relevant to include them in our analysis, in order to reach to a more general conclusion on both the P&C and Life market.

4. **Triggering Type:** Unlike the corporate and government bonds, CAT bonds do not have an underlying asset. They are priced and structured according to the events covered, which will determine the cash flow of the investor in case of occurrence. Those events are known as triggering events, and according to Cummins (2008) CAT bonds are classified under the following 5 triggering categories: Indemnity, Industry Loss Index, Modeled Loss Index, Parametric Index, and Hybrid.

   The triggering types were created in order to better meet the sponsors' need of coverage and risk transfer. According to Cummins (2008), "Index triggers tend to be favored by investors because they minimize the problem of moral hazard" The estimation of losses for the indemnity trigger could have misleading information for investors, charging them with higher losses than the actual incurred. Therefore, an index publicly known and calculated by a third party, significantly increases transparency.

   "Indices also have the advantage of being measurable more quickly after the event than indemnity triggers, so that the sponsor receives payment under the bond more quickly" (Cummins 2008). Their disadvantage lies on the risk faced by the sponsor of mismatching the funds received with the capital needed for paying the catastrophe incurred. Investors are aware of the differences that each triggering type embraces, having consequently different preferences and perceptions of risk. Therefore, the triggering type of a Cat Bond should be a relevant variable in determining its spread.

5. **Credit Rating:** The rating of the bond is given by a rating agency (S&P, Moody's, Fitch), based on an assessment of a standard criteria. We converted all ratings to the S&P's scale, and used a dummy for those bonds lying above and below the investment grade, which is "Considered as the highest speculative

grade by market participants." (S&P).

"Obtaining a financial rating is a critical step in issuing a Cat Bond because buyers use ratings to compare yields on Cat Bonds with other corporate securities. Consequently, almost all bonds are issued with financial ratings." (Cummins 2008). Most Cat Bonds are rated below investment grade, since the rating rationale focuses on likelihood of losing the investment, rather than on the financial creditworthiness of the sponsor. "Because Cat Bonds are fully collateralized, Cat Bond ratings tend to be determined by the probability that the bond principal will be hit by a triggering event. Thus, the bond ratings merely indicate the layer of catastrophic-risk coverage that is being provided by the bonds." (Cummins 2008)

We have defined 3 types of credit ratings: Investment grade (rated above or on BBB-), non investment grade (rated below or on BB+) and not rated.

6. **BB- Bonds Index:** As mentioned before, most Cat Bonds are rated below the investment grade, since the probability to be hit by its triggering event is usually uncertain. Therefore, investors can use the yield on other low rated securities as a benchmark for the spread they must expect from Cat Bonds. Braun (2012) explains the importance of capturing the influence of the corporate bond market in the Cat Bonds behavior, since "the vast majority of Cat Bonds exhibit a BB rating".

    Therefore, we have chosen the Credit Suisse's high yield II Index as a good proxy for such purpose, since it has a long historical trading, consistent with our database observations. The index measures the performance (price movements) of high-yield U.S. dollar-denominated bonds with a lowest credit rating that falls on or between BB+/Ba1/BB+ and BB-/Ba3/BB- according to the scales used by S&P, Moody's and Fitch respectively.

7. **Interest Rate:** Most authors tend to remove the floating rate component of the coupon, from their pricing assessment of Cat Bonds, choosing instead to focus on variables with a more direct effect over the spread. Nonetheless, as shown by Jin-Ping and Min-Teh (2002), "interest rate risk affects Cat Bond prices" so it is logical and necessary to include it in our research.

The proceeds from the sale of the Cat Bond are deposited on a segregated collateral account (trust account), in order to safely invest those funds and yield returns over them. That account is usually forced to be invested in money market instruments with risk free rates, since those funds represent the collateral of the bonds.

"The fixed returns on the securities held in the trust are usually swapped for floating returns based on London interbank offered rate (LIBOR) or some other widely accepted index. The reason for the swap is to immunize the insurer and the investors from interest rate (mark-to market) risk and also default risk. The investors receive LIBOR plus the risk premium in return for providing capital to the trust." (Cummins 2008).

Despite of few bonds having their coupon expressed in Treasury Bills, we have decided to work with the Libor as a proxy for the risk free rate of all observations, given its high correlation with T-bills and under the premise that Libor is a more accessible rate for investors. We used the respective 3-month USD denominated Libor at the time of issuance, since as 3 months is the usual periodicity for the coupon payments, is also the preferred investment term for the SPV (Special Purpose Vehicle) to have their funds in, according to Tower Watson (2010).

8. **Rate on Line (ROL) Index:** Researchers and industry experts agree that the reinsurance business is subject to a cycle of soft and hard market. "It is well known that reinsurance markets undergo alternating periods of soft markets, when prices are relatively low and coverage is readily available and hard markets, when prices are high and coverage supply is restricted." (Cummins and Weiss 2009). Cat Bonds as a direct substitute for traditional reinsurance contracts, should under no-arbitrage conditions, have a direct correlation with the reinsurance market movements.

Reinsurance broker Guy Carpenter, part of the Marsh & McLean Group, publishes annually a Property Catastrophe Rate on Line (ROL) at the begging of each year. The ROL is a percentage derived from taking a reinsurance contract and dividing its premium by the coverage limit. Guy carpenter compounds the

index every January with the average annual renewals from the biggest reinsurers located in Europe, The United States and The United Kingdom. A Rise in the index reflects a hard market stage, in which the policyholders must pay higher premium for the same coverage.

Even though there are country specific Rate on Line for major countries like the United States, Japan, and United Kingdom, we have decided to work with the global ROL, according to the multi-zone coverage offered by several Cat Bonds. Additionally, public data corresponding to country specific ROL is rather scarce, and there is a high correlation of individual ROLs with the global index.

Therefore, we expect that Cat Bonds spread rises with the reinsurance premiums. That should happen because the sponsor pays the SPV an initial premium for the protection, which is equivalent to a reinsurance premium. Therefore a rise in the traditional reinsurance market should be translated into a higher premium paid by the sponsor, expressed as a higher spread.

Lane and Mahul (2008) were the first to highlight the relevance of the underwriting cycle in the pricing of Cat Bonds. After them other authors like Jaeger et al (2010) and Braun (2012), have also included the ROL Index in their assessment of the returns in Cat Bonds and ILS respectively.

Classification of the Cat Bonds in the sample according to covered zone, peril, triggering type, and credit rating. For each category the number and percentage of observations as well as their average spread, and expected loss are provided, as shown in table 5.

Table 5: Descriptive Statistics for the dummy variables of Cat Bonds in the Sample

	No. Observations	Percentage	Av. Expected Loss	Av. Spread
Zones Covered				
U.S	118	61%	1.77%	7.27%
Europe	21	11%	1.77%	3.85%
Mexico	3	2%	2.62%	7.33%
Japan	17	9%	1.03%	4.08%
Multizone*	35	18%	2.42%	8.00%
	194	100%		
Perils Covered				
Earthquake	45	23%	1.26%	5.48%
Wind	83	43%	1.72%	6.04%
Mortality	5	3%	0.55%	1.34%
Multiperil	54	28%	2.78%	9.67%
Others*	7	4%	0.64%	4.87%
	194	100%		
Triggering Type				
Indemnity	58	30%	1.67%	6.75%
Industry Loss Index	75	39%	1.85%	7.33%
Modeled Loss Index	10	5%	1.41%	7.20%
Parametric	40	21%	1.62%	4.95%
Hybrid*	11	6%	3.83%	9.00%
	194			
Credit Rating				
Investment	14	7%	0.66%	3.91%
Non Investment	145	75%	1.97%	6.99%
Not Rated*	35	18%	1.76%	6.90%
*Base Dummy	194			

Source: Database constructed by the author.

