A Dead Time Compensator Based on Linear Algebra (DTCLA)

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Abstract: This paper proposes a dead time compensator based on a combination of the linear algebra control methodology (proposed by Scaglia, 2009) and an internal model structure (Smith predictor), to solve elevated dead time system problems. Moreover, the process transfer function of the nonlinear system linearized at some operation point is approximated by a second order plus dead time transfer function. The controller performance is judged by simulations and it is evaluated using the ISE performance index. We compared this approach against a typical DTC-PID controller with improved results.

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1. INTRODUCTION

The presence of time delays in many industrial processes is a well-recognized problem. Time lag, transportation lag, time delay and dead time are common phenomena in industrial processes. Time delay can be produced by measurement lag, analysis and computation time, communication lag or the transport time required for a fluid to flow through a pipe. Also, several bioprocesses concern with dynamics and certain delays related to the capability of microorganisms of reaching a good growth rate or inhibition due to the concentration of product and substrate (Quintero et al, 2008) (Amicarelli et al, 2016). Certainly, the control of some of the variables for these kinds of systems must be performed from a Nonlinear Point of View.

In Quintero et al, 2009 we demonstrate that Linear Algebra based controllers can, in an easily and understandable formulation, perform not only the trajectory tracking control but also the positioning at certain point. But the controller design must be performed under a zero error model of the nonlinear system, in certain cases too much complicated and highly nonlinear. Some approaches were also explored to control this kind of bioprocesses (di Sciascio et al, 2004).

On the other hand, the achievable performance of typical feedback control systems can decline if a process has a relatively large time delay compared to the dominant time constant (Smith and Corripio, 1997). Predictive structures and sliding mode controllers (Camacho, 2002), (Camacho, et al, 2007) have been used to solve such problems. Primarily, internal model control (IMC) and the Smith predictor (SP) are the most popular predictive structures used for time delay compensation (Marlin, 1991, Smith and Corripio, 1997). Furthermore, when the process presents an integral behaviour the original structures cannot be used since a constant load disturbance results in a steady-state error (Camacho and De la Cruz, 2004). To overcome this obstacle different approaches have been proposed (Watanabe, 1981). Simulation studies have shown that the set point and load disturbances are either very oscillatory or highly damped when the process has a large time delay (Astrom, 1994). To deal with this additional problem new structures were proposed, decoupling the disturbance from the set point response (Zhang and Sun, 1996) (Astrom et al, 1994). In general, these approaches have some problems: they are sensitive to modelling errors since the design requires the use of a process model, which can be difficult to obtain in practice. Modelling errors are unavoidable and they result in a mismatch between the model and the actual plant. Thus, the controllers designed using particular models may perform quite differently when they are implemented on the actual process (Quintero et al, 2009).

In addition, the methodology based on linear algebra and numerical methods concepts to design control algorithms is a simple and relatively new method that allows the control of highly nonlinear systems. It has been applied to the design of different class of control systems such as trajectory tracking of mobile robots and UAV’s (Capito et al, 2016), chemical plants and bioprocesses (Quintero, et al 2009, Scaglia et al, 2015) just to name some applications. Its main advantage is that the conditions for the tracking error tend to zero and control actions are obtained with low computational cost which makes it easy to implement in a microcontroller. These conditions are found by solving a system of linear equations and conditioning the system to have exact solution. For all the previous applications, the controllers were designed by using the complete nonlinear and discrete process model (Quintero et al, 2009; Scaglia et al, 2009, Serrano et al, 2016). Therefore, the obtained mathematical controller algorithm is applied to the specific process. Hence, for each different process a different controller should be obtained, in other words, it is generated a different control law in each case.

Besides, there are two problems with the use of a model as far as industrial processes are concerned. First, the development of a complete model is difficult due mainly to the complexity of the process itself, and to the lack of knowledge of some
process parameters. Second, most process models relating the controlled and the manipulated variables are of higher-order. Therefore, the traditional numerical controller procedure can produce more complex controllers and also their application is just for the process of analysis.

An efficient alternative modelling method for process control is the use of empirical models, which use low-order linear models with dead time (Marlin, 1991; Smith and Corripio, 1997). Most times, first-order-plus-dead-time (FOPDT) models are adequate for process control analysis and design. In many cases, mobile robotics as well as chemical processes, can be represented by first-order plus dead time (FOPDT) models (Capito, et al 2016).

