Prediction of Federal Funds Target Rate:  
*A Dynamic Logistic Bayesian Model Averaging Approach*  

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**Abstract**  

In this paper we examine which macroeconomic and financial variables have most predictive power for the target repo rate decisions made by the Federal Reserve. We conduct the analysis for the FOMC decisions during the period June 1998-April 2015 using dynamic logistic models with dynamic Bayesian Model Averaging that allows to perform predictions in real-time with great flexibility. The computational burden of the algorithm is reduced by adapting a Markov Chain Monte Carlo Model Composition: MC. We found that the outcome of the FOMC meetings during the sample period are predicted well: Logistic DMA-Up and Dynamic Logit-Up models present hit ratios of 87,2 and 88,7; meanwhile, hit ratios for the Logistic DMA-Down and Dynamic Logit-Down models are 79,8 and 68,0, respectively.  


Keywords: Logistic Model, Dynamic Model Averaging, Bayesian Model Averaging, Markov Chain Monte Carlo Model Composition, Federal funds target rate, Forward Rate Agreements, US Treasury Yield Curve, Overnight indexed swaps (OIS).  

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1. Introduction

Market expectations about the expected path of central bank’s target repo rate can have important consequences for financial markets and the economy as a whole. Interest rate derivatives (IRD) enable market participants to hedge against or speculate on potential movements in short-term interest rates, as a result, IRD are a rich and timely source of information on market expectations. Critical market information about market expectations can be derived from interest rate futures and forwards; being the main derivatives instruments: fed funds futures contracts, forward rate agreements (FRAs), overnight-indexed swaps (OIS), Eurodollar futures, and options on interest rate futures.

Fed funds futures are the preferred derivative instruments for anticipating the monetary stance of the Federal Reserve regarding its policy target rate: The Fed Funds. For instance, Krueger and Kuttner (1996) perform out-of-sample forecasts of future monetary policy based on one- and two-month futures prices, and conclude that predictable changes in the funds rate are rationally forecasted by the futures market. Similarly, Söderström (1999) shows that the expectations of near-term changes in the fed funds rate target extracted from the fed funds futures market seem to be useful as measures of market expectations. Gürkaynak, R. (2005) uses long-maturity federal funds futures contracts to extract policy expectations and surprises at horizons defined by future FOMC meetings. In a recent study, Crump et al. (2014) present evidence about how the paths of the policy rate constructed from fed funds futures, OIS, and Eurodollar futures are useful tools to analyze market expectations.

Although, extensive research have been conducted regarding the predictability of macroeconomic variables based on the dynamics of the term structure of interest rates; there is a lack of empirical evidence focusing on the ex-ante signals and predictive power of FRAs estimated from overnight indexed swaps, in order to anticipate the path of future monetary policy. The understanding of the dynamic evolution and forecasting of the yield curve has many practical applications: pricing financial assets and derivative securities, managing and hedging market and credit risks, as well as, conducting conventional and unconventional monetary policy strategies. A customary use of the yield curve, more common in the US, was to use its slope to forecast recessions.

After the recent financial crisis, academics are more interested in examining the important role the yield curve has for conducting monetary policy. The yield curve is important mainly for a couple of reasons: (i) it is an indicator about the expected path of future monetary policy; and (ii) the yield curve is a fundamental part of the transmission mechanism of monetary policy.
For decades the fields of finance and macroeconomics dealt with interest rates, asset prices and the yield curve in a total different way and without much interaction. As Diebold and Rudebusch (2013) point out in their book: “In macro models, the entire financial sector is often represented by a single interest rate with no accounting for credit or liquidity risk and no role for financial intermediation or financial frictions. Similarly, finance models often focus on the consistency of asset prices across markets with little regard for underlying macroeconomic fundamentals. To understand important aspects of the recent financial crisis ... a joint macro-finance perspective is likely necessary”.

Diebold et al (2006), find strong evidence of macroeconomic effects on the future yield curve and somewhat weaker evidence of yield curve effects on future macroeconomic developments. Although bi-directional causality is likely present, effects in a research done by Ang and Piazzesi (2003) seem relatively more important than those previously presented by Estrella and Hardouvelis (1991), Estrella and Mishkin (1998), and Stock and Watson (2000). Shmueli (2010) states that endogeneity does not pose an estimation bias when dealing with predictive modeling, since the focus is on association rather than causation and the prospective context, there is no need to delve into the exact role of each variable in terms of an underlying causal structure. On the contrary, criteria for choosing predictors are quality of the association between the predictors and the response, data quality, and availability of the predictors at the time of prediction, known as ex-ante availability.

Empirical results presented by S. van den Hauwe et al (2013), developing a Bayesian framework with a spike and slab variable selection methodology to model the direction of FOMC target rate decisions, show strong evidence for persistence in the target rate decisions, beyond the persistence caused by the (strongly) autocorrelated covariates. Most predictive ability is found for, first, economic activity measures like industrial production, the output gap and the coincident index, and, second, term structure variables like interest rate spreads. In addition, Survey of Professional Forecasters (SPF)-based forecasts for the US three-month T-bill rate and different survey-based variables that measure current and future consumer confidence have predictive content.