3.4 Multifactor Spread Model

We run a series of OLS regressions for our 194 observations data base, in order to identify additional variables affecting the spread of Cat Bonds in the primary market. To the best of our knowledge, a model including all 8 variables has not been contemplated in the literature before.

We use a 90% confidence interval to identify those variables with a significant impact in the spread of Cat Bonds. Later, heteroscedasticity was considered to draw more accurate conclusions from the results obtained through OLS. Finally, we evaluated the results under different periods of time, to have a more robust and a sounder model. We used EViews® to run the regressions and perform further analysis.

3.4.1 Model Specification

The final expression for calculating the spread according to the results from the regressions is:

$$Spread_i = \alpha + \beta_{EL} * EL_i + \beta_{EUR} * EUR_i + \beta_{JP} * JP_i + \beta_{Mort} * Mort_i + \beta_{MP} * MP_i +$$
$$\beta_{Ind} * Ind_i + \beta_{IL} * IL_i + \beta_{ML} * ML_i + \beta_{Inv} * Inv_i + \beta_{HY} * HY_i + \beta_{Libor} * Libor_i +$$
$$\beta_{ROL} * ROL_i + e_i$$

where:

α= Constant
EL_i= Expected loss probability attached to every bond.
EUR_i= Dummy variable representing those Cat Bonds which zone covered is Europe.
JP_i= Dummy variable representing those Cat Bonds which zone covered is Japan.
$Mort_i$= Dummy variable representing Mortality as the peril covered.
MP_i= Dummy variable representing those Cat Bonds with a Multi-peril coverage.
Ind_i= Dummy variable representing those Cat Bonds which trigger type is Indemnity.
IL_i= Dummy variable representing those Cat Bonds which trigger type is Industry Loss.

$ML_i=$ Dummy variable representing those Cat Bonds which trigger type is Modeled Loss.

$Inv_i=$ Dummy variable representing Cat Bonds with an investment grade rating.

$HY_i=$ Monthly High Yield Index.

$Libor_i=$ Monthly US denominated 3-month Libor.

$ROL_i=$ Annually Rate on Line (ROL).

$e_i=$ Random error term for Cat Bond i.

Table 6: Determinants of the Spread for Cat Bonds in the Primary Market

Variable	Coefficient	Std. Error	t-Statistic	Prob.	
Constant	0.03555	0.015	2.322	0.021	
Exp. loss	1.23032	0.092	13.395	0.000	
Europe	-0.02467	0.005	-4.684	0.000	
Japan	-0.01277	0.006	-2.230	0.027	
Mortality	-0.04188	0.010	-4.377	0.000	
Multiperil	0.01698	0.004	4.330	0.000	
Indemnity	0.00996	0.005	2.168	0.032	
Industry_Loss	0.01622	0.004	3.870	0.000	
Modeled_Loss	0.01672	0.007	2.273	0.024	
Investment	-0.01136	0.006	-1.876	0.062	*
High_Yield	-0.00003	0.000	-3.253	0.001	
Libor	-0.00188	0.001	-1.896	0.060	*
ROL	0.00011	0.000	2.044	0.042	

* Significant at a 10% confidence level

F-Statistic = 37.3462   Prob (F-Statistic)=0.000

3.4.2   Results

The findings show 12 specific significant factors contributing to the spread of Cat Bonds. As explained above, the factor with the highest coefficient is Expected Loss, because according to its definition is the variable with the explicit risk value associated to the Cat Bond.

Regarding the zones covered, both Europe and Japan are significant with negative coefficients. That is explained by the diversification contribution of those zones, according to their classification giving by Bodoff and Gan (2009), as non-peak zones. On the other hand, Mortality and Multi-peril resulted as the only significant perils. Mortality has a negative coefficient since it works as a diversifying factor, being uncorrelated to those risks covered in the P&C market. Multi-peril however is positive, since it embedded a higher risk from a more extensive coverage.

The triggering types Indemnity, Industry Loss and Modeled Loss have all positive contributions to the spread, reflecting an increased perception of risk by investors, who demand higher returns in Cat Bonds with these types of triggers than for Parametric or Hybrid triggers. A Parametric trigger is represented by a physical condition related to the catastrophic event, such as the wind speed in a hurricane, or the magnitude of an earthquake. Therefore, according to Finken et al (2009), parametric Cat Bonds are insensitive to the sponsor's risk portfolio, and the terms of the bond are not subject to adverse selection. Additionally, the triggering terms of Hybrid Cat Bonds tend to be structured under a set of conditions, being usually harder for the bond to get triggered.

Credit rating, as expected, is a determinant factor, with a negative coefficient, since bonds with an investment grade rating, involve by definition, a lower risk to investors. The high yield index proved to be significant as well, with a negative coefficient following the rationale that the higher the index (price) the lower the interest rate of those financial assets comprising the index. Furthermore, since Cat Bonds are considered high yield bonds, their return should behave in the same way, showing a fall on their spread. According to Braun (2012) "since many fixed-income investors perceive securities with the same rating to carry identical risks, there could be contagion effects that give rise to a dependence of Cat Bond on corporate bond spreads."

Interest rate (i.e. Libor) proved to have a negative impact in the spread. This is explained by the fact that the higher the Libor the higher the yield the SPV will get on the proceeds from the issuance. Therefore, the additional rate (spread), which must be provided by the sponsor to further compensate investors for the risk taken, can be lower without affecting the coupon rate offered to investors.

Finally, the proxy defined to measure the reinsurance market stage (ROL), proved to have a positive incidence on the spread. A rise in it represents an increase in the reinsurance market rates, and as Cat Bonds are a substitute to reinsurance, as market rates rise, the spread of Cat Bonds should also increase.

3.4.3 Statistical Assessment of Results

In this section, we provide a variety of statistical assessments regarding the regression run, and the final model results. We start by analyzing statistically the residuals of the regression, which at first sight appear to have normal distribution, both graphically and numerically, based on a mean of virtually zero, skewness of 0.53 and kurtosis of 3.48. However, Jarque-Bera's null hypothesis ($H_0 = The\ distribution\ is\ normal$) is rejected at a 95% confidence level, concluding a real lack of normality in the data. However, in light of the Central Limit Theorem, this fact does not affect the consistency of the results.

