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**Determining the Optimal Selling Time of Cattle:
A Stochastic Dynamic Programming Approach**

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Abstract

The world meat market demands competitiveness and optimal livestock replacement decisions can help to achieve this goal. We introduce a novel discrete stochastic dynamic programming framework to support a manager's decision-making process of whether to sell or keep fattening animals in the beef sector. In particular, our proposal uses a non-convex value function, combining both economic and biological variables, and involving uncertainty with regard to price fluctuations. Our methodology is very general, so practitioners can apply it in different regions around the world. We illustrate the model's convenience with an empirical application, finding that our methodology generates better results than actions based on empirical experience.

Key Words: Decision Analysis, Farm Management, Simulation.

JEL Classification: Q12, C51, C61.

1. Introduction

We introduce a discrete stochastic dynamic programming framework suited to supporting optimal livestock replacement decisions. Specifically, we propose a stochastic non-convex value function, which implicitly depends on a profit function that involves economic and biological variables, and incorporates selling price uncertainty. The main motivation in establishing this methodology is the scarce literature regarding formal procedures to address an important issue in beef production, namely optimal livestock replacement decisions (Frasier & Pfeiffer, 1994), this being one of the most important factors affecting farm profitability (Kalantari, Mehbarani-Yeganeh, Moradi,

Sanders, and De Vries, 2010). Unfortunately, many livestock decisions are not based on economic or financial data, but on cattlemen's intuition (Glen, 1987; Takahashi, Caldeira, & Peres, 1997).

Livestock should be replaced when performance deteriorates. Performance is affected by age, production, costs, prices, and conditions of nature, among other aspects. Evaluating the optimal factors in replacing a productive asset such as livestock involves understanding the sequential nature of replacement decisions (Glen, 1987), the biological and economic factors that affect these decisions, and the uncertainty that affects future selling price realizations. Stochastic dynamic programming is an excellent technique that accommodates all these issues and it is therefore surprising that it has been little used for evaluating livestock replacement despite the considerable potential of its application.

Literature on optimal livestock actions can be divided into research focusing on optimizing fattening strategies, research looking for an economic basis on which to determine optimal policies, and studies aiming to define the optimal fattening/replacement time. For optimizing fattening strategies, Meyer and Newett (1970) proposed a deterministic methodology, based on a dynamic programming structure, to define the optimal food ration and selling time that would maximize profits for any type of cattle. Apland (1985) and García, Rodríguez, and Ruiz (1998) used linear programming to describe the impact on a herd's productivity of interest rates and diet, respectively.

Looking for an economic basis to determine optimal policies, Bentley, Waters, and Shumway (1976) used an expression to calculate the net expected revenue for specific periods of time using prices and costs, including probabilistic uncertainty concerning the asset's productivity due to mortality or infertility. Randela (2003) proposed a method to compute the average total value of an adult cow, which could be understood as the opportunity cost for replacing an animal, allowing farmers to determine the impact of mortality.

Different methodologies have been used to define optimal times for livestock replacement. Clark and Kumar (1978) proposed a deterministic dynamic programming model to define the optimal time for selling and buying beef cattle using prices and live weight, both variables depending on time and breed. Muftuoglu, Escan, and Toprak (1980) and Göncü and Özkütük (2008) employed least squares analysis to find the optimum culling age and weight. Frasier and Pfeiffer (1994) exploited a Markovian decision analysis with dynamic programming to find the optimal replacement time for cattle breeding according to nutritional path. Takahashi et al. (1997) presented a new optimization method based on dynamic programming to establish the optimal policy for herd

shaping. Arnade and Jones (2003) used seemingly unrelated regression (SUR) together with dynamic programming to establish the cattle cycle. Kalantari et al. (2010) used stochastic dynamic programming to define the optimal replacement policy for dairy herds using milk production, parity, and pregnancy status as state variables to solve the problem. Yerturk, Kaplan, and Avci (2011) developed an analysis of variance (ANOVA) to describe fattening performance.

Cattle raising is an old economic activity, disseminated worldwide, which consists of animal handling for productive purposes such as milk and beef production. As meat has been considered the main source of protein for human nutrition (FAO, 2012a), the livestock sector plays an important role in many economies in terms of producing food supplies, and generating employment and investment in different segments of the beef industry value chain (Ramírez, 2013; Randela, 2003). However, the world beef industry has grown at decreasing rates in the last few decades (FAO, 2012a; Schroeder & Graff, 2000). Researchers hypothesize about the restructuring of global meat consumption patterns (Galvis, 2000). In fact, net returns for beef cattle feeding have been volatile since the mid-1970s (Hertzler, 1988), and a significant decay in sales and loss of the meat market share to poultry and pork has been demonstrated (Katz & Boland, 2000). Nowadays, the world's meat consumption configuration is 42% pork, 35% poultry, and 23% cattle (FAO, 2012b).