S. van den Hauwe et al (2013) propose a Bayesian forecasting scheme on a real-time basis to construct out-of-sample probability forecasts, efficiently using all information available at the time of generating the forecasts. Meetings in the period January 2001–June 2008 are predicted well; 82% of the outcomes during the out-of-sample period 2001–2008 are correctly predicted. In-sample they even achieve a hit rate of 90%.
In sum, the aim of this paper is to assess which macroeconomic and financial variables are most informative for the Central Bank’s target rate decisions from a forecasting perspective, by focusing particularly on the predictive power of FRAs derived from the short-end of the swap curve. We conduct the analysis for the FOMC decisions during the period June 1998-April 2015 using dynamic logistic models with dynamic Bayesian Model Averaging; the computational burden of the algorithm is reduced by adapting a Markov Chain Monte Carlo Model Composition: MC³. We found that the outcome of the FOMC meetings during the sample period are predicted well: Logistic DMA-Up and Dynamic Logit-Up models present hit ratios of 87.2 and 88.7; meanwhile, the hit ratios for the Logistic DMA-Down and Dynamic Logit-Down models are 79.8 and 68.0, respectively.

The outline of the paper is as follows. Section 2 describes the dynamic logistic model with dynamic Bayesian Model Averaging using Markov Chain Monte Carlo Model Composition used to estimate the direction of the Central Banks’ target rate. Subsequently, the results of some simulation exercises to show the properties of the MC³ are shown. Section 3 explains in detail the macroeconomic variables and market data used in this study. The estimation and forecasting results are presented in Section 4, and Section 5 concludes the paper.

2. Methodology

In this section we present the econometric framework that we use to perform statistical inference regarding the target repo rate decisions made by Centrals Banks. First of all, we set \( r_t \) as the prevailing rate at the end of the month \( t, t = 1, 2, \ldots, T \) being \( T \) the sample size, and \( \Delta r_t = r_t - r_{t-1} \), the variation of the rate. Then, as our main objectives are to find the determinants and predict upward \((\Delta r_t > 0)\) and downward \((\Delta r_t < 0)\) movements in the repo rate, we adopt the following definitions:

\[
\begin{align*}
    y_{tu}^t &= \begin{cases} 
    1 & \text{if } \Delta r_t > 0 \\
    0 & \text{if } \Delta r_t = 0, \Delta r_t < 0
    \end{cases} \\
    y_{td}^t &= \begin{cases} 
    1 & \text{if } \Delta r_t < 0 \\
    0 & \text{if } \Delta r_t = 0, \Delta r_t > 0
    \end{cases}
\end{align*}
\]  

(1)

(2)

where \( y_{tu}^t \) and \( y_{td}^t \) are dummy variables indicating upward and downward movements, respectively. We would like to clarify that we estimate two independent models: one for upward movements, and another one for downward movements. This is because generally agents’ expectations are between two
events: upward movements and no variation or downward movements and no variation in repo target rate.

To estimate our models, we extend the dynamic model averaging procedure for dynamic logistic regressions developed by McCormick et al. (2012). In particular, we implement a Markov Chain Monte Carlo Model Composition procedure, or MC⁵, for model selection. This adaptation reduces enormously the computational burden of the algorithm, as McCormick et al. (2012) point out in their conclusions

“... The proposed method could be adapted through an “Occam’s window” approach (Madigan and Raftery, 1994), where we evaluate only an “active” subset of the models at each time.”

McCormick et al. (2012, pp 30)

We propose a MC⁵ algorithm that goes over the model space looking for the best models reducing drastically the computational burden. Therefore, we try to find the best “active” subset of the models at each time. This econometric approach takes into consideration simultaneously three desirable statistical characteristics: dynamic parameters, dynamic Bayesian Model Averaging and an autotuning procedure all based on the best models.

The choice of the Bayesian Model Averaging methodology is based on the fact that this framework is firmly grounded on statistical theory following the rules of probability. It minimizes the sum of Type I and Type II error probabilities; its posterior point estimates minimize the mean square error, and its posterior predictive distributions perform better relative to other estimators (Raftery and Zheng, 2003). Additionally, there exists a huge difference between the BMA method and other model selection techniques, such as the Bayesian Information Criterion, as the latter method chooses just one model, whereas the former averages over all possible models. This difference makes Bayesian Model Averaging a stronger statistical tool. Although BMA enjoys a long tradition in statistics (Leamer, 1978), its application in economics has only recently come into its own (Fernandez et al., 2001; Sala-i Martin et al., 2004; Eicher et al., 2012; Moral-Benito, 2013a; Jetter and Ramirez-Hassan, 2015). Moral-Benito (2013b) provides a detailed survey on the use of BMA methods in economics.