Figure 8: Statistical characteristics of Residuals
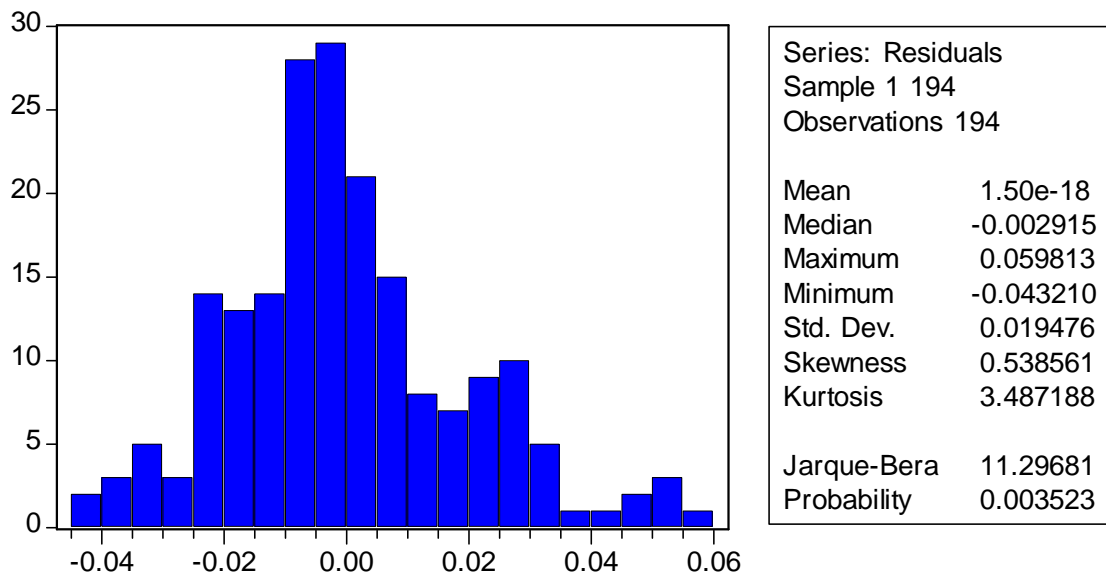
Source: EViews regression output.

The asymmetry on the results responds to a couple of outliers, recognized from the residuals. Only one significant observation was removed from the sample, correspond-

ing to the Everglades Re Cat Bond issued by Citizens Property Insurance Corp, in April 2012, for the coverage of Florida Hurricanes, with an expected loss of 2.53% and a Spread of 17.75%[3]. It is atypical since the average Spread of the Cat Bonds issued between 1997 and 2013 is only 6.8%, with an average expected loss of 1.8%.

Residuals present a high variation outside the confidence interval. After a careful analysis we concluded there is no specific trend identified on those observations, being the high variation of residuals random, as a response to the volatile nature of Cat Bonds. Furthermore, since an OLS approach in cross section data was used, it becomes relevant the evaluation of the homoscedasticity in our sample. A standard and general methodology used to evaluate heteroscedasticity is done through the White Test. The Results for our sample yielded an F-statistic p-value of 0.33, which according to a 95% confidence level, does not lead to reject the null hypothesis. Thus, we can conclude that the residuals from the regression are homoscedastic, being the regression's results valid under an OLS approach.
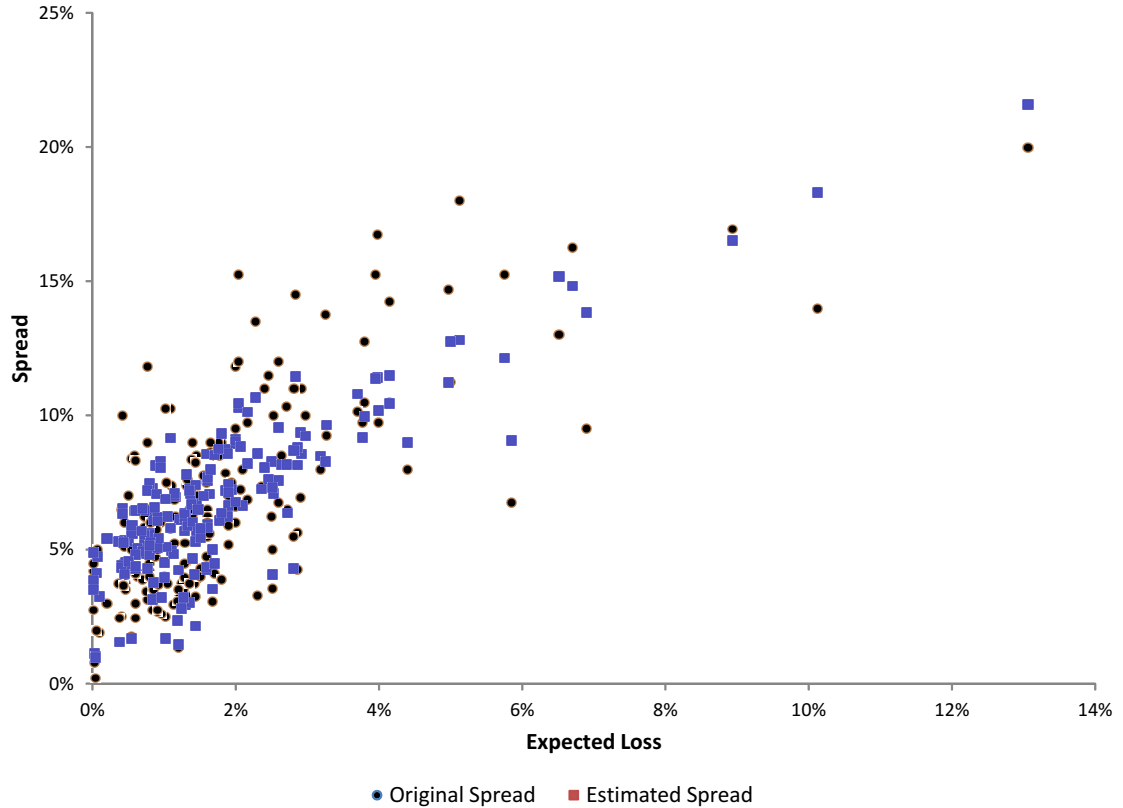
3.4.4 Robustness

The multifactor spread model proposed was evaluated against the original spreads, drawing conclusions about the accuracy of the model, and its fitness with the actual real data. Results proved an absolute average deviation of 1.54% from the original spread.

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[3]AON Benfield. Insurance-Linked Securities 2012: Second Quarter Update

Figure 9: Original Vs Estimated Spread applied to the database (194 observations)

Besides of an accurate fit with the original data, a reliable model has to be robust regardless of the period of time. Therefore, to assess our model robustness and validity, we have defined 2 subsamples to run the model on, according to the definition of *hard market cycle* for the reinsurance industry.

Such stage of the cycle occurs after a massive natural's catastrophe happens, in which the reinsurance market undergoes a period of hardening rates and lower capacity. According to Guy Carpenter (2013), the last 3 periods of hard market were characterized by "demand driven in response to capacity needs and market hardening following major catastrophe events such as WTC (2001), Katrina (2005) and the New Zealand/Tohuku Earthquakes (2011)".

Due to the low volume of Cat Bonds issued between the hard market cycle comprising 2002 and 2003, we have defined for the purpose of this paper, the following two subsam-

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ples: January 2006 – December 2009, when the reinsurance market experienced a hard stage, as well as well as the financial turmoil triggered by the subprime mortgage crisis took place. Also the period of January 2012-December 2013, in which a hard reinsurance market cycle occurred, after the costly catastrophes from Japan's Earthquake and Tsunami in 2011.

