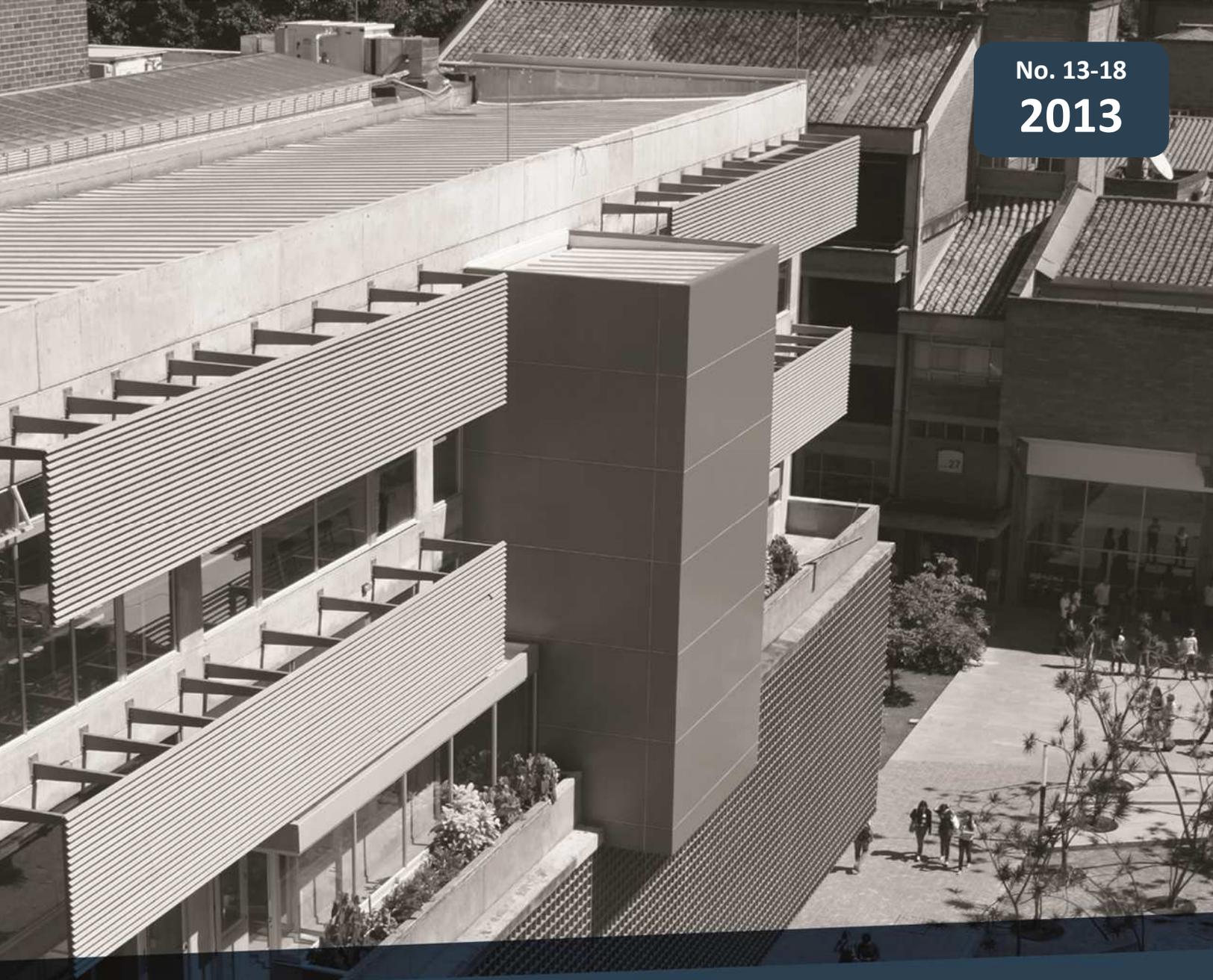


No. 13-18

2013



**PROFIT EFFICIENCY OF U.S. COMMERCIAL BANKS: A DECOMPOSITION**

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Documentos de trabajo

# Economía y Finanzas

Centro de Investigaciones Económicas y Financieras (CIEF)



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# Profit efficiency of U.S. commercial banks: a decomposition.<sup>☆</sup>

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## Abstract

This paper presents new evidence regarding the relation between profit, revenue, and cost efficiencies of U.S. commercial banks. Building on the widely used nonstandard profit function (NSPF) approach, we show (i) why estimation of NSPF would be wrong and (ii) how revenue and cost efficiencies contribute to profit efficiency. Using data from U.S. commercial banks from 2001 to 2010, we find that losses due to profit inefficiency represents about 8.2% of banks' equity of which 3.5% is due to revenue inefficiency and 4.7% to cost inefficiency. Cost efficiency weighs more than revenue efficiency in estimated profit efficiency. However, compared with cost inefficiency, revenue inefficiency affects more overall profitability.