Specifically, we use the linear logistic model where a Bernoulli response, say, $y_t$, is related to a set of covariates, $x_t' = (x_{1t}, x_{2t}, \ldots, x_{qt})$, using the logit link (Cameron and Trivedi, 2005).
\[
\logit(p_t) = \ln \left( \frac{p_t}{1 - p_t} \right) = x_t' \beta_t
\]  

(3)

Where \( p_t \equiv \Lambda(x_t' \beta_t) = P[Y_t = 1|x_t' \beta_t] = \frac{\exp(x_t' \beta_t)}{1 + \exp(x_t' \beta_t)}, \) and \( \beta' = (\beta_1, \beta_2, ..., \beta_q) \) is a vector of coefficients to estimate at time \( t \). This model is specified by a Normal initial distribution for the \( q \)-dimensional state vector at time \( t = 0, \beta_0 \sim N_q(m_0, C_0) \), where \( m_0 \) and \( C_0 \) emerge from a Maximum Likelihood estimation of a logistic model using an initial sample training.

Assuming \( \beta_t = \beta_{t-1} + w_t \), where \( w_t \sim N_q(0, W_t) \), \( \pi(\beta_{t-1}|y_{1:t-1}) \sim N_q(m_{t-1}, C_{t-1}) \), and the prediction equation is \( \pi(\beta_t|y_{1:t-1}) \sim N_q(m_{t-1}, R_t) \), where \( R_t = C_{t-1} + W_t \) (Petris et al., 2007). However, McCormick et al. (2012) define the covariance matrix of the prediction equation as \( R_t = C_{t-1}/\lambda_t \), where \( \lambda_t \) is called the forgetting parameter which takes values slightly less than one. This parameter allows to incorporate more information from past time periods when the process is stable.

Following the updating property of Bayesian inference, we have that \( \pi(\beta_t|y_{1:t-1}) \propto f(y_{1:t-1}|\beta_t) \pi(\beta_t) \), where \( f(y_{1:t-1}|\beta_t) \) is the likelihood function, and \( \pi(\beta_t) \) is the prior distribution of the state vector. Therefore,

\[
\pi(\beta_t|y_{1:t}) \propto f(y_{1:t}|\beta_t) \pi(\beta_t)
\]

\[
= f(y_t|y_{1:t-1}, \beta_t) f(y_{1:t-1}|\beta_t) \pi(\beta_t)
\]

\[
= f(y_t|y_{1:t-1}, \beta_t) \pi(\beta_t|y_{1:t-1})
\]

\[
= f(y_t|\beta_t) \pi(\beta_t|y_{1:t-1})
\]

(4)

where the last line uses the fact that conditionally on \( \beta_t, y_{1:t} \)'s are independent and \( y_t \) depends on \( \beta_t \) only.

We see from equation (4) that the predictive density acts as the prior. Unfortunately, the likelihood function of a logistic process does not allow an analytical expression for the posterior distribution. Then, McCormick et al. (2012) approximates this expression using a Normal distribution where the mean is the mode of equation (4).

Using ideas from a Newton-Raphson algorithm (Judge et al., 1988),

\[
m_t = m_{t-1} - \left[ \nabla^2 l(m_{t-1}) \right]^{-1} \nabla l(m_{t-1})
\]

(5)

where \( l(\beta_t) = \ln \{f(y_t|\beta_t)\pi(\beta_t|y_{1:t-1})\} = y_t \ln p_t + (1 - y_t) \ln (1 - p_t) - \frac{q}{2} \ln (2\pi) - \frac{1}{2} \ln |R_t| - \frac{1}{2} \{ (\beta_t - m_{t-1})' R_t^{-1} (\beta_t - m_{t-1}) \} \). Then,
In addition, McCormick et al. (2012) approximates the state variance using \( C_{t-1} = \left[ -\nabla^2 l(m_{t-1}) \right]^{-1} \).

The forgetting parameter is such that, where \( f(y_t | y_{1:t-1}) \) is the predictive likelihood. This integral does not have a closed-form, so a Laplace approximation is used.

A Bayesian approach allows an elegant solution to model uncertainty. This is due to the fact that a model is a random element from a Bayesian perspective. Thus, given \( K \) possible predictors, the number of possible models becomes \( 2^K \). Setting \( \mathcal{M}_t = \{M_t^{(1)}, M_t^{(2)}, \ldots, M_t^{(2^K)}\} \) denoting the set of considered models, each model depending on a vector of parameters \( \beta_t^{(k)}, k = 1, 2, \ldots, 2^K \). Now the state space at each time consists of the pair \((M_t^{(k)}, \beta_t^{(k)})\), then using standard probabilistic rules,

\[
\pi(\beta_t | y_{1:t-1}) = \sum_{k=1}^{2^K} \pi(\beta_t | y_{1:t-1}, M_t^{(k)}) \pi(M_t^{(k)} | y_{1:t-1})
\]

where \( \pi(\beta_t | y_{1:t-1}, M_t^{(k)}) \) is the predictive distribution for model \( k \), and

\[
\pi(M_t^{(k)} | y_{1:t-1}) = \frac{\left[ \pi(M_{t-1}^{(k)} | y_{1:t-1}) \right]^{\alpha_t}}{\sum_{l=1}^{2^K} \left[ \pi(M_{t-1}^{(l)} | y_{1:t-1}) \right]^{\alpha_l}}
\]

Where \( \alpha_t \leq 1 \) is known as the model forgetting factor,

\[
\alpha_t = \arg\max_{\alpha_t} \sum_{l=1}^{2^K} f^{(l)}(y_t | y_{1:t-1}) \pi(M_t^{(l)} | y_{1:t-1})
\]

This strategy avoids to calculate the \( 2^K \times 2^K \) transition matrix.