The results were evaluated under the Wald test, suggesting that for each one of the subsamples evaluated most of the variables' coefficients are equal to those from the original regression.

Table 7: Wald Test's results for two defined subsamples

	Original Coefficients	2006-2009		2012-2013	
		Coefficient	Wald Test	Coefficient	Wald Test
Expected Loss	1.23032	0.86920	**	1.28955	
Europe	-0.02467	-0.02879		-0.01983	
Japan	-0.01277	-0.02089		0.00298	
Mortality	-0.04188	-0.04622		-0.01954	
Multiperil	0.01698	0.01189		0.01067	
Indemnity	0.00996	0.00436		0.01013	
Industry Loss Index	0.01622	0.01291		0.01215	
Modeled Loss Index	0.01672	0.04939	*	-0.02744	***
Investment	-0.01136	-0.01427		-0.02141	
High Yield Bond Index	-0.00003	-0.00011		0.00002	
Libor	-0.00188	-0.00296		0.11078	
ROL	0.00011	0.00004		0.00586	

*Wald Test is rejected at 10%

**Wald Test is rejected at 5%

**Wald Test is rejected at 1%

For the period 2006 - 2009, the coefficients of Expected Loss and Modeled Loss Index are significantly different from those of the multifactor spread model. For the period 2012 - 2013, the only not significant variable is Modeled Loss Index. The reason why the mentioned factors do not remain stable during the years when the reinsurance market is on its hard cycle, could be an area for further research.

From the results, we conclude that the multifactor spread model has an adequate robustness regarding its validity across different periods of time, during which it remains sound, regardless the volatile market conditions. The variables significantly different may indicate that there are some factors with a heavier weight on the spread, as a consequence of atypical market conditions, changing investors' preferences and their risk aversion to Cat Bonds.

Part IV

CAT BONDS' SPREAD IN THE SECONDARY MARKET –

CASE STUDY AND RESULTS

4.1   Background

Given the relevance that Cat Bonds are taking in the financial markets, as well as their appeal for different types of investors, it becomes relevant to understand the price dynamics of Cat Bonds in the secondary market.

Previous authors focusing on valuing Cat Bonds in the secondary market, have concentrated on modeling the occurrence of catastrophic events in order to value the bond as a function of the expected probability of loss. Cox and Pedersen (1997) propose an expression in which catastrophe reinsurance premium is expressed as a high-yield bond, depending on the risk free rate, the probability of a catastrophe (default) and an amount to be reinsured. The authors use stochastic probability and construct a binomial tree approach for pricing Cat Bonds. Their methodology is supported by an intuitive and theoretical basis, and no empirical results were contrasted. Finally, the lack of inclusion of other factors (others than the probability of a catastrophe) that are priced by investors, narrows the pricing drivers' scope of Cat Bonds, leading us to seek alternative approaches for this research.

Other authors instead, have taken the Poisson distribution as the basis for explaining the behavior of a catastrophic event. That is the case of Baryshnikov et al (2001), who affirm that some catastrophic events have a power law distribution, for which they develop a model using the Poisson distribution. Jin-Ping and Min-Teh (2002), also use a Poisson distribution for modeling catastrophe probabilities. Their main innovation is the assessment of how interest rates affect the Cat Bond price, and offer a solution for modeling them stochastically.

Loubergé et al (1999) on the other side proposed a valuation based on the resemblance

on financial options. They develop a valuation methodology exclusively for those Cat Bonds with an Industry Loss Index. They analyze Cat Bonds as financial portfolios combining a straight bond and catastrophe options. "Using option pricing theory and simulation analysis in a stochastic interest rate environment, we show that investors attracted by the potential for diversification benefits should not overlook the optional features when including these securities in an asset portfolio."

As previously mentioned, authors have focused on the probability of occurrence of catastrophic events, as the exclusive variable in determining the pricing of Cat bonds in the secondary market. We propose a multifactor model proving that several factors are relevant to the bond's spread.

4.2   Data

Due to the recent appearance of Cat Bonds, and the fact that all investors that currently invest in them are institutional, the liquidity of these kind of assets is rather low. There is not a price marked in the market, as there are for other high liquid assets. Instead, financial agencies and reinsurance brokers, gather information from the transactions made, to create an indicative market price for every cat bond outstanding.

The information used in this research was taken from the information provided by Lane Financial L.L.C in their Annual Review for the four quarters, Q2 2012 to Q1 2013. They provide secondary market prices of Cat bonds, in a quarterly basis. Every public outstanding Cat bond has an average market indication of its spread for each of the 4 quarters taken.

Information from Lane Financial L.L.C was used for constructing a data base with 324 observations, corresponding to 81 Cat Bonds outstanding from June 2012, through March 2013. Each Cat bond has an indicative market spread for 4 periods, being the database a temporary sample of the secondary market of Cat bonds.
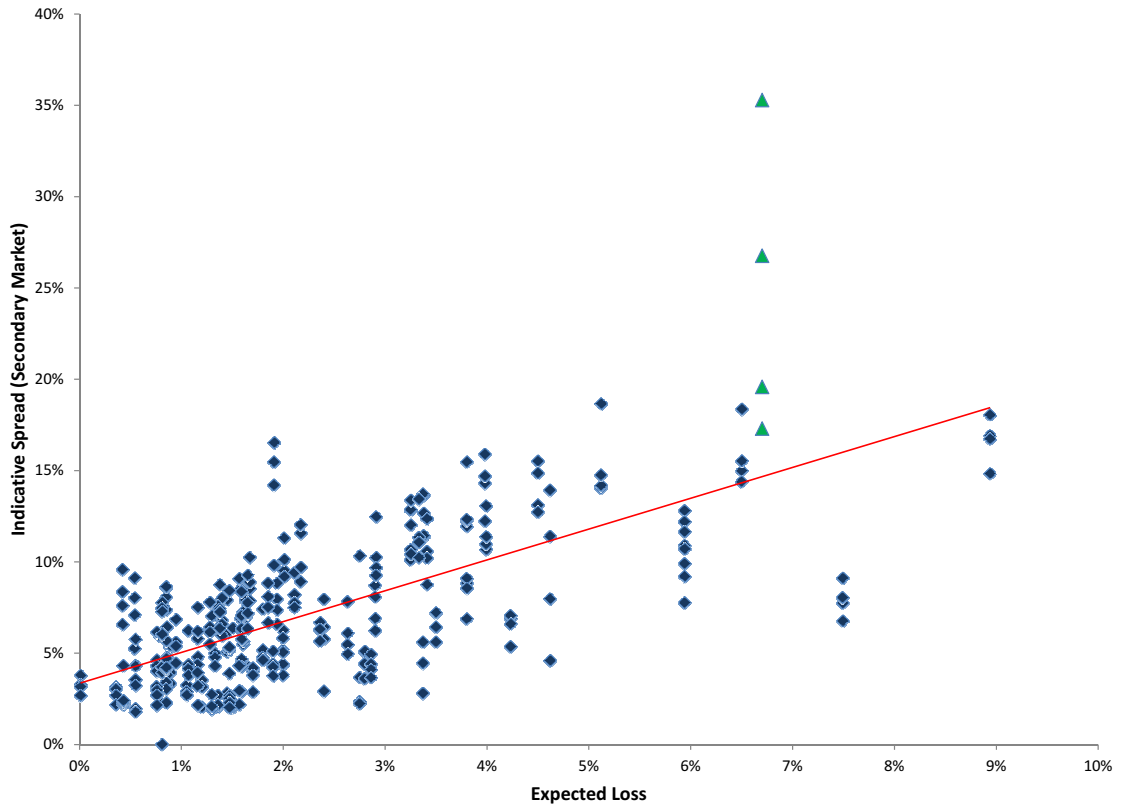
Figure 10: Expected Loss Vs Indicative Spread