The worldwide beef market suffers many pitfalls. First, supply fluctuations, volatility in prices (Glen, 1987; Kalantari et al., 2010), and foodborne illnesses attributed to red meat (Katz & Boland, 2000) have meant that consumers' preferences have shifted to other meat types (Galvis, 2000). Second, there is a separation between production and processing processes in contrast to substitute industries that are strongly integrated (Katz & Boland, 2000). In particular, asymmetry in the supply chain (Lafaurie, 2011), lack of coordination between production and commercialization (Schroeder & Graff, 2000), and poor vertical integration (Galvis, 2000) are crucial factors that must be addressed in the beef sector.

Third, cattlemen avoid changes necessary to improve competitiveness due to rigidity in regulations (Katz & Boland, 2000), input prices, cost structures, volatile selling prices, and poor economic incentives (Kalantari et al., 2010). All these factors reduce their capacity to develop technical changes to increase efficiency (Galvis, 2000). In addition, it is clear that the industry's dependence on natural conditions, the influence of climate change, interdependence with other human activities, and increasing requirements to become a global competitor, as well as health requirements for the exportation of meat (Takahashi et al., 1997), demand a strong reorientation to

achieve competitiveness (Crespi & Sexton, 2005), improve the flow of information (Schroeder & Graff, 2000), valorize whilst taking into account value-generating factors (Scoones, 1992) and increase productivity.

In this dynamic and challenging competitive environment, proposing methodological approaches that can help to improve the performance of the beef sector is a valuable contribution from an economic and financial perspective.

The paper is organized as follows: Section 2 presents the theoretical framework, including our methodological proposal. Section 3 sets out an empirical application with its results. Section 4 provides concluding remarks and future research paths.

2. Theoretical Framework

Dynamic programming is a versatile optimization method developed by Bellman (1957), which uses the principle of optimality to reduce the number of calculations required to determine the optimal decision path (Kirk, 1970). Bellman's principle of optimality postulates that:

“An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.” (Bellman, 1957, p. 83)

The principle of optimality applies to problems characterized by an optimal substructure, that is, when a problem's solution can be defined as a function of optimal solutions to minimize the size of sub-problems or problems with overlapping sub-problems, so the same problem is solved several times when a recursive solution arises. The idea behind the method is to find a functional form for each problem through the principle of optimality, thereby establishing a recurrence that generates an algorithm solving the problem. The recursive expression essentially converts a T -period problem into a two-period problem with the appropriate rewriting of the objective function. This expression is known as the value function and the mapping from the state to actions is summarized in the policy function.

For the purposes of the dynamic programming problem, it does not matter how the decision sequence was taken from the initial period; all that is important is that agents are rational and act optimally in each period of time (Guerequeta & Vallecillo, 1998). Indeed, the state variables summarize all the information from the past that is required to make a decision. The main features

of the dynamic programming method are its versatility in modeling both continuous and discrete variables, and its capability to introduce uncertainty; this is the only general approach for sequential optimization under randomness (Bertsekas, 2005). As the livestock replacement problem can be represented as a multi-stage decision process involving uncertainty (Frasier & Pfeiffer, 1994), dynamic programming is a natural modeling tool for solving it (Glen, 1987).

Because complexities in finding a closed form solution are common in dynamic programming problems, numerical methods such as the value function iteration procedure, the policy function iteration method, and projection methods are used to solve them. The value function iteration procedure starts from Bellman's equation and computes the value function by iterations on an initial guess; albeit slower than methods that operate on the policy function rather than the value function, it is trustworthy as it has been proved that under certain conditions – a continuous, bounded real-valued payoff and a continuous, compact non-empty constraint – there is a unique value function that solves the problem. Thus, the solution of the Bellman equation can be reached by iterating the value function starting from an arbitrary initial value (Adda & Cooper, 2003; Stokey & Lucas, 1989).

To compute the value function using this procedure, we must define functional forms and discretize state variables. In the case of stochastic dynamic programming problems, the formulation of which includes expected values for the future, we can approximate an order one autoregressive random shock, which comes from a continuous distribution, to a discrete Markov chain using the technique presented by Tauchen (1986). This method simplifies computation of expected values in the value function iteration framework and has the advantage that we can discretize before implementing the numerical method, avoiding the calculation of a cumbersome integral in each iteration.

2.1. Formulation of the model

Determining the optimal selling time for livestock is a basic problem that farmers face. We define this as the time at which farm managers maximize the net expected present value of financial profits associated with livestock management, $\Pi(q_t, p_t)$, where the state variables are q_t , the animal's weight (kilograms), and p_t , the price per kilogram (US dollars).