The posterior model probability is equal to
\[ \pi(M_t^{(k)}|y_{1:t}) = \frac{\pi(M_t^{(k)}|y_{1:t-1}) f^{(k)}(y_t|y_{1:t-1})}{\sum_{i=1}^{2^K} \pi(M_t^{(i)}|y_{1:t-1}) f^{(i)}(y_t|y_{1:t-1})} \]  

(12)

And the dynamic model averaging prediction is

\[ \hat{y}_t = \sum_{k=1}^{2^K} \pi(M_t^{(k)}|y_{1:t}) \hat{y}_t^{(k)} \]  

(13)

Where \( \hat{y}_t^{(k)} \) is the forecast for the model \( k \) at time \( t \).

One huge issue remains unsolved in this setting. The number of possible models, \( 2^K \), can be enormous. For instance, if the number of controls is 20, there are more than 1 million models. To avoid such computational burden, we implement an adaptation of the Markov Chain Monte Carlo Model Composition, MC\(^3\), an algorithm adopted from the original mechanism developed by Madigan et al. (1995).

The MC\(^3\) procedure is an algorithm for drawing candidate models over the space \( \mathcal{M} \), based on a Metropolis-Hastings algorithm (Metropolis et al., 1953; Hastings, 1970). It simulates a chain of models, \( M_t^{(k)} \) (for \( k = 1, 2, \ldots, S \)), where the mechanism samples candidate models from a particular distribution, and accepts them with a probability. If a candidate model is not accepted, the chain remains in the current model (Koop, 2003).

In particular, we initially build a design matrix \( \mathbf{X}_{J \times K} \), selecting predictors using a Bernoulli distribution with probability \( p \). Such that each row of \( \mathbf{X} \) defines a candidate model, and our goal is to find the \( J \) best models. It is a good idea to use a Beta-Bernoulli prior for the model size because the resulting prior model distribution is considerably less tight and should thus reduce the risk of unintended consequences from imposing a particular prior model size (Ley and Steel, 2009). Thus, \( \pi(p) \sim \text{Be}(\alpha, \beta) \) which implies an expected initial model size equal to \( K\alpha/(\alpha + \beta) \). Setting \( \alpha = 1 \) and \( \beta = 1 \), which implies a prior Uniform distribution for \( p \), the prior expected model size is equal to \( K/2 \), the model prior is completely flat over model sizes.

We calculate the average posterior model probability for these initial models, \( \pi(M_T^{(k)}|y_{1:T})^{Ave} = 1/T \sum_{t=1}^{T} \pi(M_t^{(k)}|y_{1:t}) \), and find the model that has the minimum posterior model probability, \( M_t^{(Min)} \). Then, a candidate model \( M_t^{(c)} \) is drawn randomly from the set of all models excluding the initial models, and we estimate its posterior model probability. We accept this candidate with probability

\[ \alpha(M_T^{(Min)}, M_T^{(c)}) = \min \left\{ \frac{\pi(M_T^{(Min)}|y_{1:T})^{Ave}}{\pi(M_T^{(c)}|y_{1:T})^{Ave}}, 1 \right\} . \]  

(14)
2.1 Simulation Exercises

In this section we present the results of a limited Monte Carlo experiment to show the ability of our Model Composition strategy to solve a variable selection problem. In particular, we evaluate the performance of our algorithm to detect the hidden data generating process (d.g.p.) using different number of iterations $S = \{100, 500, 1000, 5000, 10000\}$.

The data generating process is given by
\[
y_t^* = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \beta_5 x_{5t} + \epsilon_t
\]
\[
y_t = \begin{cases} 
1, & y_t^* > 0 \\
0, & y_t^* \leq 0 
\end{cases}
\]

Where $x_{it} \sim i.i.d \mathcal{N}(0,1)$ and $\epsilon_t \sim i.i.d \mathcal{L}G(0,1)$, $i = 1, \ldots, 5$, $t = 1, 2, \ldots, 5000$.

As we can see from Table 1, the data generating process changes through time.

<table>
<thead>
<tr>
<th>$t$</th>
<th>\beta_0</th>
<th>\beta_1</th>
<th>\beta_2</th>
<th>\beta_3</th>
<th>\beta_4</th>
<th>\beta_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 1, \ldots, 4000</td>
<td>1</td>
<td>-2</td>
<td>-1</td>
<td>1 + t/5000</td>
<td>2.5</td>
<td>-1</td>
</tr>
<tr>
<td>t = 4001, \ldots, 5000</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1 + t/5000</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

Our design matrix includes 9 additional regressors that are not part of the d.g.p., such that $x_{it} \sim i.i.d \mathcal{N}(0,1)$, $i = 6, \ldots, 14$, $t = 1, 2, \ldots, 5000$. In addition, we use as training sample 60% of data.

Our setting implies that there are $2^{15}$ possible models, that is, 32768 models. Our goal is to find the 20 best models, and determine if these encompass the true data generating process.

