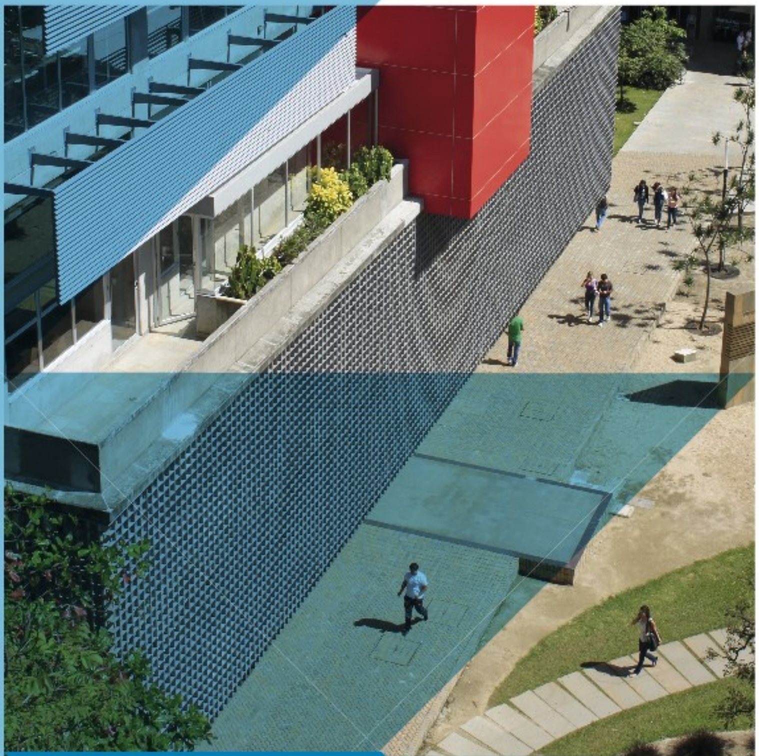

Escuela de Economía y Finanzas

Documentos de trabajo
Economía y Finanzas

Centro de Investigación
Económicas y Financieras

No. 18-04
2018

Dynamic Macroeconomics: A Didactic Numeric Model

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Abstract

Teaching Dynamic Macroeconomics at undergraduate courses relies exclusively on intuitive prose and graphics depicting behaviours and steady states of the main markets of the economy. But when the case of forward-looking agents and the macroeconomic implications of their actions are discussed, intuitions and graphical representations offered to students may lead to unsupported conclusions. This happens even if the teacher and students use the chapter upon a dynamic macroeconomic model of one of the most didactic and ordered texts ever published: Williamson (2014). In this paper we try to sustain this assertion3.

JEL Classification: A22, A23, C61, C63, E00, E17

Key Words: Dynamic Macroeconomics, Forward-looking agents, General equilibrium, Competitive markets, Total factor productivity, Public expenditure multiplier.

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[3] The present document draws back on Posada (2017) but it has several modifications being the principal one the elimination of ad hoc assumptions related to the labor supply.

I. Introduction

Williamson (2014; chap. 11) exposes a macroeconomic intertemporal model (two periods: present and future) in a prose version including graphics. This is one of the most relevant contributions of his text and, perhaps, the main component of its advantage vis á vis other Macroeconomics handbooks for undergraduate students. But the author does not provide a mathematic version of the model, not even in the mathematical appendix.

Such omission compels the reader to attempt to apply the model playing (usually) with simultaneous displacement of two curves (supply and demand) in two frames (labor and output markets), and only for the present period. The results out of such game depend on the intensity of the displacement of the different curves, and, in some cases, it is not clear why more or less intensity is applied to one or other of these movements.

Up to where our knowledge reached there is not a mathematical version of the model which could help to a more profound understanding of it, and doing numerical simulations. The aim of this document is presenting a such version. The proposed model can be used to make simulations, exercises, etc., in a spreadsheet such as Excel.

The document has seven sections, being this the first one. On section II general characteristics of the model are described; along the III section a partial equilibrium specification based on the household decisions is presented; in the IV section the macroeconomic (dynamic general equilibrium) model and solution are presented; the V contains a model´s calibration and results of the numeric exercises with it; section VI is an intuitive and graphical analysis of the results; finally, section VII presents the conclusions. Two annex show several elements of the matrix system´s mathematical approach, and the numerical inverse matrix required to do the simulations.

II. Basic characteristics of the model

The model is a simplified representation of the paradigm of a decentralized closed economy in which the representative agents act rationally, are completely informed and have perfect foresight (similar households, homogeneous firms and government). The agents are price takers (competitive markets). So, the markets work efficiently without help of economic policies. The model describes a stable equilibrium situation of an economy with 5 markets: present labor, future labor, present (aggregate) product, future product and credit. Nevertheless, this fifth market does not play a relevant role (the credit system is "passive" or "accommodating"): equilibrium in the first 4 markets implies that the fifth (credit market) will also be. The exogenous variables of the model are 7: present public expenditure, future public expenditure, present capital (at the beginning), present total factor productivity (TFP), future TFP, capital depreciation rate and taxes per household in the present period. On the other hand, the endogenous variables are 15. The model's core is a set of 10 equations (and 10 endogenous variables; the other 5 variables can be deduced straightforward using the solution of the model´s core and simple equations), and the economy's equilibrium can be understood as the set of equilibrium values of the endogenous variables consistent with the situation in which all agents optimize, subject to restrictions, and all markets are cleared. If sum lump taxes (and subsidies) are assumed (and, then, do not influence on incentives), general equilibrium results will be equivalent to a Pareto optimum. The model contemplates only one tax (on each period); this is a sum lump tax on households.