Specifically, at each point in time, the agent chooses whether to sell or to wait another period. Given that this problem fits within the family of problems called optimal stopping problems (Chow,

Robbins & Sigmund, 1971), we can describe it as a dynamic stochastic discrete choice problem, which can be expressed as a two-period problem using Bellman's equation.

Formally, let $V(q_t, p_t)$ represent the value function of having an animal in state (q_t, p_t) . We can express this as the maximum value between keeping the animal and selling it, and thus:

$$V(q_t, p_t) = \max\{V^k(q_t, p_t), V^s(q_t, p_t)\} \quad (1)$$

where, $V^k(q_t, p_t)$ and $V^s(q_t, p_t)$ represent the value functions of *keeping* and *selling* the animal in state (q_t, p_t) , respectively.

This problem has a non-convex value function, which is common in economic applications but is unusual in dynamic programming applications given the complexity of introducing it in the dynamic programming framework.

We define δ as the probability of death, $E[V(\cdot | I_t)]$ as the expected value function conditioned by the information available in period I_t , and $\Pi(\cdot)$ as the present value of profit from selling the animal. Then, the value of keeping the animal is the expected value function of the next period conditioned on the available information at time t , multiplied by the survival probability. The value of selling the animal is the present value of the profit. Thus:

$$V^k(q_t, p_t) = (1 - \delta)E[V(q_{t+1}, p_{t+1} | I_t)] \quad (2)$$

$$V^s(q_t, p_t) = \Pi(q_t, p_t) \quad (3)$$

The net present value of profit at time t is the present value of income, discounted at rate r , minus the initial inversion made when the producer bought the animal at $t = 0$, and the present value of the costs per kilogram earned in each keeping period. Hence:

$$\Pi(q_t, p_t) = \beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^t \beta^s \tilde{c}(q_s - q_{s-1}) \quad (4)$$

where $\beta = (1 + r)^{-1}$ and \tilde{c} is the average cost per kilogram.

Let a_t represent the age of the cattle; a_t is implicitly a control variable as it maintains a straight relation with the state variable weight, q_t , and the real control variable, which is the time an investor should keep the animal.

We assume that the weight of the cattle, q_t , is a function of the age and a Gaussian stochastic perturbation. We also introduce square age to gather the concavity in weight evolution. Empirical evidence suggests that animals gain more weight when they are calves.

In addition, we model price per kilogram, p_t , as the product between two components. The first component is the expected price conditioned on the weight. The second component (u_t) is an autoregressive Gaussian process; this represents changes around the expected price. Modeling prices in a multiplicative form, rather than an additive form, simplifies the interpretation and analysis of price shocks. For instance, $u_t = 1$ implies a neutral situation. We introduce these shocks because prices are a source of uncertainty that affects business profitability.

The functional forms that define the state variables q_t and p_t are:

$$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t \quad (5)$$

$$p_t = E[\bar{p}_t | q_t] u_t \quad (6)$$

$$\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t \quad (7)$$

$$u_t = \mu(1 - \phi) + \phi u_{t-1} + \xi_t \quad (8)$$

where, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, and $\xi_t \sim N(0, \sigma_\xi^2)$.

3. Empirical Application

3.1. Estimation

To apply our methodological approach, we estimate equation (5) using 24 representative fattening cattle that were weighed at different ages since they were weaned at the age of 10 months. This dataset comes from an extensive cattle farm, providing a sample size of 162 observations, meaning that the farmer weighed each animal approximately seven times. Also, we found that farm managers sold these animals at a weight of 440 kg on average. In addition, we use average weight and market prices between October 2010 and May 2013 to estimate equations (7) and (8).

Table 1 shows the estimation results of equation (5). The coefficients have the expected signs, gathering the concavity in age (we show the regression diagnostics in Appendix 1). Figure 1 shows the relation between age and weight for the representative animal; as we can see, weight increases at a declining rate.

Table 1. Parameter estimates: age versus weight

Weight $q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$		
Observations	162	
R^2	0.681	
Parameter	Value	Standard error ^a
η_1	26.43***	0.878
η_2	-0.34***	0.046

***Significant at the 0.01 level

^a. Robust standard errors

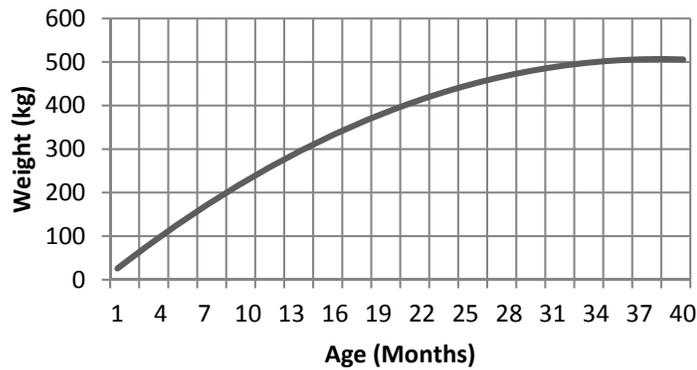


Fig. 1 Average relation between age and weight

We obtain the parameters of price in two phases: in the first stage, we estimate equation (7); then, we calculate u_t using equation (6) to estimate an autoregressive model with drift (equation (8)). Table 2 displays the estimation results. The coefficients are significant at the 0.05 level and correspond to those expected based on theory (we show the regression diagnostics in Appendix 1).