Figure 10 shows a clear lineal path around which the Expected Loss and Indicative Spread of Cat Bonds interact. There is one markedly outlier observed, corresponding to Successor X-Class V F4, a Swiss Re Cat Bond issued in November 2011. The bond was issued with an expected Loss of 6.7% and Spread of 16.25%, which is a value within normal ranges compared to other Cat Bonds with similar Expected Loss. However, in the secondary market the Bond experienced a high volatility in its indicative spread, being 19.6% in June 2012, 17.3% in September 2012, followed by a steep rise of 26.8% in December 2012 and an outstanding 35.3% in March 2013.

The bond's volatility was caused by the occurrence of Hurricane Sandy in October 2012, threatening the trigger to be set. The Cat Bond covers Wind in the United States and Europe, for which after the Hurricane Sandy, investors speculated on whether the losses incurred would be high enough to set the trigger, losing the principal and remaining coupons. Such speculative period cause investors to perceive a higher risk, causing the

spread in the secondary market to trade above twice its initial spread only 16 months later.

4.3 Methodology

Taking into account our objective of assessing different variables to identify those determinants in the secondary spread of Cat Bonds, we have decided to use panel data as the methodological approach.

Panel data is defined as multi-dimensional data in which variables are observed for each individual, across several points in time. A panel has the following form:

$$X_{it} \quad i = 1, \ldots, N \quad t = 1, \ldots, T$$

Where i is the individual dimension and is the time dimension.

The structure of panel data typically is that the number of cross-section units (N) is large, and the number of time periods (T) over which individuals are observed is rather small.

Before exploring Panel Data it is worth noting that the simplest case of using longitudinal data arises from ignoring the panel structure of the data, represented by the following equation:

$$y = \beta x + e$$

The simplicity of this model comes from the following assumption about the error term:

$$e_{it} \sim iid \left(0, \sigma^2\right)$$

That is, for a given x, there is no serial correlation between observations and, furthermore, errors are not heteroscedastic.

However, ignoring the panel structure of the data by assuming that the error terms are independent and identically distributed (iid) leads to results that are not appropriate in many cases. For determining whether the best option for the model is OLS or Panel data, one can apply the Breusch- Pagan Test.[4]

Breusch-Pagan is a Lagrange-Multiplier test that evaluates the non existence of unobserved heterogeneity, i.e. whether the pooled OLS is an appropriate model or not. It was developed by Breusch and Pagan (1980) and is based on the following hypotheses:

$$H_0 : \sigma_c^2 = 0$$

Where σ_c is the variance of the unobserved heterogeneity in any point.

The following Lagrange multiplier (LM) yields the test statistic for the Breusch–Pagan test:

$$LM = \frac{nT}{2(T-1)} \left[\frac{\sum_{i=1}^{n}(T^- e_i)^2}{\sum_{i=1}^{n}\sum_{t=1}^{T} e_{it}^2} - 1 \right]^2 \sim \chi_1^2$$

Under the null hypothesis, the statistic is distributed as a chi-squared with one degree of freedom.

In the case there is unobserved heterogeneity, we must work with panel data, which model is expressed as follows:
$$y_{it} = \beta x_{it} + e_{it}$$

Where the structure of the error term is as follows:

$$e_{it} = C_i + U_{it}$$

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[4]Breusch, T. S., & Pagan, A. R. The Lagrange multiplier test and its applications to model specification in econometrics. In: The Review of Economic Studies. (1980).

That is, the disturbance term is decomposed in two parts:

1. C_i is the between entity error, which represents the unobserved hetereogeneity, reflecting the existence of other relevant variables that are not observed, but might be correlated with the observed variables. The general idea behind this is that there are individual-specific characteristics that are difficult or even impossible to observe or measure. One of the more typical examples for such an unobservable characteristic is the intelligence or the abilities of individuals.
It is assumed that those characteristics vary across individuals but are constant over time.

2. U_{it} corresponds to the common stochastic error term in, for example, classical linear regression models.

The above expressions show the assumption under which panel data works: $U_i t$ is assumed to be uncorrelated with x_{it} and C_i, and it is furthermore assumed to vary unsystematically across individuals and time. That is called **Strict Exogenity** and is expressed as:

$$E\left[U_{it} \setminus x_{it}, \ldots, x_{iT}, C_i\right] = E\left(U_{it}\right) = 0$$

There are two models for estimating parameters from a panel data set: Fixed Effects Model and Random Effects Model.

The crucial assumption that distinguishes the fixed effects model from the random effects model is whether C_i may or may not be correlated with the set of explanatory variables, x_{it} :

- **Fixed Effects Model:** C_i is correlated with x_{it}. The C_i are assumed to be n unknown parameters.
  This approach is relevant when there is an expectation that the averages of the dependent variable will be different for each cross-section unit, or each time period, but the variance of the errors will not.

$$E(C_i \setminus x_{i1}, \ldots, x_{iT}) \neq E(C_i)$$

- **Randon Effects Model:** C_i is uncorrelated with x_{it}. The C_i are treated as drawings from a distribution with mean μ and variance σ_i^2 which are independent from the explanatory variables in x_{it}. Unlike the fixed effects model, the variation across entities is assumed to be random and uncorrelated with the predictor or independent variables included in the model.

  This approach is relevant when there is an expectation of no omitted variables, or that the omitted variables are uncorrelated with the explanatory variables of the model.

$$E(C_i \setminus x_{i1}, \ldots, x_{iT}) = E(C_i)$$

For evaluating whether a model should be worked under fixed or random models, we use the Hausman test[5], which is a statistical test assessing the following hypothesis:

$$H_0 : C_i \ is \ uncorrelated \ with \ x$$

The statistic is given by:

$$m = q(\hat{V}_1 - \hat{V}_0)^- \hat{q}$$

where $(\hat{V}_1)$ and $(\hat{V}_0)$ represent consistent estimates of the asymptotic covariance matrices of the estimators $(\hat{\beta}_1)$ and $(\hat{\beta}_0)$ respectively. Note that $(\hat{\beta}_1)$ is the estimator of Random Effects and $(\hat{\beta}_0)$ is the estimator of Fixed Effects:

$$q = \hat{\beta}_1 - \hat{\beta}_0$$

The m-statistic is then distributed χ^2 with k degrees of freedom, where is the rank of the matrix $(\hat{V}_1 - \hat{V}_0)$.

4.3.1   Stationary Data

In order to have adequate results from the application of panel data as a methodological approach for time series, we must assess before the data behavior in order to determine whether is stationary or non-stationary.