Since the model's core describe the mechanisms and functions of four "active" markets (each of them related to one merchandise (present output, future output, present employment and future employment), it considers 3 relative prices: present real wage, future real wage and real interest rate. The real interest rate, which is determined as the other 2 relative prices of the model, contributes to the equilibrium of the four "active" markets, and, hence, the credit market.

III. The partial equilibrium

Households decisions depend on maximizing a two periods utility function related to present and future consumption and leisure. Additionally, there is a quantity of time in each period, h, to distribute between work, n, and leisure, l. The trade-off between consumption and leisure is evidenced since the more leisure the families choose, the less working hours they sell; consequently, they obtain less income and less consumption.

So, let us consider this representative household problem:

$$\max\ U(c, c', l, l') = \ln(c^{\gamma} l^{1-\gamma}) + \beta \ln\left(c'^{\gamma} l'^{1-\gamma}\right); 0 < \gamma \ and\ \beta < 1$$

Where c, c', l, l', β stand for present consumption, future consumption, present leisure, future leisure and intertemporal discount factor, respectively.

In addition to the temporal restrictions ($h = n + l; h = n' + l'$), the maximization is subject to the intertemporal budget constraint. This one is derived from the budget constraints corresponding to each period:

$$c \leq wn + \pi - t - s$$

$$c' \leq w'n' + \pi' - t' + s(1 + r)$$

Where w represents the real wage, π benefits obtained out of the firms (households own the firms), t taxes, s savings, and r the real interest rate4.

Consequently, the intertemporal budget constraint (with the temporal restrictions) is:

$$c + \frac{c'}{1 + r} \leq w(h - l) + \pi - t + \frac{w'(h - l') + \pi' - t'}{1 + r}$$

In the partial equilibrium, present labor supply, future labor supply and savings can be found out according to the first order conditions. Nevertheless, the demand in each of these markets can´t be found; for this reason, relative prices (real wage and interest rate) must be imposed exogenously.

The maximization is done on a leisure (hence working)-consumption decision in order to obtain the labor supply curve. The first order conditions (F.O.C) obtained out of the maximization problem are:

$III.1)$ $\dfrac{\gamma}{c} - \lambda = 0$

$III.2)$ $\beta \dfrac{\gamma}{c'} - \dfrac{\lambda}{1 + r} = 0$

$III.3)$ $\left(\dfrac{1 - \gamma}{h - n}\right) - w\lambda = 0$

$III.4)$ $\beta \left(\dfrac{1 - \gamma}{h - n'}\right) - \dfrac{w'\lambda}{1 + r} = 0$

$III.5)$ $c + \dfrac{c'}{1 + r} - wn - \dfrac{w'n'}{1 + r} - \pi - \dfrac{\pi'}{1 + r} + t + \dfrac{t'}{1 + r} = 0$

Where λ is the Lagrange multiplier (associate to the intertemporal budget constraint).

[4] Again, variables with (´) represent future values.

There is a five equations-variables system (including λ) for which equilibrium can be found. However, in order to figure out easily the effects of an exogenous shock on the endogenous variables the following procedure is suggested. First differentiating the equations system:

$$III.I) \quad -\frac{\gamma}{c^2}dc - d\lambda = 0$$

$$III.II) \quad -\beta\frac{\gamma}{c'^2}dc' - \frac{1}{1+r}d\lambda = -\frac{\lambda}{(1+r)^2}dr$$

$$III.III) \quad \frac{1-\gamma}{(h-n)^2}dn - wd\lambda = \lambda dw$$

$$III.IV) \quad \beta\frac{1-\gamma}{(h-n')^2}dn' - \frac{w'}{1+r}d\lambda = \frac{\lambda}{1+r}dw' - \frac{\lambda w'}{(1+r)^2}dr$$

$$\begin{aligned} III.V) \quad & dc + \frac{dc'}{1+r} - wdn - \frac{w'}{1+r}dn' \\ & = ndw + \frac{n'}{1+r}dw' + d\pi + \frac{d\pi'}{1+r} - dt - \frac{dt'}{1+r} - \frac{(c' - w'n' - \pi' + t')}{(1+r)^2} \end{aligned}$$

After, a matrix system can be written as follows:

$$A_{5x5}dEn_{5x1} = dEx_{5x1}$$

Where dEn_{5x1} represents the vector with endogenous variables (including λ differentiated), whereas dEx_{5x1} is the vector of exogenous variables differentiated. Once the matrix system is obtained, the procedure to find the resulting endogenous variables after a shock on the exogenous variables is straightforward since:

$$A_{5x5}{}^{-1}A_{5x5}dEn_{5x1} = A_{5x5}{}^{-1}dEx_{5x1}$$

$$dEn_{5x1} = A_{5x5}{}^{-1}dEx_{5x1}$$

IV. The general equilibrium

For the general equilibrium, three agents (households, firm´s managers and government) interact on each period (present and future) in the different markets. Families act similarly as proposed in the partial equilibrium scenario, maximizing a utility function which depends on consumption and leisure subject to the same budget constraints presented before. According to these statement, families offer labor time in the labor market and demand goods for consumption in the product market both in the present and future periods5.