Figure 2 exhibits the price prediction conditioned on weight. As we can see, the price per kilogram decreases at decreasing rates: as the animal weighs more, the marginal value for gaining a kilogram is lower; that is, the relative price of a kilogram is higher when the animal is younger.

Table 2. Parameter estimates: price equations (US\$/kg)

Price $p_t = E[\bar{p}_t q_t]u_t$		
First stage		
$\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$		
Observations	180	
R^2	0.250	
Parameter	Value	Standard error ^a
γ_0	1.7799***	0.0514
γ_1	-0.0014***	0.0003
γ_2	1.32×10^{-6} ***	4.35×10^{-7}
Second stage		
$\frac{p_t}{E[\bar{p}_t q_t]} = u_t = \mu(1 - \phi) + \phi u_{t-1} + \xi_t$		
Observations	95	
R^2	0.122	
Parameter	Value	Standard error ^a
μ^b	1.002***	0.007
ϕ	0.354***	0.099

***Significant at the 0.01 level

^a. Robust standard errors

^b. Do not reject the null hypothesis of $\mu = 1$ at the 0.05 level

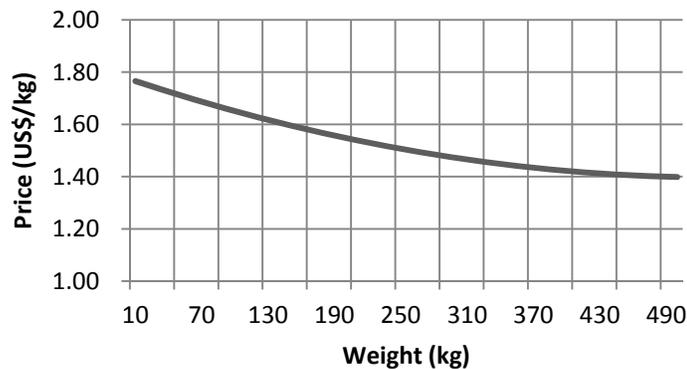


Fig. 2 Average relation between price and weight

We set the mortality rate at 2%, which is consistent with empirical evidence for the livestock sector in the region (FEDEGAN, 2006). The average cost per kilogram of cattle weight in this farm is US\$0.5. The monthly interest rate is equal to 1%, corresponding to an annual interest rate of 12.7%, which is the average annual interest rate for a credit loan in the country.

3.2. Dynamic programming

We must use a numerical technique to approximate the solution because the problem presented in section 2.1 does not have a closed solution. This is a valid mechanism as the problem fulfills the conditions to ensure that the value function can be achieved by iteration (that is, the operator T , mapping from a guess concerning the value function to another value function, is contracting mapping). Therefore, we implement the value function iteration procedure to compute the value function from an initial guess. To solve the dynamic problem using the value function iteration method, we follow four steps: first, the specification of functional forms; second, the discretization of both control and state variables; third, the computation of iterations and definition of tolerance parameters; finally, the evaluation of the value and the policy functions.

We performed the first step in section 2.1, in which we specified all the functional forms, including the payoff functions for selling and keeping the animal. To complete the second step, we discretize the control variable age a_t into 36 points, with each point representing a month; thus, the time horizon is set over three years, which is the maximum time that animals stay on the farm in our study case. Taking the age discretization, we can discretize the weight and expected price through equations (5) and (7). As the multiplicative random shocks of the price come from a continuous distribution that follows a Gaussian autoregressive process of order one with parameters (μ, ϕ, σ_ξ) , we implement Tauchen's (1986) procedure to avoid the calculation of an integral for the expected value function in each iteration. This method approximates an autoregressive process of order one using a Markov chain to create a discrete state space of the shock process, discretizing it into N optimal points and defining the transition matrix $\pi_{ij} = P[u_t = u_i | u_{t-1} = u_j]$ by calculating the transition probabilities between points. Therefore the Markov chain mimics the autoregressive process (Adda & Cooper, 2003; Tauchen, 1986; Tauchen & Hussey, 1991). We show the pseudo-code in Appendix 2.