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[5]Hausman, J. A. Specification tests in econometrics. In: Econometrica: Journal of the Econometric Society. 1978.

Stationarity is the quality of a process in which the statistical parameters (i.e. mean and standard deviation) are constant in time. More formally, a strictly stationary stochastic process is one where given $t_1, ..., t_l$ the joint statistical distribution of $x_{t1}, ..., x_{tl}$ is the same as the joint statistical distribution of $x_{t1+T}, ..., x_{tl+T}$ for all l and T . The above means that all moments of all degrees (expectations, variances, third order and higher) of the process, anywhere are the same.

Also, a weaker form of stationarity is the one in which only the mean and variance (first and second moments) of a stochastic process are time invariant, and the autocovariance depends only on the lags. This weak form is the one used in time series analysis.

To evaluate stationarity, we use a Unit Root Test, whose null hypothesis states that a series is non-stationary, in which case there is a unit root (attribute of a statistical model of a time series whose autoregressive parameter is one).

4.3.2 Dynamic Panel Data Models

In cases where the dependent variable in a panel data model depends on its value in a prior point in time, we must work with a dynamic model, which has the following structure:

$$y_{it} = \gamma y_{it-1} + \beta x_{it} + C_i + U_{it}$$

From the above expression, y_{it-1} does not meet the strict exogeinity condition. Dynamic models assume instead sequential exogeinity, in which U_{it} is uncorrelated with current and past regressors, including C_i.

Given the above, Fixed Effects and Random Effects models become inconsistent estimators, for which we must use Generalized Method of Moments (GMM). The former is a statistical method that combines observed data with the information in population moment conditions to produce estimates of the unknown parameters of a model. Is used in models in which data is dynamic.

One of the estimators used under the GMM methodology is the Blundell-Bond's[6], which is a Statistical tool for estimating whether the lagged-levels of the dependent variable are significant within a model. The idea on the estimator is to create a set of estimating equations for the regression, by making sample moments match the population moments defined.

With time series, we can add moment conditions by assuming that past values of explanatory variables, or even past values of the dependent variable, are uncorrelated with the error term, even though they do not appear in the model.

4.4   Application

Panel data is a multi-dimensional data in which certain variables are observed for an individual, through several periods of time. Therefore, it allows assessing the impact of the variables in a dynamic setting.

Some of the benefits of panel data are:

- To better identify effects in variables: Special cases of Panel data are Cross-section which assesses different variables for one period of time, as well as time-series which evaluates one variable in different periods of time. Panel data combines multiple variables in a time series framework, allowing us to better explain the dynamics of variables in a dynamic setting.

- To construct and test more complicated behavioral models than purely cross-section or time-series data.

However some restrictions also appear, being the design and data collection, for assuring a balanced panel, which must also meet the suppositions of a higher number of variables than periods of time.

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[6]Blundell, R., & Bond, S. Initial conditions and moment restrictions in dynamic panel data models. In: Journal of econometrics. 1998

According to the literature we reviewed, there is a very restricted number of authors who have previously assessed Cat Bonds under a panel data approach. Tao (2011), and Cummins and Weiss (2009), proved Cat Bonds as a zero beta security, by developing a comparative analysis with other financial securities, using panel data. Also, Gürtler et al (2012) explores the impact of the financial crisis on Cat bonds, in a dynamic stage using panel data. However, to the best of our knowledge, panel data for assessing the spread of Cat bonds in the secondary market has not been explored yet, for which we consider relevant developing an approach under a multi-dimensional framework in order to understand the complex dynamics of Cat bonds spread traded in the market.

The dependent variable we will be assessing is the average market indicative spread in the secondary market (Indicative spread), which is the spread to maturity as a function of the built-in price of market agents, according to their appreciation of the Cat Bond price.

The price of a bond in the secondary market can trade at premium or discount, and in either case, such price can be reflected in an indicative spread rate. Since Cat bonds usually have a floating coupon rate, the price movements of such bonds are reflected into the fixed component of the rate (spread). Therefore, the average market indicative spread of Cat bonds reflects the price perception of investors over that security.

4.5   Explanatory variables

There has not been a wide exploration of variables affecting the spread in the secondary market. Following our work on the spread in the primary market, our initial hypothesis here is also based on the inclusion of internal and external variables as significant explaining the spread movements in the market. Those variables are the following:

1. **Spread at Issue:** Following the definition of initial spread, we have already observed it depends on several factors which will ultimately all reflect the risk of the Cat Bond, translated into the spread of the security. Therefore, we expect investors to price this factor along the life of the bond, affecting its price on the secondary market.

2. **Expected Loss:** As previously explored, expected loss proved to be the most relevant factor in the initial spread of a Cat Bond. And since is the factor explicitly representing the risk associated to the Bond, we expect to remain relevant for investors when pricing Cat Bonds.

3. **Credit Rating:** The rating of a Cat Bond proved to be relevant in the initial spread, since is a reflection of the standardize definition of risk, represented to investor. We will assess whether this factor remains valid in the pricing over the secondary market.

   We converted all ratings to the S&P's scale, and used a dummy for those bonds lying above and below the investment grade. We have defined 3 types of credit ratings: Investment grade (rated above or on BBB-), non-investment grade (rated below or on BB+) and not rated.

4. **Time to Maturity Factor:** Since bonds are sensitive to time, knowing how the longer the term the higher its risk, we have defined a proxy for reflecting the time to maturity each bond has in every period of time assessed, which under the bonds dynamics should be relevant in the indicative spread.

   Our proxy is represented by the following Factor, based on the reciprocal of time to maturity:
   $$\frac{1}{(T-t)}$$

   Being T the expiration date and t the date of assessment.
   One of the methods for pricing securities consists in discounting to time zero, the security's future cash flows. For example the price of a Bond is the present value of its coupons and principal. The notion of Present Value is a function of time and interest rate in the market, becoming the foundation of pricing.

   Following the role of a bond's maturity as a discount factor, we have defined that proxy for measuring the impact of the number of days remaining to maturity over the indicative spread, by resembling a discount factor.

5. **BB- Bonds Index:** As previously used in our assessment of spread in the primary market, we consider the Credit Suisse's high yield II index as the proxy for

an indicative spread on non-investment grade securities. We expect this factor to not only be relevant in the initial spread, but also in the indicative spread, influencing investors' perception of Cat Bonds' price.

6. **Interest Rate:** Once more, the general notion of interest rate that affects the price of bonds, and the direct impact on Cat bonds coupons of floating rate securities, lead us to consider a risk free interest rate as a price determinant in the secondary market. Our proxy is the 3-month US dollar denominated Libor correspondent to every quarter under assessment.

7. **Swiss Re Cat Bond Total Return Index (SCATTRR):** The reinsurer Swiss Re has develop an index to track the total rate of return for a basket of outstanding USD denominated Cat Bonds, priced by them. It is one of the most used proxies to understand the performance of Cat bonds 'price movements in the secondary market.

By definition of the former variables, the indicative spread has no influence on them, meeting one of the assumptions of panel data analysis: Strict Exogeneity.

4.6   Preliminary Assessments

Before running the regression with panel data, there are certain factors relevant to evaluate, in order to meet all conditions in the data needed when working with such a method.