On the other side, the firm´s manager maximizes the firm's value as shown:

$$\max\left(\pi + \frac{\pi'}{1+r}\right)$$

Where:

$$\pi = zk^{\alpha}n^{1-\alpha} - wn - I$$

$$\pi' = z'k'^{\alpha}n'^{1-\alpha} - w'n' + (1-\delta)k'$$

$$k' = I + (1-\delta)k$$

[5] In what follows, it will be assumed that the society is composed of only one firm and one household thanks to other two assumptions: a) representative agents, and b) production with constant returns to scale. These two hypothesis simplify the analysis and allow for to know significant macroeconomic variables (consumption per family, consumption per person, product per worker, product per person, etc.) that are independent of the size of the population, the size of the firm or the number of firms.

And z is the total factor productivity (TFP), k is the initial capital stock, δ is the capital depreciation rate l, and I is the gross investment.

The manager demands labor time both in present and future labor markets and demands credit for investment in the credit market. All the benefits (π, π') are included in the budget constraint of the representative household, and are determined endogenously in the general equilibrium system.

Finally, the government spends as much as it collects in both periods but the model allows for indebtedness in the first period; for this reason, Ricardian equivalence is fully met. Namely:

$$G = T$$

$$G = g + \frac{g'}{1+r}$$

$$T = t + \frac{t'}{1+r}$$

$$g + \frac{g'}{1+r} = t + \frac{t'}{1+r}$$

Where G is total government expenditure in present value, g is government expenditure, T is tax collection in present value, and t tax collection. It should be noted that (to make the model more empirically intuitive) both present and future government expenditures are exogenously determined as well as present tax collection, whereas future tax collection is endogenously determined in order to carry out the Ricardian equivalence.

The equilibrium in the output market (present and future) is determined as follows:

$$c + g + I = zk^{\alpha}n^{1-\alpha}$$

$$c' + g' = z'k'^{\alpha}n'^{1-\alpha} + (1-\delta)k'$$

With the equations presented, after the optimization process, there is a 10 equations (and 10 variables) system obtained[6]:

[6] The presentation method used next follows that of chapter 1 of Sargent (1987).

$IV.1)\quad \frac{\gamma}{c} - \lambda = 0$

$IV.2)\quad \beta\frac{\gamma}{c'} - \frac{\lambda}{1+r} = 0$

$IV.3)\quad \left(\frac{1-\gamma}{h-n}\right) - w\lambda = 0$

$IV.4)\quad \beta\left(\frac{1-\gamma}{h-n'}\right) - \frac{w'\lambda}{1+r} = 0$

$IV.5)\quad c + g + I - zk^{\alpha}n^{1-\alpha} = 0$

$IV.6)\quad c' + g' - (1-\delta)k' - z'k'^{\alpha}n'^{1-\alpha} = 0$

$IV.7)\quad Pmgn_n - w = 0$

$IV.8)\quad Pmgn_{n'} - w' = 0$

$IV.9)\quad Pmgn_{k'} - \delta - r = 0$

$IV.10)\quad g + \frac{g'}{1+r} - t - \frac{t'}{1+r} = 0$

Where:

$k' = I + (1-\delta)k$

$Pmgn_n = (1-\alpha)zk^{\alpha}n^{-\alpha}$

$Pmgn_{n'} = (1-\alpha)z'k'^{\alpha}n'^{-\alpha}$

$Pmgn_{k'} = \alpha z'k'^{\alpha-1}n'^{1-\alpha}$

After obtaining the equilibrium of the system, we must proceed to differentiate the system of equations:

$IV.I)\quad -\frac{\gamma}{c^2}dc - d\lambda = 0$

$IV.II)\quad -\beta\frac{\gamma}{c'^2}dc' - \frac{1}{1+r}d\lambda + \frac{\lambda}{(1+r)^2}dr = 0$

$IV.III)\quad \frac{1-\gamma}{(h-n)^2}dn - wd\lambda - \lambda dw = 0$

$IV.IV)\quad \beta\frac{1-\gamma}{(h-n')^2}dn' - \frac{w'}{1+r}d\lambda - \frac{\lambda}{1+r}dw' + \frac{\lambda w'}{(1+r)^2}dr = 0$