We use the parameters given in section 3.2 to run the code. In addition, we discretize age and price shocks into 36 and 500 points, respectively. Simulation exercises show that the autoregressive process is well approximated and that 500 points are sufficient to reach an equilibrium point in the resulting value function. The method takes 21 iterations to converge to the value function V , which we present in Figure 3.

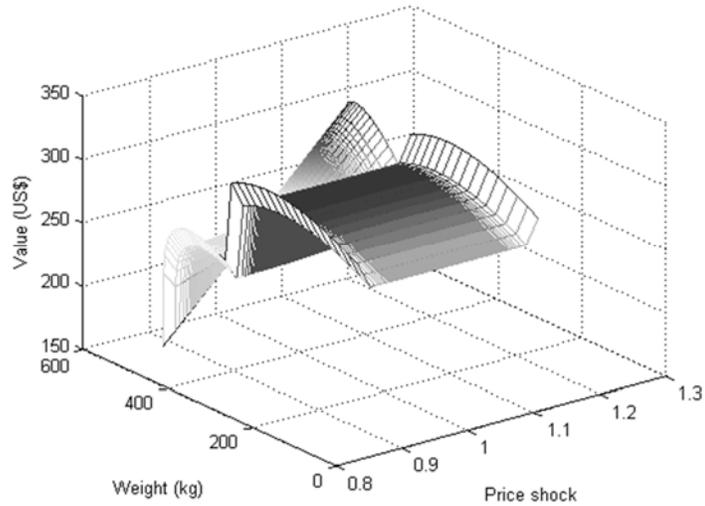
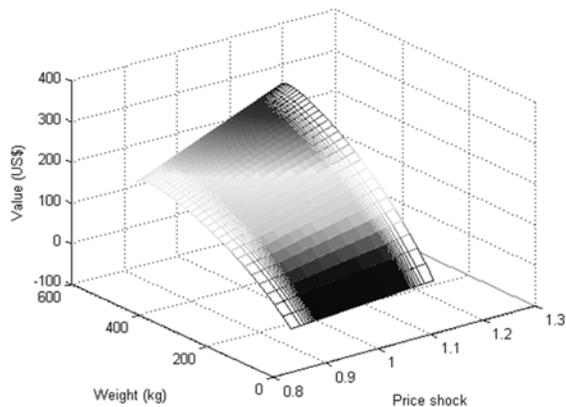
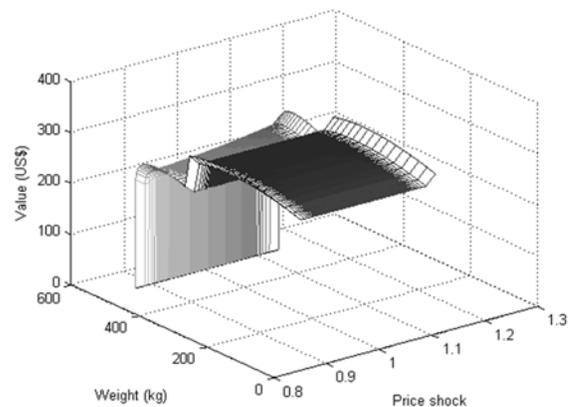


Fig. 3 Value function

Figure 4 presents the selling and keeping value functions V^s and V^k . In panel (a) we can see that when the animal weighs less, that is, when it is younger, the selling function is lower, even negative, meaning that farm managers should wait another period to sell. On the other hand, when there is a positive price shock ($u_t > 1$), the farmer should sell. We observe in panel (b) the keeping value function. In particular, we observe that when the animal is younger, the keeping value function is higher, so the farmer should wait to sell.



(a) Selling value function



(b) Keeping value function

Fig. 4 Selling and keeping value functions

The policy function defines whether the farmer should sell or wait at time t according to the cattle weight and selling price features. Specifically, the policy function takes the value one if the selling value function is higher than the keeping value function. Figure 5 shows the policy function, from

which we deduce that the investor should wait for a positive price shock and a weight of around 300 kg. However, if the animal weighs more than 500 kg, it is not necessary to wait for a favorable price shock to sell.

The value function is formed by blending both selling and keeping value functions, taking the maximum of these at each point of the grid; that is, the value function represents the potential farmer's profit for each configuration of the state variables. However, it is important not to interpret the value function as present value cash profits as there are some configurations of the state variables for which the value function denotes the expected profits of waiting another period. The policy function allows us to determine where the value function actually displays selling profits. Figure 6 displays the net present value of the farmer's profit, that is, the value function of selling cattle.