The internal model control design is based on the idea of forming an ideal and simple controller based on the transfer function of the process model, and it has presented satisfactory results, as in (Camacho et al 2002). When controlling chemical processes, it is important to simplify the process model, because these processes are often represented by several state space equations, and then the controllers must be resistant to modelling errors and uncertainties.

Guevara et al, 2016 designed a Linear Algebra (and Numerical Methods) based controller (NMCr) designed on a first order plus dead time (FOPDT) model of the actual process. The overall idea is to develop a general NMCr, which can be used for different self-regulating industrial processes, if they have a similar open loop response, such as an FOPDT model. Hence, this work summarizes the easily and simplicity of numerical methods procedures along to a reduced order model of the process to obtain a simple and versatile controller. The performance of the proposed controller in this article is tested by simulations, but the performance degrades when the controllability relationship \( \frac{\tau}{\tau_0} \) increases.

This paper proposes from a Smith Predictor structure the synthesis of a Deadtime compensator that uses the linear algebra methodology. This new approach uses a second order plus dead-time model obtained from the reaction curve of the nonlinear process. This novel proposal has the capability of solving elevated dead time problems. The proposed scheme is tested and compared against the previous approach built from reduced first order plus dead time model of the process (Guevara et al, 2016).

This work is organized as follows: section 2 presents the fundamentals of the linear algebra based controllers and IMC control and section 3 explains the Dead Time Compensator Based on Linear Algebra (DTCLA). Then section 4, presents some study cases where the DTCLA scheme is presented and compared with a regular DTC-PID. Finally, conclusions are presented in section 5.

2. BASIC CONCEPTS

This section presents briefly some basics of the technique developed by Scaglia in 2009 and then basic ideas of Internal model control (Francis and Wonham, 1976). These concepts are the foundation for the approach that is presented in section 3. It is important to mention that the internal model structure is used as a Smith predictor.

2.1 Linear Algebra methodology

The numerical methods and linear algebra tracking control methodology has been proved over highly nonlinear systems for bioprocesses (Quintero et al, 2009) (Scaglia et al, 2009, 2014), and also in under-actuated mechanical systems, dynamic nonlinear systems controlled with a kinematic model for tracking and positioning control tasks for fast and slow dynamics systems such as mobile robots (Scaglia et al, 2009, 2010, 2015; Serrano et al 2014, Rosales et al 2015). Stability and robustness demonstrations can be found in (Scaglia et al, 2010, 2015) and parameters optimization through Monte Carlo methods (Serrano et al, 2016). The methodology for calculating the control actions can be summarized in the following steps:

1. To describe the system using state equations and then approximate them using a numerical method.
2. To set out the control actions calculation as a problem of solving a system of linear equations.
3. To choose the state variables to be followed and under what conditions the system of linear equations, of the previous point, has exact solution, and then to define the reference trajectory of the remaining state variables.
4. To solve the system of equations.

2.2 Internal Model Structure as Smith Predictor

This controller structure can be seen in Francis and Wonham, 1976. In this structure \( G_P(s) \) represents the process, \( G_C(s) \) is the controller transfer function. \( G_m(s) \) is the process model, \( e_m \) represents the modelling error, \( yr \) is the process output, \( y_m \) is the model output and \( y_m^\text{em} \) is the output of the non-invertible part of the model. To implement it, it is necessary to obtain the process model and separate it into two parts: a non-invertible \( G_m(s)^+ \) and an invertible part \( G_m(s)^- \), as shown in (1).

\[
G_m(s) = G_m(s)^-G_m(s)^+
\]  
(1)

To obtain this controller, only the invertible part \( G_m(s)^- \) is used, because the non-invertible part \( G_m(s)^+ \) can present problems such as being unstable, not causal or cannot be found.

3. A DEAD TIME COMPENSATOR BASED ON LINEAR ALGEBRA (DTCLA)

In this section is shown the development of the proposed Dead Time Compensator Based on Linear Algebra (DTCLA). This proposal is based on the improvement of the control scheme by (Guevara et al, 2016) in which the linear algebra controller design is based on a first order plus dead time model of the system. For this new approach, it is assumed that the process presents an overdamped dynamics and the parameters of a transfer function are obtained from the reaction curve method identification (Smith and Corripio, 1997). The process model is represented as a second order system plus delay transfer function, which is divided into two components to be added to the control scheme (Astrom et al, 1994).
In figure 1, it is represented the proposed scheme, it can be seen that the controller based on linear algebra needs as the signal to its calculations the output of the invertible part of the transfer function from the internal model structure in a similar way as in the Smith predictor architecture. Therefore, the Linear Algebra method is applied to the part of the reduced model without time delay.