$IV.V)\ -Pmgn_n dn + dc + dI = Pmgn_k dk + Fdz - dg$

$$IV.VI)\quad -Pmgn_{n\prime}dn' + dc' - [Pmgn_{k'} + (1-\delta)]dI$$
$$= (Pmgn_{k'}(1-\delta) + (1-\delta)^2)dk + F'dz' - (Pmgn_{k'}k + I + 2k(1-\delta))d\delta - dg'$$

$$IV.VII)\quad Pmgn_{nn}dn - dw = -Pmgn_{kn}dk - \frac{Pmgn_n}{z}dz$$

$$IV.VIII)\quad Pmgn_{n\prime n\prime}dn' - dw' + Pmgn_{k'n'}dI$$
$$= -Pmgn_{k'n'}(1-\delta)dk - \frac{Pmgn_{n'}}{z'}dz' + Pmgn_{k\prime n\prime}d\delta$$

$$IV.IX)\quad Pmgn_{k\prime n\prime}dn' - dr + Pmgn_{k'k'}dI$$
$$= -Pmgn_{k'k'}(1-\delta)dk - \frac{Pmgn_{k'}}{z'}dz' + (1 + Pmgn_{k'k'}k)d\delta$$

$$IV.X)\quad \frac{dt'}{1+r} - \frac{t'-g'}{(1+r)^2}dr = dg + \frac{dg'}{1+r} - dt$$

Where:

$Pmgn_k = \alpha z k^{\alpha-1} n^{1-\alpha}$

$Pmgn_{kk} = (\alpha-1)\alpha z k^{\alpha-2} n^{1-\alpha}$

$Pmgn_{k\prime k\prime} = (\alpha-1)\alpha z' k'^{\alpha-2} n'^{1-\alpha}$

$Pmgn_{nn} = (1-\alpha)(-\alpha) z k^{\alpha} n^{-\alpha-1}$

$Pmgn_{n\prime n\prime} = (1-\alpha)(-\alpha) z' k'^{\alpha} n'^{-\alpha-1}$

$Pmgn_{kn} = (1-\alpha)\alpha z k^{\alpha-1} n^{-\alpha}$

$Pmgn_{k\prime n\prime} = (1-\alpha)\alpha z' k'^{\alpha-1} n'^{-\alpha}$

With these 10 equations, a matrix system can be written similarly as the one obtained in the partial equilibrium system as:

$$A_{10x10} dEn_{10x1} = dEx_{10x1}$$
$$A_{10x10}{}^{-1} A_{10x10} dEn_{10x1} = A_{10x10}{}^{-1} dEx_{10x1}$$
$$dEn_{10x1} = A_{10x10}{}^{-1} dEx_{10x1}$$

V. Calibration and results

Table 1: U. S. Economy. Initial steady state (before shocks)

Values in trillions of 2011 US dollars (h and n are given in millions of people, and wages in millions of dollars)		
Parameters	Exogenous variables	Endogenous variables (equilibrium)
$\gamma = 0.9$	$k = 49.28$	$c = 10.96$
$\beta = 0.710$ (1)	$g = 3.16$	$c' = 14.83$
$\alpha = 0.4$ (2)	$g' = 3.75$	$n = 132.75$
$\delta = 0.95$ (3)	$t = 3.16$	$n' = 149.74$
$h = 150$	$z = 0.175$	$I = 1.5$
$h' = 172$	$z' = 0.525$	$t' = 3.75$
		$r = 0.91$ (4)
		$w = 0.071$ (5)
		$w' = 0.074$
		$y = 15.63$
		$y' = 18.58$ (6)

(1) We suppose a 14 years period between the present and the future; β per year = 0.975
(2) The share of the labor income coming from wages and salaries was 0,6 in 2011; source: U. of California (Davis) & U. of Groningen; Penn World Tables 9.0
(3) δ per year =0.05
(4) r per year= 0.047
(5) 71,000 dollars per worker
(6) GDP growth rate per year = 0.012
Data retrieved from the FRED (Federal Reserve System data) for 2011.

Source: Own elaboration, and FRED data.

With the data from Table 1, the impact matrix is this one:

Table 2: Impact matrix

	dk	dg	dg'	dt	dz	dz'	$d\delta$
dc	*0.13*	*-0.69*	*-0.17*	*0.00*	*66.64*	*1.26*	*-33.82*
dc'	*0.08*	*-0.40*	*-0.58*	*0.00*	*38.35*	*31.97*	*-31.07*
dn	*-0.06*	*1.04*	*0.25*	*0.00*	*-5.94*	*-1.89*	*50.57*
dn'	*-0.01*	*0.57*	*0.83*	*0.00*	*-54.54*	*-5.28*	*-60.70*
dI	*-0.01*	*-0.23*	*0.19*	*0.00*	*22.24*	*-1.40*	*37.40*
dw	*0.00*	*0.00*	*0.00*	*0.00*	*0.40*	*0.00*	*-0.01*
dw'	*0.00*	*0.00*	*0.00*	*0.00*	*0.01*	*0.14*	*-0.35*
dr	*-0.01*	*0.07*	*-0.05*	*0.00*	*-6.65*	*3.89*	*1.88*
dt'	*0.00*	*1.90*	*1.00*	*-1.90*	*0.00*	*0.00*	*0.00*
$d\lambda$	*0.00*	*0.01*	*0.00*	*0.00*	*-0.50*	*-0.01*	*0.25*

Source: Own calculations

What the impact matrix presents, as can be seen, is the reaction of the endogenous variables to a shock of a unit of each of the exogenous variables. The impact matrix is not linear, which means that the magnitude of the reaction completely depends on the initial values of the model; nevertheless, the signs are kept despite the calibration.