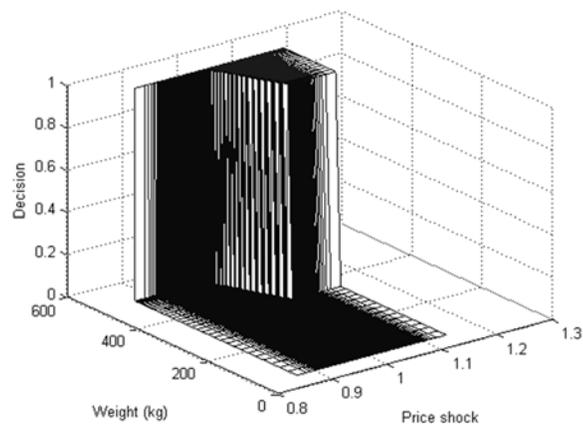


Fig. 5 Policy function

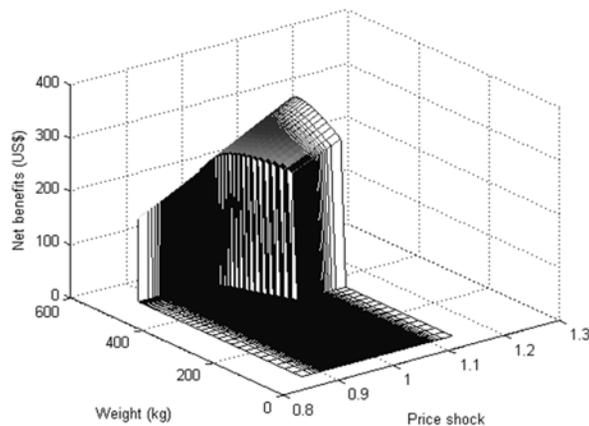


Fig. 6 Value function if the animal is sold

Variable u_t is an unknown price shock that investors cannot predict, so for the decision-making process managers will always expect that shocks take the value of one, which is the mean or neutral situation. Table 3 summarizes the maximum value for each function when $u_t = 1$. It is remarkable that the maximum found for the value function equals the maximum of the keeping value function although the maximum in the selling function is lower. This is explained by the fact that prices have a stochastic component and the calculation when the animal is younger generates expected values that are slightly higher than the real values once the animal gains weight.

In addition, we can see in this table that the present value of cash profits (US\$238.98) is lower than the maximum obtained in other functions. This happens because the configuration that generates the highest value in the selling value function produces a higher value in the keeping value function. Thus, it is better for the owner to wait another period in the hope of a positive price shock in the future, which will represent higher profits, but risking a negative price shock, which represents lower profits.

To summarize, a neutral price situation would imply that managers should sell animals with a weight of 497.6 kg. This generates the maximum attainable present value of profit per animal, i.e., US\$238.98.

Table 3. Maximum values and variable configuration: neutral price situation

Function	Maximum Value (US\$)	Variable Configuration		
		Age a_t (Months)	Weight q_t (Kg)	Price p_t (US\$)
Selling - V_s	241.64	29	480.53	1.44
Keeping - V_k	295.29	12	268.20	1.51
Value - V	295.29	12	268.20	1.51
Value*	238.98	32	497.60	1.44

*Value function if the animal is sold

As stated above, farm managers sell animals weighing 440 kg in our study case. In a neutral price scenario, this weight represents a net present value of US\$235. This is close to the optimal strategy proposed in our framework (US\$238.98), although we obtain a 1.7% higher net return using our proposal.

Let us analyze this 1.7% net return excess: It takes 32 months to achieve an animal weighing 497.6 kg, while it takes 24.4 months to have an animal weighing 440 kg, that is, there is a difference of 7.6 months. This implies an annual net return excess equal to 2.69% $((1 + 1.70\%)^{12/7.6})$. The total factor productivity growth for last few years in the entire economy and the agricultural sector has been estimated at 1.4% and 1.1%, respectively (DNP, 2011). Thus, we find that our methodological approach can generate significant improvements in competitiveness.

Stochastic discrete problems, such as the one that we present, have the feature that a threshold function, representing the point at which the decision of whether to sell or not is indifferent, can be computed. In the model, we can define the threshold p^* as the price at which the choice to sell or keep the animal is indifferent. Thus, if $p > p^*$, the policy function d takes the value of one, that is, the investor should sell.

We can calculate the threshold by equating V^s and V^k , and solving for p^* the following:

$$\begin{aligned}
 V^s(q_t, p_t) &= V^k(q_t, p_t) \\
 \Pi(q_t, p_t) &= (1 - \delta)E[V(q_{t+1}, p_{t+1}|I_t)] \\
 \beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^t \beta^s \tilde{c}(q_s - q_{s-1}) &= (1 - \delta)E[V(q_{t+1}, p_{t+1}|I_t)] \\
 p^* &= \frac{(1 - \delta)E[V(q_{t+1}, p_{t+1}|I_t)] + q_0 p_0 + \sum_{s=1}^t \beta^s \tilde{c}(q_s - q_{s-1})}{\beta^t q_t} \quad (9)
 \end{aligned}$$

Figure 7 depicts the price threshold in a neutral situation. If the price is higher than the threshold given a weight q_t , the investor should sell. For instance, if the price is higher than US\$2.1 per kg for fattening animals that weigh 250 kg, the farm manager should sell those animals.