Some of the explanatory variables previously defined, are time invariant (i.e. Spread at issue, Expected Loss, and Credit Rating). The other variables are time variant, for which it becomes relevant to evaluate whether any of those is an integrated variable.

We use the Levin, Lin & Chu Unit Root test for panel data, in order to evaluate stationarity. Its null hypothesis is the existence of a Unit Root, which represents non-stationarity.

The test was run for the Indicative spread, Time to Maturity Factor, BB- Bond Index, Libor and SCATTRR, with the following p-values:

Table 8: Unit Root Test results for assessed factors

	P-Value
Indicative Spread	0.0000
Time to Maturity Factor	0.0500
BB- Bond Index	0.0000
Libor	0.0000
SCATTRR	0.0000

We conclude all former series are stationary.

Additionally, we made a prior assessment of Breusch-Pagan test, in order to determine whether there is unobserved heterogeneity, which would imply working with Panel Data instead of OLS.

The F-statistic from the test yielded the following p-value:

$$Prob > F = 0.0000$$

Therefore, we reject the null hypothesis and conclude our data has unobserved heterogenity, being Panel Data the most convenient methodological approach.

4.6.1 Fixed Effects or Random Effects?

Now that we have concluded Panel Data is the best methodology for assessing our data, and after meeting its assumptions, we must now define which of the models will better suit our data.

We use a Hausman test for evaluating between fixed and random effects, under the null hypothesis that the preferred model is random effects. Results are as shown in table 9.

Table 9: Hausman Test Results

| | Coefficients | | | |
| | (b) | (B) | (b-B) | sqrt(diag(V_b-V_B)) |
	fixed	random	Difference	Standard Error
High_Yield	0.0003	0.0003	0.0000	0.0000
Time to Maturity Factor	-1.7310	-1.6966	-0.0344	0.3079
Libor	0.0192	0.0190	0.0002	0.0014
SCATTRR	-0.0023	-0.0023	0.0000	0.0001

b = consistent under Ho and Ha;
B = inconsistent under Ha, efficient under Ho

Test: Ho: difference in coefficients not systematic
chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
= 0.02
Prob>chi2 = 0.9993

According to the p-value yielded, we conclude the preferred model for our data is Random Effects.

4.7 Multifactor spread model

We run a GLS regression for the data base constructed form 81 registers. The 324 observations are set as panel data.

From results, using a 5% confidence level, the final expression for calculating the spread of a Cat Bond in a secondary market is:

$$Spread_i = \alpha + \beta_{Spread} * Spread_i + \beta_{EL} * EL_i + \beta_{Maturity} * TTMFactor_{it} + \beta_{HY} * HY_{it} + \\ \beta_{SCATTRR} * SCATTRR_{it} + C_i + U_{it}$$

Where:
α = Constant

$Spread_i =$ Initial spread at time of issuance of every bond

$EL_i =$ Expected loss probability attached to every bond.

$TTMFactor_{it} =$ Time to maturity factor for every bond in every period of time.

$HY_{it} =$ High Yield Index corresponding to every observation.

$SCATTRR_{it} =$ Swiss Re Cat Bond index return for every observation.

$C_i =$ Unobserved heterogeneity.

$U_{it} =$ Error.

Table 10: Determinants of the Spread for Cat Bonds in the Secondary Market

Indicative Spread	Coefficient	Standard Error	t-statistic	Prob.
Constant	0.1679	0.0728	2.3100	0.0210
Spread at Issue	0.8669	0.0780	11.1200	0.0000
Expected Loss	0.3800	0.1595	2.3800	0.0170
Time to Maturity Factor	-1.6966	0.5711	-2.9700	0.0030
High_Yield	0.0003	0.0001	2.4100	0.0160
SCATTRR	-0.0023	0.0007	-3.1400	0.0020

Source: Stata regressions results

4.7.1 Results

The results show 5 specific factors contributing to the indicative spread given by the market in any moment of the secondary market. The variable with the highest coefficient is the time to maturity factor, since as explained before; it has a direct relation in the pricing of Bonds. The coefficient is negative because of the definition of the factor $(1/(T-t))$, in which our proxy represents the reciprocal of time to maturity. Therefore, the lower the factor, the longer days remain for the bond to expire, being higher the risk (maturity risk) and rising the spread.

Regarding internal factors, the Initial Spread and Expected Loss are both significant with positive coefficients. Initial Spread is positive, since as mentioned, the higher the issuance rate, the higher the risk embedded by the bond, for which a higher perception of risk persists. Similarly, the higher the expected loss defined for every Cat Bond, the

higher the spread in the secondary market.

Credit rating proved to be not significant for the secondary market. From our previous work regarding the factors determining the spread of Cat Bonds in the primary market, we conclude that credit rating is important when defining the initial spread of a Cat Bond, after which that risk information is reflected in the initial spread. Also the interest rate (i.e Libor), proved to be relevant only for the spread in the primary market, after which investors under a floating coupon reflect their price perceptions on the Spread to Maturity, which is the factor receiving all the pricing information.

In regards to the external factors, both the High Yield index and Cat Bond Index proved to be relevant in the indicative spread. High Yield Index has an inverse effect in the primary and secondary market, having a negative effect for the spread in the primary market and positive for the secondary market. In the former it worked as a benchmark for other securities with non-investment grade, for which a rise in the index (price) was translated in a fall in the interest rate for such securities. Therefore, following the resemblance to Cat Bonds, the spread at issue from these securities fell as well.

In the secondary market the index also works as a benchmark, for comparing the attractiveness of Cat Bonds against other non-investment grade securities. Therefore, a rise in the index (price) reflects a fall in the indicative spread of these securities, which reflects a higher demand for those securities. Now, according to the definition of substitute goods, investors would be demanding lower Cat Bonds, which will lower the price of Cat Bonds, raising their spread to maturity.

Finally, the Swiss re Cat Bond total return index (SCARRTT) used as a proxy for the development of the secondary market of Cat Bonds, has a negative effect proving the lower the index (price), the higher the interest rate, which must be directly reflected on the yield of Cat Bonds.

4.7.2 Fitness of the Regression Model

The multifactor spread model for secondary spreads is now evaluated against the original indicative spreads, in order to conclude about the accuracy of the model and fit with the real data. Results proved and average absolute deviation of 1.75%.