As shown in Table 2, Ricardian equivalence is fulfilled since present taxes has no real impact on the economy because increasing present tax collection will only change the consumers saving decisions leaving unaffected the optimal equilibrium value of the other endogenous variables. Furthermore, it must be considered that since there is not external sector, total investment is equal to the sum of public plus private savings. An increase in the present tax collection unchains a private indebtedness in order to maintain the initial consumption level; here is where Ricardian equivalence analysis take place: how is investment not affected since there is a reduction of private savings? Since public expenditure is kept the same as it is determined exogenously, the public sector is the one which is financing the mentioned indebtedness of the private sector; therefore, the equality between savings and investment is sustained.

Table 3 is another impact matrix but only shows the results related to more realistic changes in three of the exogenous variables.

Table 3: Impact matrix (lower shocks)

	$dz = 0.1$	$dz' = 0.1$	$d\delta = 0.01$
dc	6.64	0.13	-3.4
dc'	3.84	3.2	-3.1
dn	-0.59	-0.19	5.6
dn'	-5.45	-0.53	-6.1
dI	2.22	-0.14	3.7
dw	0.04	0	-0.001
dw'	0.001	0.01	-0.04
dr	-0.66	0.39	0.19
dt'	0	0	0
$d\lambda$	-0.05	-0.001	0.03

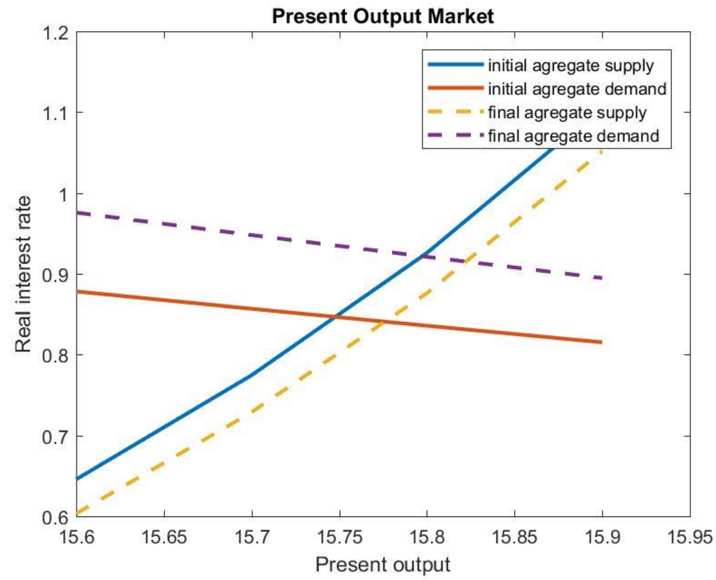
Source: Own calculations

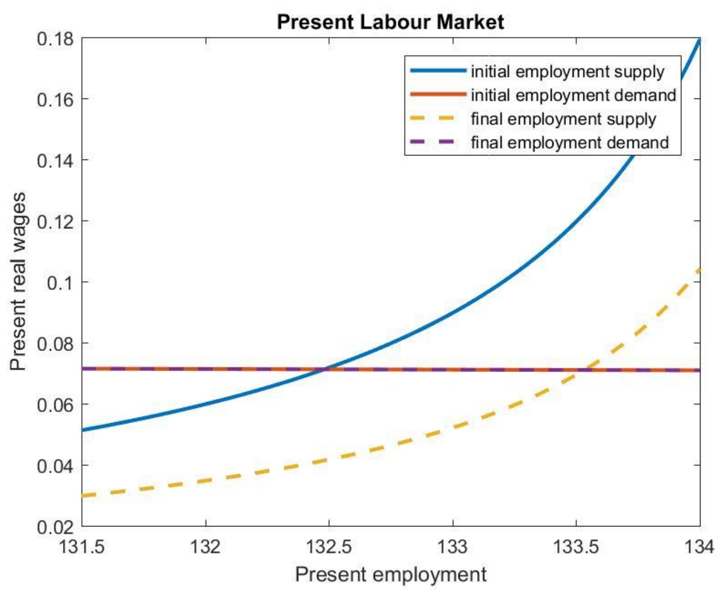
VI. Alternative impacts and responses: intuitive and graphical analysis