We can see in Table 2 the Posterior Inclusion Probability of each variable. Among the regressors that are part of the d.g.p., $x_{1t}$ has the minimum PIP (0.65) followed by $x_{4t}$ (0.70) using 100 iterations. We expected this outcome because these regressors have coefficients equal to zero in the second part of the sample. Regarding $x_{2t}$, $x_{3t}$ and $x_{5t}$ their PIP increase as the number of iterations increase. On the other hand, there is a decrease of the PIP of regressors that are not part of d.g.p. when the number of iterations increase. These are good signals of the capacity of our strategy to identify the true d.g.p. as the number of iterations increase.

In Table 3 are shown the top 20 best models of our simulation exercise using 10000 iterations. We can see there that most of the models include the first five regressors, which in turn are part of the d.g.p. The models number 18, 10, 4, 6 and 8 have the highest average Posterior Model Probabilities. The average PMP are 0.14, 0.08, 0.07, 0.07 and 0.06, respectively.
Figure 1 depicts the PMP for the top 5 models. The model number 18, which is the true d.g.p. in the first sub-sample, has the highest PMP in this data subset. In the second subset, the model 6 has the highest PMP followed by the model 8. Those models exclude variables $x_{1t}$ and $x_{4t}$, which are not part of the d.g.p in this segment, whereas maintain $x_{2t}$, $x_{3t}$ and $x_{5t}$. However, the model 6 includes the variable $x_{13t}$, and the model 8 includes $x_{9t}$. These variables are not part of the d.g.p. Despite of this, the Model Composition strategy works so well to identify the true d.g.p.

Table 2: Posterior Inclusion Probability

<table>
<thead>
<tr>
<th>Variable</th>
<th>100</th>
<th>500</th>
<th>1000</th>
<th>5000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1t}$</td>
<td>0.65</td>
<td>0.85</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$x_{2t}$</td>
<td>0.80</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$x_{3t}$</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>$x_{4t}$</td>
<td>0.70</td>
<td>0.75</td>
<td>0.75</td>
<td>0.70</td>
<td>0.85</td>
</tr>
<tr>
<td>$x_{5t}$</td>
<td>0.75</td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>$x_{6t}$</td>
<td>0.45</td>
<td>0.25</td>
<td>0.20</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$x_{7t}$</td>
<td>0.50</td>
<td>0.40</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$x_{8t}$</td>
<td>0.55</td>
<td>0.35</td>
<td>0.30</td>
<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td>$x_{9t}$</td>
<td>0.45</td>
<td>0.35</td>
<td>0.50</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$x_{10t}$</td>
<td>0.35</td>
<td>0.25</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$x_{11t}$</td>
<td>0.60</td>
<td>0.45</td>
<td>0.25</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$x_{12t}$</td>
<td>0.45</td>
<td>0.30</td>
<td>0.25</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$x_{13t}$</td>
<td>0.35</td>
<td>0.45</td>
<td>0.45</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>$x_{14t}$</td>
<td>0.35</td>
<td>0.25</td>
<td>0.15</td>
<td>0.20</td>
<td>0.10</td>
</tr>
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</table>

Table 3: Best Models: 10000 iterations

<table>
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<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>PIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1t}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.75</td>
</tr>
<tr>
<td>$x_{2t}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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We can see in Figure 2 the posterior means of the coefficients that are part of the d.g.p. We observe that they follow the true process. This indicates that our methodology captures the dynamic of the d.g.p.
In addition, we found that the posterior means of the other variables have means approximately equal to zero (available upon request).

3. Data and target rate characteristics

We investigate the federal funds target rate at a monthly frequency for the period from June 1998 until April 2015. This sample period covers Alan Greenspan’s term from August 11, 1987 to January 31, 2006, as well as Ben S. Bernanke’s term from February 1, 2006 to Jan. 31, 2014, and Janet L. Yellen’s term from February 3, 2014 until today. Since during this time frame the Federal Reserve’s monetary policy objectives have not change, we rely on a specific macroeconomic and financial variables that are most informative for predicting target rate changes, along with Forward Rate Agreements derived from...
the short-end of the OIS curve and U.S. Treasury Bills. For instance, in its discussions, the Committee (FOMC) considers factors such as trends in prices and wages, employment and production, consumer income and spending, residential and commercial construction, business investment and inventories, foreign exchange markets, interest rates, money and credit aggregates, and fiscal policy.

The FOMC meets eight times per year at previously set dates. Our sample period covers 203 months in total, the announced target rates are displayed in Fig. 3, with summary statistics being provided in Table 4. During the sample period the target rate varied considerably, between a minimum of 0.25% since December 2008 to April 2015 and a maximum of 6.50% during the second half of year 2000. Fig. 3 clearly shows that decisions of the same type appear in clusters. For example, periods of sustained declines of the target rate occurred from January 2001 until June 2003, and from September 2007 until December 2008. To a large extent this target rate declines coincide with U.S. recessions as declared by the NBER. Similarly, multiple consecutive decisions to increase the target rate were made during August 1999 until December 2000, and from June 2004 until August 2007.