Figure 11: Original Indicative Spread Vs Estimated Spread
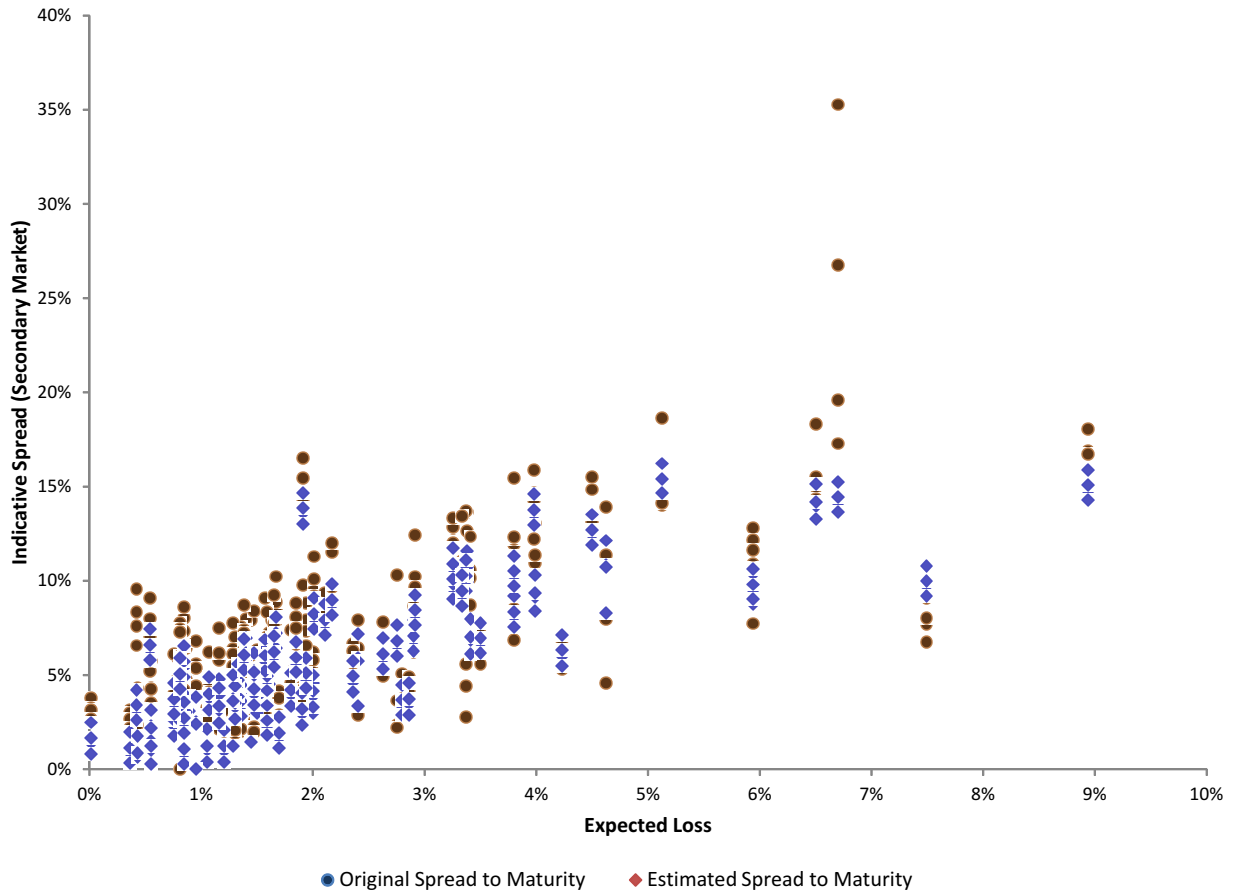
The model has a better adjustment for those Cat Bonds with lower expected loss, which are generally the ones trading at lower spreads. However, the spread in the secondary market of those Cat Bonds with a high expected Loss and a longer time to maturity, tends to have a very volatile behavior, for which our model tends to either under or overestimate the spread to maturity.

Table 11: Descriptive Statistics for the deviation of the Modeled Indicative Spread Vs Original Spread

Deviation	0% - 1%	1%-2%	2%-6%	>6%
Average Indicative Spread	6.44%	6.65%	8.41%	9.51%
Average Initial Spread	8.97%	8.18%	9.08%	11.83%
Average Expected Loss	2.46%	2.08%	2.39%	3.50%
Average Time to Maturity Factor	0.0017	0.0015	0.0020	0.0011
Number Observations	90	134	94	6
% Sample	28%	41%	29%	2%

Table 11 shows a clear relation of high deviations (above 6%), with higher initial spreads, Expected Loss and days to maturity. Future research can better focus on the particular performance of highly risk Cat Bonds, in order to better identify potential additional factors influencing the price perception of investors in the secondary market.

4.8 Static Model Assessment

Our hypothesis assumes that the indicative spread for each Cat Bond depends on the initial spread exclusively. However the question on whether the indicative spread on t is influenced by the indicative spread in t-1 arises.

We therefore evaluate whether the model is static or dynamic, estimating the regression using the one-step GMM system estimator of Blundell-Bond.

Table 12: Regression Output using GMM system estimator (Blundell-Bond)

Indicative Spread	Coefficient	Standard Error	t-statistic	Prob.
L1.	0.0584	0.5033	0.1200	0.9080
Constant	-0.1677	0.2156	-0.7800	0.4370
Initial Spread	1.5726	1.0298	1.5300	0.1270
Expected Loss	2.0325	1.2391	1.6400	0.1010
Investment	3.1787	2.2984	1.3800	0.1670
Non Investment	0.3995	0.2888	1.3800	0.1670
Time to Maturity Factor	5.0873	4.6742	1.0900	0.2760
High_Yield Index	0.0003	0.0004	0.7300	0.4620
SCATTRR	-0.0029	0.0014	-2.1600	0.0310

Source: Stata results from Blundell-Bond estimator

From table 12 we can see that the variable L1, corresponding to the lag, has a p-value of 0.9, which is not significant, concluding our model is static and the results shown in the last regression remain valid.

Part V

CONCLUSIONS

Previous authors exploring the dynamics of Cat Bonds' spread have focused exclusively in certain relevant factors, being the expected loss in the primary market, and the probability of occurrence of a catastrophe in the secondary market. Those models fall short in providing a robust model for explaining the spread of Cat Bonds, arising therefore the need to widen the scope of factors impacting these securities. Our research seeks to satisfy that need by proposing two multifactor spread models, with several internal and external variables proving to be significant in determining the spread of Cat Bonds.

Results suggest that 12 and 5 variables respectively have a significant impact over the spread of Cat Bonds in the primary and secondary market. As expected and previously researched, expected loss is the single most important determinant factor in the primary market, since is the direct risk measure embedded to the bond. On the other hand, time to maturity was the most relevant factor in the secondary market, since it has a direct relation and high impact in the pricing of Bonds.

Although in the secondary market the Expected Loss remained significant, is no longer the most relevant factor, suggesting its impact on the spread of Cat Bonds is heavily absorbed in the primary market, where such probability is first known, remaining later constant until maturity. Therefore its impact in the secondary market decreases considerably.

Besides the Expected Loss factor, the other variable relevant for both models is the high yield index. In the primary and secondary market the sign and magnitude of this factor's coefficient is equal, suggesting benchmarks over the performance of other non-investment grade bonds, is a relevant determinant in the spread of a Cat Bond throughout its life.

Our proposed models show to have a high accuracy on replicating the spread of Cat Bonds. Furthermore, unlike most authors, our models have a general application, rele-

vant both for the P&C and Life market of Cat Bonds. Therefore, we have developed not only a pricing tool, but an insightful model for understanding the variables that directly affect the pricing dynamics of Cat Bonds, in order for investors to make sound decisions.

Some areas for further research should focus in identifying additional factors impacting the spread of Cat Bonds in the secondary market, especially given the fact that these securities are relatively new, and their increasing attractiveness in financial market are continuously changing their dynamics and structure, to better fit the market players' needs.

Part VI

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