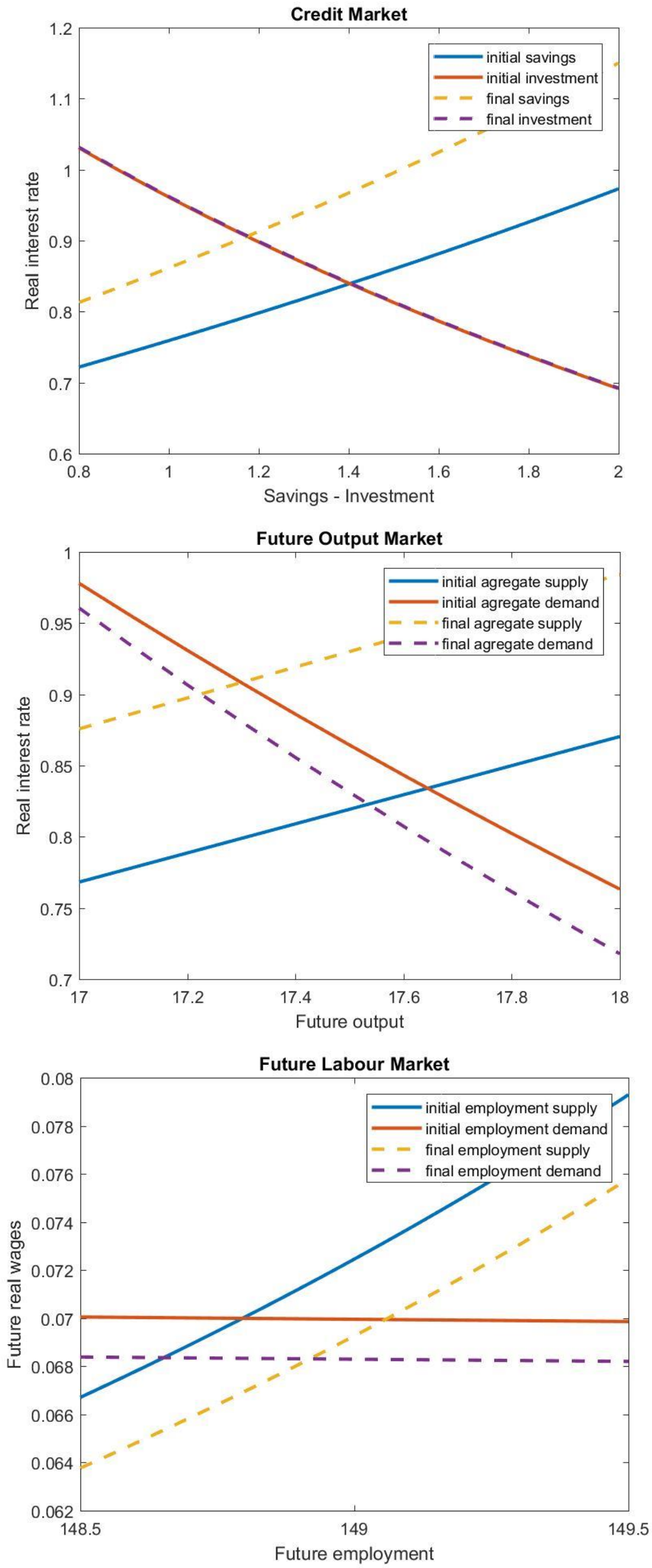
A) An increase in present public expenditures ($\Delta^{+} g = 1$)

In Figure 1, graphics about all markets are presented in order to understand the reason of the model´s numerical performance. An exogenous positive shock to the public expenditure immediately impacts the labor supply increasing it in order to recover the non-labor lost income (note that present labor demand does not react at all). The representative family agent reduces the consumption and increases the private savings because (s)he will be taxed in the future. Nevertheless, and at the same time, the public sector has a bigger increase in the deficit which reduces the savings supply as shown in the credit market graph. The product market presents an increase in the equilibrium output pulled by the major increases of the present public expenditure (although both private consumption and investment decrease but in a smaller measure) and of the labor supply. Both present and future real wages decrease due to the positive impact on the labor supply, and the real interest rate increases due to the decrease of the savings supply. Since there is an investment fall, there will be a lower future capital, which implies a reduction of the future aggregate product supply, and so a lower future output.

Figure 1: Effects of a present public expenditure shock







Source: own elaboration

The resulting public expenditure multiplier is:

$$\frac{\Delta^{+}y}{\Delta^{+}g} = 0.07$$

B) <u>An increase in the future public expenditure</u> ($\Delta^{+}g' = 1$)

An increase on the future expenditure makes react the present labor market the same way as the increase in present public expenditure. On the other hand, as agents know they will be taxed in the future, they will save up as they would on an increase in present public expenditure; nevertheless, government does not go into a deficit which implies an increase on the savings curve. The investment curve increases due to expected increase on future labor; interestingly, savings-investment equilibrium increases significantly with this shock. The product market presents an increases on the demand curve (due to an increase on the investment bigger than the decrease on consumption) and on the supply curve. Both demand and supply increase in the future labor market; added up to an increase on future capital, the future aggregate product supply curve increases.

The impact on real wages occur the same way as in the present expenditure shock. However, real interest rate decrease as the savings curve has the opposite impact as the one presented on the first exercise.

The resulting public expenditure multiplier is:

$$\frac{\Delta^{+}y + \frac{\Delta^{+}y'}{1+r}}{\frac{\Delta^{+}g'}{1+r}} \approx \frac{\Delta^{+}y(1+r) + \Delta^{+}y'}{\Delta^{+}g'} \approx 0{,}47; \quad r = \left(\frac{1}{2}\right) * (r\ before\ change + r\ after\ change);$$

C) <u>An increase both in present and future public expenditures</u>

$(\Delta^{+}g = 1) + (\Delta^{+}g' = 1)$

A permanent positive shock to the public expenditure increases significantly both present and future labor supply. There is an increase on the private savings in order to pay for the future taxes; nevertheless, the deficit generated by the government is bigger than the private savings increase, which causes a reduction on the savings curve. Moreover, since there is an increase on the expected future labor, investment curve increases. Both present and future aggregate demand are pulled on due to public expenditure increases, in spite of the reduction on consumption.