![Diagram of the proposed scheme](image)

Fig. 1 Proposed scheme for the DTCLA.

First of all, let us show the model of the process, the system model to approximate the process is described by (2):

$$G_m(s) = \frac{k_m e^{-t_0s}}{(\tau_{m1}s+1)(\tau_{m2}s+1)}$$

Where:

$$\begin{align*}
G_m^{-}(s) &= \frac{k_m}{(\tau_{m1}s+1)(\tau_{m2}s+1)} ;
G_m(s) &= e^{-t_0s}
\end{align*}$$

where:

- $k_m$: Static gain.
- $\tau_{m1}$: Time constant 1.
- $\tau_{m2}$: Time constant 2.
- $t_0$: Dead time.

Rearranging some terms in Eq. (2), the invertible part $G_m^{-}(s)$, can be represented as:

$$G_m^{-}(s) = \frac{y_m(s)}{u(s)} = \frac{k_m K_B}{s^2+K_A s+K_B}$$

Where:

$$\begin{align*}
K_A &= \frac{\tau_{m1}\tau_{m2}}{\tau_{m1}+\tau_{m2}}
K_B &= \frac{1}{\tau_{m1}\tau_{m2}}
\end{align*}$$

To facilitate the next calculations, let us call $y_m(s) = \chi(s)$. Now for controller design (4) can be represented in differential equation form as follows:

$$\dot{\chi} + K_x \dot{\chi} + K_B \chi = k_m K_B u$$

Representing this second order equation in state variables form, the following expression is obtained:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K_B & -K_A \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ k_m K_B \end{bmatrix} [u]$$

Where $X_1$ and $X_2$ are the state variables of the system, being the derivate of the controlled variable and $u$ is the controller output. From the state variables representation, the following equations are obtained:

$$X_1 = X_2$$

$$k_m K_B u = \dot{X}_2 + K_A X_2 + K_B X_1$$

Applying Euler approximation to discretise the derivative of the state variables, the following equations are obtained:

$$\begin{align*}
X_2 n &= \frac{X_1 n+1 - X_1 n}{T} \\
k_m K_B u n &= \frac{X_2 n+1 - X_2 n}{T} + K_A X_2 n + K_B X_1 n
\end{align*}$$

Where, $T$ is the sampling time. The sampling time for the discrete controller will be in terms of the time constants of $\tau_{m1}$ and $\tau_{m2}$.

$$\min(\tau_{m1}, \tau_{m2})/15 < T < \min(\tau_{m1}, \tau_{m2})/4$$

To reduce the resulting expressions, the difference between the reference $X_{1 ref n}$ and the actual output $X_n$ will be represented by $e_n$. Assuming that the current value of variable $X_2 n$ is the same required for a next discrete time $X_{2 n+1}$ (see remark 1). Then it can replace (14) in (12) to obtain:

$$k_m K_B u_n = \frac{X_{1 ref n+1} - X_{1 n} - K_x e_n}{T^2} - \frac{X_2 n}{T} + K_A X_2 n +$$

Finally, from previous equation, the control law $u_n$, is obtained:

$$u_n = \frac{1}{k_m K_B T^2} \left[ X_{1 ref n+1} - X_{1 n} + K_x e_n - T X_2 n \right] +$$

$$\frac{X_1 n}{k_m} + \frac{K_A X_2 n}{k_m}$$

Where:

$$X_2 n = \frac{X_1 n - X_{1 n-1}}{T}$$

One of the main advantages of the proposed scheme is that it presents a fixed architecture facilitating its implementation, therefore the proposed controller that can be easily used for nonlinear and different kinds of processes that fit the premise...
Remark 1: \( x_{2(n+1)} \) can be estimated using the Taylor’s formula:

\[
\begin{align*}
x_{2(n+1)} &= x_{2(n)} + \frac{d^2 x_2}{dt^2} T + \frac{d^2 x_2}{dt^2} \frac{T^2}{2} + \cdots + C \\
\end{align*}
\]