In addition, a long-lasting period of No-change is also evident during the sample, which is the result of the sustained monetary expansion policy adopted by the FOMC, whereby the target rate has been set within the 0.00%-0.25% range. In 2008, with short-term interest rates essentially at zero, the Federal Reserve undertook non-conventional monetary policy measures to provide additional support to the economy. Between late 2008 and October 2014, the Federal Reserve purchased longer-term mortgage-backed securities (MBS) and notes issued by certain government-sponsored enterprises, as well as longer-term Treasury bonds and notes. The primary purpose of these purchases was to help to lower the level of longer-term interest rates, thereby improving financial conditions for the economy as a whole. Thus, this nontraditional monetary policy measure operated through the same broad channels as traditional policy, despite the differences in implementation of the policy due to its nontraditional mechanism.
Our data consists of a set of macroeconomic and financial variables that are considered potential predictors for the FOMC target rate decisions. These variables can be categorized in 2 groups. The first group comprises measures related to observed inflation, industrial production, as well as the expectations (surveys conducted by Bloomberg) of y/y GDP and Industrial Production. These variables are most closely related to the monetary policy objectives of the Federal Reserve. The second group of variables consists of financial market data, where the estimation of Forward Rate Agreements (FRAs) is derived from both: the short-end of the OIS curve and US Treasury yield curve, specifically the 3-month and 6-month tenors (T-bills), which by definition are highly liquid-zero coupon securities. OIS rates were calculated using the arithmetic average over a month of daily closing data (mid-market rates from Bloomberg) and FRAs using the traditional non-arbitrage approach.

\[
(1 + R_{0,T})^T = (1 + R_{0,t_1})^{t_1}(1 + R_{t_1,t_2})^{t_2}, \text{ where } t_1 + t_2 = T.
\]
Due to the presence of multicollinearity we could not add more independent variables. In principle, we had 22 regressors composed of macroeconomic and financial variables, as well as survey measures and professional forecasts, yet after we run the VIF multicollinearity test a set of variables were eliminated, therefore we end up with a reduced number of regressors. Gujarati and Porter (2008), and Carsey and Harden (2013) state that simply checking the bivariate correlation among independent variables is not sufficient to diagnose multicollinearity when there are three or more independent variables in the model; therefore, variance inflation factors (VIFs) should be computed for each independent variable. Since multicollinearity reduces the amount of information available to estimate the coefficients, increasing the sample size helps to solve this problem by adding more information in the form of more data points back into the analysis.

Table 5 lists the complete set of 7 potential predictors we use in our analysis. It is worth mentioning that variables measured at a quarterly frequency, such as Expected y/y GDP (data gathered from Bloomberg’s survey), are transformed to monthly observations by keeping the value constant for the three months within the quarter. This approach is a common market practice because during the three-month period no new information about the variable is revealed. Since the first group of predictors is subject to revisions after their initial release, the currently available time series is different from the one that was the FOMC’s members’ disposal at the time they met; therefore, we decided to employ data as available on a real-time basis in order to make our empirical analysis as realistic as possible.

Table 5 presents the set of candidate predictor variables in the BMA model for the FOMC decision on the federal funds target rate. The columns headed \( Pr[y_k = 1|y] : PIPs \) give the posterior inclusion probabilities.
probabilities (PIPs) in the dynamic model for Up and Down movements in the fed funds rate on the full sample period June 1998 - April 2015.

4. Empirical Results

4.1. Estimation Results

We consider estimation results using the full sample period from June 1998 until April 2015, focusing on the question which variables appear to be the most informative for FOMC target rate decisions. Table 6 presents the key properties of the marginal posterior distributions of the Logistic DMA (Up & Down) in the left panels and the Dynamic Logit (Up & Down) in the right panels. These results were obtained by applying the MC3 simulation scheme of Section 2. Using the full sample period from June 1998 until April 2015. The effects of the most relevant predictors (that is, with the highest posterior inclusion probabilities $Pr[y_k = 1 | y]$) are shown conditional on inclusion: Posterior expectation (mean), posterior standard deviation (St.D.) and the 5th and 95th percentiles of the respective marginal distributions.

| Parameter  | Mean  | St.D. | 5th | 95th | Pr[Yk = 1 | y] |
|------------|-------|-------|-----|------|-------------|
| CPI.YoY    | 0.006 | 0.011 | 0.000 | 0.024 | 0.050 |
| Ind.Pcc    | 1.377 | 0.036 | 1.346 | 1.442 | 1.000 |
| EInd.Pcc   | 0.016 | 0.027 | 0.001 | 0.063 | 0.050 |
| EGDP.YoY   | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| FS3x6      | 3.943 | 0.128 | 3.714 | 4.055 | 0.950 |
| FUST3x6    | -3.691 | 0.151 | -3.845 | -3.426 | 1.000 |
| X6mFFunds  | 6.084 | 0.285 | 6.006 | 6.418 | 0.950 |

| Parameter  | Mean  | St.D. | 5th | 95th | Pr[Yk = 1 | y] |
|------------|-------|-------|-----|------|-------------|
| CPI.YoY    | 1.597 | 0.020 | 1.574 | 1.646 | 1.000 |
| Ind.Pcc    | 2.166 | 0.009 | 2.154 | 2.176 | 1.000 |
| EInd.Pcc   | 0.936 | 0.032 | 0.918 | 0.998 | 1.000 |
| EGDP.YoY   | 0.014 | 0.002 | 0.013 | 0.018 | 1.000 |
| FS3x6      | 6.123 | 0.056 | 6.029 | 6.196 | 1.000 |
| FUST3x6    | -6.127 | 0.040 | -6.186 | -6.046 | 1.000 |
| X6mFFunds  | 5.455 | 0.186 | 4.992 | 5.591 | 1.000 |