The impact on real wage persist the same as the ones presented on both exercises before. Nevertheless, real interest rate increases significantly; this is because of the reduction of the savings curve and the increase of the investment curve.

The public expenditure multipliers are:

$$\frac{\Delta^{+}y}{\Delta^{+}g} = 0.09$$

$$\frac{\Delta^{+}y'}{\Delta^{+}g'} = 0.02$$

The results show that the public expenditure multiplier is positive but smaller than one, no matter when the government shock is held.

D) <u>An increase in present Total Factor Productivity</u> ($\Delta^{+}z = 0.1$)

A shock on present total factor productivity (see Table 3) increases both present labor supply and demand, and so aggregate supply output curve. Savings curve increases due to an income increase bigger than of consumption´s (consumption smoothing), which implies that both consumption, saving and investment increase. Since it becomes more expensive working in the future due to a lower future TFP / present TFP ratio, future labor supply decreases, but future labor demand increases since there is much more future capital thanks to more savings. Finally, there is a significant increase in both present and future output equilibrium.

Real wage increase significantly both in present and future; the real interest rate, meanwhile, decreases pushed by the savings supply increase.

E) <u>An increase in future Total Factor Productivity</u> ($\Delta^{+}z' = 0.1$)

A shock at the future TFP presents a seemingly surprise results (Table 3): present output decreases; this is due to a reduction on labor supply as it will become more attractive working on the future. On the credit market, a reduction on the savings–investment equilibrium is expected since the agents will increase their consumptions because of optimistic perspectives on the future productivity.

Future labor increases despite the fact that the income effect on leisure is dominant (which means that labor supply is reduced); however, the increase on future labor demand is bigger than the (absolute value of the) reduction on the labor supply curve. Additionally, there is a reduction on future capital caused by the reduction on investment and, hence, a negative impact on the future aggregate supply curve. Nevertheless, this curve will be displaced on the right because of the future increase in the TFP. On the other hand, the aggregate demand curve increases significantly more as the household perceives a bigger wealth (the present value of his incomes). So, the interest rate increases, and product increases only on the future period.

E) An increase on the depreciation rate ($\Delta^{+}\delta = 0.01$)

An increase on depreciation rate impacts positively the (gross) investment curve (Table 3), and the savings one; moreover, since there is a reduction on the family income, present and future labor supply increase. Aggregate product supply increases whereas consumption decreases pushed by the household wealth loss. Notwithstanding, we can observe an increase on present aggregate product. Savings-(gross) investment equilibrium increases, but future capital is reduced. Future aggregate product supply gets slightly wider due to increasing future labor, while future aggregate product demand is significantly reduced as future consumption is contracted. Real interest rate is increased but present and future real wages fall.

VII. **Conclusions**

According to the model presented in these pages, the evolution of the economy in general and that of the aggregate product in particular may have temporal phases more or less intense or more or less prolonged depending on changes in present or future public expenditure or in present taxes, in the present or future total factor productivity, and in the depreciation rate of capital.

In order to put this conclusion in concrete terms, let look at two of the majors stylized facts of the United States, European Union and Japan in the years 2011-16. These were slow economic growth, and low (and declining) real interest rate (some economists called it “secular stagnation”). The model can replicate this situation by perturbing two of the exogenous variables, that is with a positive but really weak shock on present TFP, accompanied with a negative shock on the expected (future) TFP.

Regarding the pedagogical issue, an intertemporal numeric general equilibrium model like the presented in these pages is seem to us a useful tool for learning Dynamic Macroeconomics, even in the undergraduate courses. In fact, the proposed model can be used to make simulations and exercises in a spreadsheet such as *Excel* or any other program like *MatLab*.

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Annexes

A. 1. Partial equilibrium:

$dEn =$

dc
dc'
dn
dn'
$d\,\lambda$

$A=$

$-\frac{\gamma}{c^2}$	0	0	0	-1
0	$-\beta\frac{\gamma}{c'^2}$	0	0	$\frac{-1}{1+r}$
0	0	$\frac{1-\gamma}{(h-n)^2}$	0	$-w$
0	0	0	$\beta\frac{1-\gamma}{(h-n')^2}$	$\frac{-w'}{1+r}$
1	$\frac{1}{1+r}$	$-w$	$\frac{-w'}{1+r}$	0

$dEx =$

0
$-\frac{\lambda}{(1+r)^2}dr$
λdw
$\frac{\lambda}{1+r}dw' - \frac{\lambda}{(1+r)^2}dr$
$-dc - \frac{dc'}{1+r} + d\pi + \frac{d\pi'}{1+r} + ndw + \frac{n'dw'}{1+r} - \frac{(c' - w'n' - \pi' + t')}{(1+r)^2}dr$