Where, \( C \) is the complementary term (Hildebrand, 1987). So, if the sampling time is small, \( x_{2(n+1)} \) can be approximated in one of following ways:

\[
\begin{align*}
x_{2(n+1)} &\approx x_{2(n)} \\
x_{2(n+1)} &\approx x_{2(n)} + \frac{d^2 x_2}{dt^2} \frac{T}{2} \\
x_{2(n+1)} &\approx x_{2(n)} + \frac{x_{2(n)}^2 - x_{2(n-1)}^2}{\tau} + \frac{x_{2(n)}^2 - 2x_{2(n-1)}^2 + x_{2(n-2)}^2}{\tau} \frac{T^2}{2} \\
\end{align*}
\]

4. SIMULATION RESULTS AND DISCUSSION

The process model will be described by the following transfer function:

\[
G_p(s) = \frac{1}{(s + 1)(0.5s + 1)(0.25s + 1)(0.125s + 1)e^{-9.7s}}
\]

4.1 Guevara et al, approach

From reaction curve procedure a FOPDT model is obtained.

\[
G_{m1}(s) = e^{-10.38s} \frac{1}{(1.305s + 1)}
\]

The controllability relationship is given by \( \frac{\tau_0}{\tau_m} = \frac{10.38}{1.305} = 7.95 \). The time delay can be replaced by a pole located in \(-1/10.38\). Thus, the former transfer function can be modified as indicated in (Camacho et al, 2003),

\[
G_{m2}^{-}(s) = \frac{1}{(1.305s + 1)(10.38s + 1)}
\]

Following the procedure as was described in (Guevara et al, 2016), the best results obtained can be seen in the following figure:

Fig. 2. System response without error.

It also can be seen that the system reacts to the events in a time instant equal to the delay, following the setpoint changes and rejecting the constant disturbances with zero error.

To test the robustness of the system 1000 tests were calculated through Monte Carlo simulations as in (Serrano et al, 2016), varying the parameters \( k_A, k_B, k_m \) with a uniform distribution. Results can be seen in Fig. 3.

From Fig. 3 can be seen that system follows the reference signal even with a constant disturbance in presence of modeling errors. The proposed control structure allows following different continuous trajectories for each change in the set point.

The previous model can be decomposed in two parts

\[
G_{m3}^{-}(s) = \frac{1}{(0.9173s + 1)(0.7120s + 1)}
\]

\[
G_{m3}^{+}(s) = e^{-9.95s}
\]

The controller design was based on \( G_{m3}^{-}(s) \) as process model and the procedure was the same as indicated in (17). In Fig. 2 can be seen the response of the system when the set point goes from 1 to 1.5 in time equal to 175 seconds. It is also necessary to establish certain boundaries for stability and robustness against disturbances. For this reason, the system was perturbed with magnitude 0.3 in time 100 seconds until the end of the simulation, from Fig. 2 can be seen that the controller can lead the system to the reference very fast.
Fig. 4 shows the system response when the set point is a continuous signal variant in time and also it is present a load disturbance, the system is capable of following the reference signal. If the reference is known each time instant, it is possible to advance the reference. The system response with an advanced reference can be seen in Fig. 5 and Fig. 6, with variable and constant set points respectively, also the system follows the reference signal in a precise way without the delay shown in Fig. 2 and 4. The closed loop system reacts to the disturbance because of neither the magnitude nor the time instant are known.

Fig. 5 System response advancing the reference signal and constant disturbance.

Fig. 6 System response advancing the reference signal and constant disturbance.

Fig. 7 Comparison of the DTC with a PID controller and a LA controller following a variable reference.

which leads to an easy design with better and faster control actions. The scheme showed accurate results and presented a good performance against disturbances of big magnitude. Also, the robustness of the proposed controller was achieved through Monte Carlo simulations to find the accurate boundary for the parameters. This controller is a good option for elevated dead time systems. Therefore, it can be used for processes with a controllability relationship greater than two. The proposed approach can track variable time references, therefore it can be applied in robotics, teleoperation systems, bioprocesses, chemical processes and so on. The proposed approach was compared against a typical deadtime compensator for the same kind of systems and presented better results. The deadtime compensator based on linear algebra presents an easy algorithm which does it very suitable for industrial implementation.

5. CONCLUSIONS

In this work was proposed a dead time compensator that combines the Smith Predictor control architecture and the Linear algebra controller methodology. For designing purposes, a second order plus dead time model was used. To get the controller only the invertible part of the model was used.

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