Several conclusions emerge from these posterior results. For instance, based on the marginal posterior inclusion probabilities $Pr[y_k = 1 | y]$, ($k = 1, ..., K$) only a limited number of predictor variables appear to be informative for the target rate decisions. We show these probabilities for the seven most frequently variables in the rightmost columns of the left panels in Table 6. For the Logistic DMA-Up model we find that four variables have a conditional posterior inclusion probability that exceeds 0.50: common
level threshold suggested by Eicher et al (2012) and Kass and Raftery (1995). The four variables are the following: (i) Industrial Production (Ind.Pcc), (ii) 3x6 FRAs derived from the OIS curve (FS3x6), (iii) 3x6 FRAs computed from the U.S. Treasury yield curve (FUST3x6), and (iv) the spread between the six-month T-bill rate and the federal funds rate (X6mFFunds). Meanwhile, for the Logistic DMA-Down model we find that only two variables comply with the aforementioned condition: (i) CPI year-over-year (CPI.YoY), and (ii) Industrial Production (Ind.Pcc).

The coefficient of the 3x6 FRA computed from 3-month and 6 month US T-bills may seem strange at first. However, we can interpret this variable as a proxy for short-run inflation expectations, as pointed out in, e.g., Estrella and Mishkin (1997). This derivative contract, which is highly liquid and volatile, reflects practitioner’s expectations regarding the direction of the fed funds target rate. For instance, proprietary traders and portfolio managers are well aware of the damage that changes in the U.S. Treasury yield curve can inflict on a fixed-income portfolio. However, those who anticipate these yield curve shifts will find that not only FRAs, but also U.S. Treasury futures may be used to design a variety of trades that can serve both risk management and yield enhancement purposes.

Economic activity measures have a positive effect, that is, larger values of these variables imply a higher likelihood of a target rate increase. This corresponds well with the idea that the FOMC tries to temper economic activity during expansionary periods by setting a higher target rate, in order to prevent from becoming too high. Therefore, a positive trend in terms of industrial production (Ind.Pcc) is indicative of economic growth. In addition, we find that market expectations embedded in interest rate derivatives contracts such as FRAs derived from the OIS and the US Treasury yield curve, as well as expectations from the spread X6mFFunds, represent short-term market expectations about inflation and economic activity to which the FOMC does react.

The following set of graphs show the dynamics of the correspondent posterior means for each regressor variable throughout the time, where the CPI.YoY (Down) and EInd.Pcc (Up) present the highest variability, meanwhile the rest of the variables are quite stable. Table 6 provides the main descriptive statistical measures for each regressor variable. We computed the posterior expectation (mean), posterior standard deviation (St.D.) and the 5th and 95th percentiles of the respective marginal distributions for the Logistic DMA and the Dynamic Logit models.
The DMA provides us with the dynamic nature of the betas, therefore, it is common to observe some variability. Nonetheless, as pointed out before, this is not the case when we model the behavior of the betas under the dynamic logit model, because the betas for each regressor variable are practically moving within a range showing high stability.
4.2. Forecasting

We examine the predictive ability of our Logistic DMA model (Up and Down) in the following way. First, we compute the probabilities of a decrease, no-change or increase of the target rate for each month in the full sample period June 1998-April 2015. Second, we compare the fitted values with the realized FOMC’s decision, as described before. Figures 4 and 5 show the probability estimated for each FOMC event during the sample period. From Fig. 4 we can conclude that Up movements in the target repo rate
are very well anticipated by the Logistic DMA-Up model, with probabilities ranging from 0.40 up to 0.80. The case for Down movements is quite different, because only from August 2007 to November 2008 the Logistic DMA shows high levels of probabilities in the range of 0.25 to 0.865.

Table 7 summarizes the corresponding hit rates for each model, as well as their respective cutoff points. Hit rates are equal to the percentage of correctly predicted target rate decisions. In-sample refers to hit
rates for probability estimates obtained when the models are estimated on the full sample period June 1998-April 2015. For example, the Logistic DMA-Up and Dynamic Logit-Up models present high hit ratios of 87.2 and 88.7, respectively. Meanwhile for the Logistic DMA-Down and Dynamic Logit-Down models are 79.8 and 68.0, respectively.