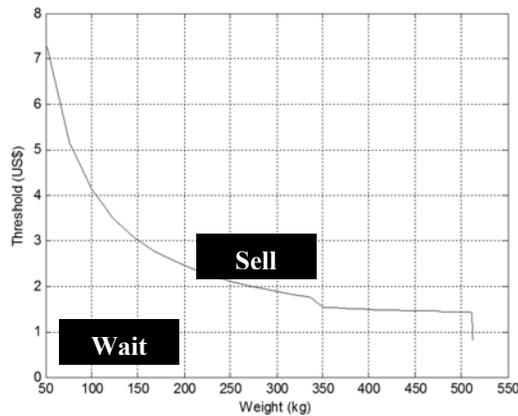


Fig. 7 Price threshold

Finally, an important feature of the dynamic programming framework is its facility to simulate models using the policy function to determine the optimal choice for each period. Furthermore, when we can describe the problem as a stochastic discrete model, simulations are simplified as the policy function is mapped using the threshold function. As a consequence, we can use simulations to describe multiple agents' behavior and the market's configuration patterns through time.

To perform model simulations representing a stock of S animals, we have to define a price shock for each animal at each point in time simulating the S autoregressive process. Then, we can calculate the selling price at each point in time by multiplying the shock and the expected price at that point. Thus, if the price is higher than the threshold, farm managers should sell animals of that specific weight. We use this framework to find the percentage of cattle at age a_t in the herd that farm managers should sell in a rational environment. Appendix 3 shows the pseudo-code.

Figure 8 illustrates our simulation exercise using a herd composed of $S = 10,000$ animals. We observe in this figure the percentage of sales according to weight. For example, our model predicts that in a rational market, 12% of the animals that weigh 351 kg or 30% of the animals that weigh 417 kg are sold at market. In addition, we observe that farm managers should sell 100% of the cattle weighing more than 510 kg. Finally, a clear consequence of our framework is that farm managers should sell 50% of the livestock weighing 497.6 kg.

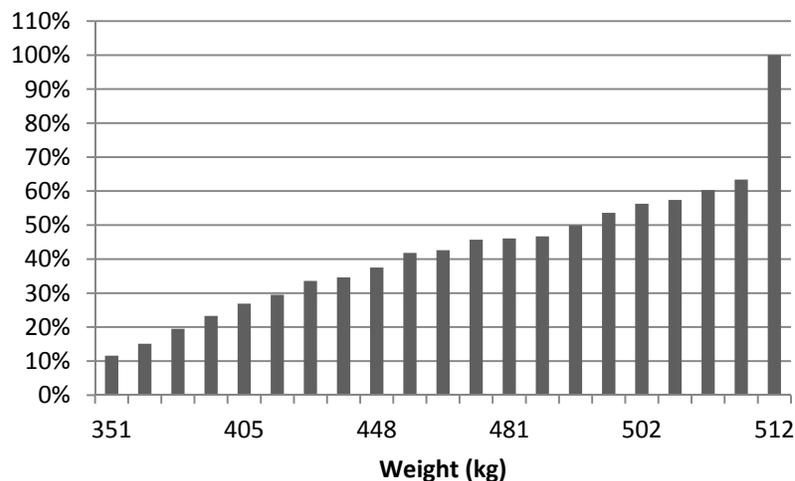


Fig. 8 Simulated sales according to age

4. Conclusions

We introduce a flexible stochastic dynamic program that allows the investor to support decisions concerning the best time to sell fattening cattle. Our proposal contains both economic and biological variables, and involves uncertainty derived from future price realizations. This dynamic program makes it possible to find the optimal time by comparing financial outcomes rather than other biological or technical measurements that are common in the literature; our approach makes it easier to interpret the results as financial profit is a classic figure that investors use to evaluate investments. In addition, our proposal allows us to perform different simulation exercises to identify livestock life cycles in the market.

Our methodological approach is very general, so practitioners can use it in different regions by using appropriated parameter estimates. Moreover, its economic and financial foundations, as well as its mathematical, statistical, and computational framework, can be used as a basis to model other economic sectors.

We find in our study case that although common sense and empirical experience are priceless assets, techniques based on scientific principles can help to improve the level of competitiveness of the livestock sector.

Future work lies in improving our estimation strategy. In particular, we would like to estimate our model using the structure of our stochastic dynamic program. However, we require an excellent micro dataset, as well as a macro dataset, to achieve this objective. Unfortunately, we have not yet found such a resource.