A. 2 . Impact Matrix for the partial equilibrium model

In Table 1 calibration for the parameters and exogenous variables are presented, in addition to the resulting equilibrium out of the calibration to the partial equilibrium model. With this presented, the impact matrix will be shown:

$$Impact\ matrix$$

	dt	dt'	$d\pi$	$d\pi'$	dw	dw'	dr
dc	-0.5056	-0.3139	0.5056	0.3139	75.8427	47.0792	-2.9468
dc'	-0.6353	-0.3944	0.6353	0.3944	95.3001	59.1573	4.7471
dn	0.7902	0.4905	-0.7902	-0.4905	119.6089	-73.5779	4.6053
dn'	0.9259	0.5747	-0.9259	-0.5747	-138.8799	173.9665	-6.9179
$d\lambda$	0.0039	0.0024	-0.0039	-0.0024	-0.5816	-0.3610	0.0226

B. General Equilibrium

$$dEn =$$

dc
dc'
dn
dn'
dI
dw
dw'
dr
dt'
$d\lambda$

$A=$

$-\frac{\gamma}{c^2}$	0	0	0	0	0	0	0	0	-1
0	$-\beta\frac{\gamma}{c'^2}$	0	0	0	0	0	$\frac{\lambda}{(1+r)^2}$	0	$\frac{-1}{1+r}$
0	0	$\frac{1-\gamma}{(h-n)^2}$	0	0	$-\lambda$	0	0	0	$-w$
0	0	0	$\beta\frac{1-\gamma}{(h-n')^2}$	0	0	$\frac{-\lambda}{1+r}$	$\frac{w'\lambda}{(1+r)^2}$	0	$\frac{-w'}{1+r}$
1	0	$-Pmgn_n$	0	1	0	0	0	0	0
0	1	0	$-Pmgn_{n'}$	$-(Pmgn_{k'}+1-\delta)$	0	0	0	0	0
0	0	$Pmgn_{n'n'}$	0	0	-1	0	0	0	0
0	0	0	$Pmgn_{n'n'}$	$Pmgn_{k'n'}$	0	-1	0	0	0
0	0	0	$Pmgn_{k'n'}$	$Pmgn_{k'k'}$	0	0	-1	0	0
0	0	0	0	0	0	0	$-\frac{t'-g'}{(1+r)^2}$	$\frac{1}{1+r}$	0

$$dEx =$$

0
0
0
0
$Pmgn_k dk - dg + Fdz$
$(Pmgn_{k'}(1-\delta) + (1-\delta)^2)dk - dg' + F'dz' - (Pmgn_{k'}k + I + 2(1-\delta)k)d\delta$
$-Pmgn_{kn}dk - \frac{Pmgn_n}{z}dz$
$-Pmgn_{k'n'}(1-\delta)dk - \frac{Pmgn_{n'}}{z'}dz' + Pmgn_{k'n'}kd\delta$
$-Pmgn_{k'k'}(1-\delta)dk - \frac{Pmgn_{k'}}{z'}dz' + (Pmgn_{k'k'}k + 1)d\delta$
$dg + \frac{dg'}{1+r} - dt$

$A =$

-0,00748	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	-1,00000
0,00000	-0,00290	0,00000	0,00000	0,00000	0,00000	0,00000	0,02261	0,00000	-0,52496
0,00000	0,00000	0,00034	0,00000	0,00000	-0,08206	0,00000	0,00000	0,00000	-0,07064
0,00000	0,00000	0,00000	0,00014	0,00000	0,00000	-0,04308	0,00167	0,00000	-0,03870
1,00000	0,00000	-0,07064	0,00000	1,00000	0,00000	0,00000	0,00000	0,00000	0,00000
0,00000	1,00000	0,00000	-0,07372	-1,90490	0,00000	0,00000	0,00000	0,00000	0,00000
0,00000	0,00000	-0,00021	0,00000	0,00000	-1,00000	0,00000	0,00000	0,00000	0,00000
0,00000	0,00000	0,00000	-0,00020	0,00000	0,00000	-1,00000	0,00000	0,00000	0,00000
0,00000	0,00000	0,00000	0,00744	-0,28071	0,00000	0,00000	-1,00000	0,00000	0,00000
0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,52496	0,00000

$$A^{-1} =$$

-40,80	57,84	138,84	16,29	0,69	0,17	-11,39	-0,70	1,34	0,00
53,44	-142,93	79,90	412,23	0,40	0,58	-6,56	-17,76	-2,55	0,00
-138,84	-86,48	2621,54	-24,36	-1,04	-0,25	-215,12	1,05	-2,00	0,00
-76,00	-286,47	-113,64	6057,31	-0,57	-0,83	9,32	-260,93	3,62	0,00
30,99	-63,95	46,34	-18,01	0,23	-0,19	-3,80	0,78	-1,48	0,00
0,03	0,02	-0,56	0,01	0,00	0,00	-0,95	0,00	0,00	0,00
0,01	0,06	0,02	-1,19	0,00	0,00	0,00	-0,95	0,00	0,00
-9,27	15,82	-13,85	50,11	-0,07	0,05	1,14	-2,16	-0,56	0,00
0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,90
-0,69	-0,43	-1,04	-0,12	-0,01	0,00	0,09	0,01	-0,01	0,00