As Youden (1950) and Sebastian et al (2010) describe, the cutpoint analysis presented in last column of Table 7 involves locating the optimal value that minimizes prediction errors associated with binary outcomes, where both, the sensitivity and specificity statistical measures of the performance of the binary classification are maximized. Sensitivity (also called the true positive rate-TP) measures the proportion of positives which are correctly identified as such (e.g., the model assigns a high probability of occurrence to the event in which the FOMC decides to increase/decrease the target repo rate, and the final outcome is True). This measure is complementary to the false negative rate-FN. On the other hand, specificity (also called the true negative rate-TN) measures the proportion of negatives which are correctly identified as such (e.g., the model assigns a low probability of occurrence to the event in which the FOMC decides to increase/decrease the target repo rate, and the final outcome is True), and is complementary to the false positive rate-FP. Both measures can be represented graphically as a receiver operating characteristic curve or ROC curve. Fig. 6 portrays the corresponding ROC curves for each model considered in this analysis.

<table>
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<tr>
<th>Model</th>
<th>Data</th>
<th>In-sample hit rate</th>
<th>Cutoff point</th>
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<tr>
<td>Logistic DMA-Up</td>
<td>Real time</td>
<td>87.2 (177 / 203)</td>
<td>0.186</td>
</tr>
<tr>
<td>Logistic DMA-Down</td>
<td>Real time</td>
<td>79.8 (162 / 203)</td>
<td>0.186</td>
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<tr>
<td>Dynamic Logit-Up</td>
<td>Real time</td>
<td>88.7 (180 / 203)</td>
<td>0.164</td>
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<tr>
<td>Dynamic Logit-Down</td>
<td>Real time</td>
<td>68.0 (138 / 203)</td>
<td>0.073</td>
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5. Conclusions

We model the discrete changes in the federal funds target rate during the period June 1998-April 2015. We focus on the direction change as decided by the FOMC during their meetings held approximately eight time per year, using Logistic DMA models. The key modelling issue that we address concerns the question which indicators can help predict the FOMC decisions. For this reason we use a self-contained variable selection procedure. We consider a set of 7 potential predictors, including macroeconomic and financial market series. Our goal is to assess which macroeconomic and financial variables are most informative for the Central Bank’s target rate decisions from a forecasting perspective, by focusing particularly on the predictive power of FRAs derived from the short-end of the OIS curve.

Our empirical results show strong evidence for persistence in the target rate decisions. For the Logistic DMA-Up model, the most predictive ability is found for, first, economic activity measures like industrial production, and second, term structure variables such as 3x6 FRAs and interest rate spreads. Nonetheless, for the Logistic DMA-Down, the most predictive ability rests on the following macroeconomic variables: industrial production (Ind.Pcc) and CPI year-over-year (CPI.YoY). In this case, term structure variables do not present evidence of predictive power. FOMC meetings during the sample period June 1998-April 2015 are predicted well: Logistic DMA-Up and Dynamic Logit-Up models present high hit ratios of 87.2 and 88.7. Meanwhile, the Logistic DMA-Down and Dynamic Logit-Down models have medium-high hit ratios: 79.8 and 68.0, respectively.
Another contribution of this paper is that we propose a Bayesian model averaging (BMA) scheme with dynamic betas, that takes into account model uncertainty by going through all combinations of models that can arise within a given set of variables on a real-time basis to construct in-sample probability forecasts, efficiently using all information available at the time of generating the forecasts. The Bayesian model approach makes sure that we can appropriately deal with parameter and model uncertainty to end up with a parameter-and model-free forecast.

Finally, we identify the following areas where future empirical research could be conducted in order to improve the results and the methodology presented in this paper: (i) design and algorithm that takes into account the trinomial case (multinomial classification), where the three possible states of the FOMC decisions are very well captured, (ii) perform ‘pattern net’ analysis associated with neural networks classification and compare the results with the output provided by our model: the Bayes classification approach, and (iii) perform this analysis for the Economic and Monetary Union (EMU) and contrast the results by running out-of-sample estimates in order to stress the model and gauge its predictive power throughout the time.
References


Annex

A. Definition of macroeconomic variables

- **Repo Rate:** “The federal funds rate is the short term interest rate targeted by the Federal Reserve’s Federal Open Market Committee (FOMC) as part of its monetary policy. In December 2008, the target “fed funds” level was replaced by a target range…”**. It is released monthly by the FED, without revisions or lags.

- **CPI (YoY):** The CPI is released on a monthly basis by the Bureau of Labor and Statistics (BLS) as a non-seasonally adjusted indicator. It has no monthly revisions, but annual changes may be introduced in February with the release of the January print, going back up to five years††. This indicator is lagged one month.

- **Industrial Production:** The Industrial Production is released monthly by the FED as the indicator that shows the industry’s output, including everything that is produced within the country. This print can have revisions for the previous three months and it has a yearly revision on the fall that can go back several years‡‡. The Industrial production is lagged by two months. The expectation of Industrial Production is gathered from surveys conducted by Bloomberg.

- **GDP (YoY):** The Gross Domestic Product for the US is released on a quarterly basis by the Commerce Department of the Bureau of Economic Analysis (BEA), as a seasonally adjusted annual rate (SAAR) indicator. This print is revised twice after its release, with each revision being one month apart from the other. The GDP is lagged by three months. The expectation of y/y GDP is gathered from surveys conducted by Bloomberg.

** Description of the federal funds rate taken from Bloomberg


‡‡ Ibid., p.145