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Appendix 1. Statistical tests

	Equation	Jarque–Bera Normality Test	White's Heteroskedasticity Test
Weight	$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$	1.1 (0.578)*	3.77 (0.012)
Price	First component: $\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$	320.74 (0.00)	3.51 (0.0319)
	Stochastic component: $u_t = \frac{p_t}{E[\bar{p}_t q_t]} = \mu(1 - \phi) + \phi u_{t-1} + \xi_t$	17.20 (0.00)	0.69 (0.504)*

^{a.} * Do not reject null hypothesis

^{b.} *p*-value appears in parenthesis

Appendix 2. Pseudo-code for the value function iteration method applied to the optimal selling time problem.

```

optimalSellingTime()
  Define animal information
  Read  $p_0, a_t, t$ 
   $a_0 \leftarrow a_t + t$ 
  Define parameters
  Read  $\delta, r, \tilde{c}$ 
   $\beta \leftarrow (1 + r)^{-1}$ 
  Initialize  $\eta_1, \eta_2, \gamma_0, \gamma_1, \gamma_2, N, \mu, \phi$ 
  Discretize Variables
  Discretize AR  $u \leftarrow$  Tauchen procedure( $N, \mu, \phi$ )
  Save probability transition matrix  $\pi$ 
  Discretize Age  $a \leftarrow a_0: 1: a_0 + 36$ 
   $q \leftarrow \eta_1 a + \eta_2 a^2$ 
   $q_0 \leftarrow q(1)$ 
   $E[\bar{p}|q] \leftarrow \gamma_0 + \gamma_1 q + \gamma_2 q^2$ 
   $p \leftarrow uE[\bar{p}|q]$ 
  Iterate Value Function
  Define maxIter, tol
  for  $i_q = 1$  to size(a) - 1
    for  $i_p = 1$  to size(u)
       $t \leftarrow i_q$ 
      Initialize  $V(i_q, i_p) \leftarrow \beta^t q(t)p(i_p, t) - q_0 p_0$ 
    end for
  end for
  for  $i = 1$  to maxIter
    for  $i_q = 1$  to size(a) - 1
      for  $i_p = 1$  to size(u)
         $t \leftarrow i_q$ 
         $\delta_q = q(t + 1) - q(t)$ 
         $c(t) \leftarrow \beta^t \tilde{c}$ 
         $sum_c(t) \leftarrow \sum_{s=1}^t c(s)$ 
         $V_s \leftarrow \beta^t q(t)p(i_p, t) - q_0 p_0 - sum_c(t)$ 
         $V_k(i_q, i_p) \leftarrow (1 - \delta)\pi(i_p, :)V(i_q + 1, :)$ 
         $V_{aux} \leftarrow \max(V_s, V_k)$ 
      end for
    end for
    error  $\leftarrow \max((V_{aux} - V)/V)$ ;
    if error < tol then break else  $V \leftarrow V_{aux}$  end if
  end for
  Calculate Policy Function
  Policy function  $d \leftarrow V_s > V_k$ 
end optimalSellingTime

```

Appendix 3. Pseudo-code for simulating sales behavior applied to the optimal selling time problem.

```
Simulations()
  Define information
  Define number of periods  $a$ 
  Read threshold function given  $u = 1 T_{a \times 1}$ 
  Read expected price  $E_p_{a \times 1}$ 
  Define parameters
  Initialize number of simulations  $S$ 
  Initialize AR Parameters  $\mu, \phi, \sigma_u$ 
  Simulate AR
  Define Burn-in iterations  $B$ 
   $e_{(B+a) \times S} \leftarrow$  generate shocks  $\sim N(0, \sigma_u^2)$ 
  Initialize  $u(1, :) \leftarrow \mu(1 - \phi) + e(1, :)$ 
  for  $t = 2: (B + a)$ 
    for  $s = 1$  to  $S$ 
       $u(t, s) \leftarrow \mu(1 - \phi) + \phi u(t - 1, s) + e(t, s)$ 
    end for
  end for
  Drop  $B$  first simulations of  $u$ 
  Simulate agent's behavior
  for  $t = 1: a$ 
    for  $s = 1$  to  $S$ 
       $p(t, s) \leftarrow u(t, s) E_p(t)$ 
      if  $p(t, s) \geq T(t) \rightarrow \text{sell}(t, s) = 1$  else  $\text{sell}(t, s) = 0$  end if
      if  $\text{sell}(t, s) = 1 \rightarrow \text{Csell}(t, s) = 1$  else  $\text{Csell}(t, s) = 0$  end if
      if  $t > 1$ 
        if  $\text{Csell}(t - 1, s) = 1 \rightarrow \text{Csell}(t, s) = 1$  end if
      end if
    end for
  end for
end Simulations
```