*Keywords:* Profit Efficiency, Revenue efficiency, Cost efficiency, Nonstandard Profit Function, Stochastic Frontier

*JEL Classification No.:* D24, G21, L13

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<sup>☆</sup>The authors thank participants at the 4th International Finance and Banking Society Conference 2012, the North America Productivity Workshop 2012, the 38th Annual Conference of the Eastern Economic Association 2012, and the 48th Annual Meeting of the Missouri Valley Economic Association 2011.

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## 1. Introduction

Over the past fifteen years, the Nonstandard Profit Function (NSPF) approach of [Humphrey and Pulley \(1997\)](#) has become the dominant method to estimate profit efficiency in banking.<sup>2</sup> In this paper, we show that the NSPF econometric model used in the empirical literature is misspecified and may yield misleading results. Based on [Humphrey and Pulley's](#) framework, we propose an alternative method that solves this misspecification problem and makes explicit the conditions under which NSPF efficiency measures capture both revenue and cost efficiencies.

Our paper contributes to the banking literature by providing new evidence on profit efficiency of U.S. commercial banks and its relation with revenue and cost efficiencies based on our proposed alternative method. Our findings suggest that cost inefficiency weighs more than revenue inefficiency in estimated banks' profit efficiency. Combined annual losses for all U.S. commercial banks due to cost and revenue inefficiencies represent about \$31.5 and \$23.5 billion, respectively. However, for any given efficiency level, profit efficiency responds more to revenue efficiency changes than to cost efficiency changes. This helps explaining our finding that, on average, banks tend to be more revenue than cost efficient.

Using our alternative method to measure banks' profit efficiency, we find that while revenue and cost efficiencies tend to be negatively correlated, both correlate positively with profit efficiency. In contrast, using the misspecified econometric model, profit and cost efficiencies tend to be negatively correlated, a result for which researchers have no compelling arguments (e.g., [Rogers 1998](#) and [Berger and Mester 1997](#)).

Since [Humphrey and Pulley's](#) publication, more than fifty published empirical studies have adopted the NSPF approach to investigate different issues in banking such as bank performance, bank productivity, deregulation, competition, market power, bank size, and cross-country comparisons. In contrast, only one study uses the standard neoclassical profit function ([Kumbhakar, Lozano-Vivas, Lovell, and Hasan, 2001](#)) and two others use it only for comparison purposes ([Berger and Mester, 1997](#) and [Vivas, 1997](#)).

Proponents of the NSPF approach argue that the neoclassical assumption of perfect competi-

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<sup>2</sup>The NSPF is also known as the alternative profit function. We use the acronym NSPF to contrast it to the standard neoclassical profit function.

itive markets is unsuitable for the banking industry (e.g., [Berger, Humphrey, and Pulley, 1996](#); [Humphrey and Pulley, 1997](#), and [Berger and Mester, 1997](#)). In particular, [Humphrey and Pulley](#) argue that banks may have more flexibility in choosing output prices than output quantities. Further, [Berger and Mester](#) maintain that the NSPF is more appropriate for modeling banks' maximizing behavior. The widespread use of the NSPF supports their arguments.

In the NSPF framework, banks maximize profits choosing output prices and input quantities while taking input prices and output quantities as given. The solution of the maximization problem yields the NSPF which is a function of input prices and output quantities. To estimate profit efficiency, researchers append an efficiency term in an ad hoc basis to the NSPF, which is then estimated using, mainly, the stochastic frontier technique. Building on [Humphrey and Pulley](#)'s work, we show that such an approach, although intuitive, is algebraically incorrect and therefore the specification of the NSPF is wrong.

Our paper is organized as follows. In Section 2, we solve the bank's profit maximization problem using [Humphrey and Pulley](#)'s framework. Unlike [Humphrey and Pulley](#), we explicitly introduce two potential sources of profit inefficiency: *input inefficiency* and *output price inefficiency*. Then, we show that the bank's profit maximization problem is equivalent to solving the nonstandard revenue maximization problem ([Berger et al., 1996](#)) and the standard cost minimization problem, separately. That is, unlike the neoclassical profit function, the correct NSPF specification is given by the difference between the nonstandard revenue function and the standard neoclassical cost function in which parameters are completely different. Thus the revenue and cost functions and inefficiencies therefrom can be estimated separately from each of them. The profit efficiency measure is shown to be a composite of both cost and revenue efficiencies, and it can be computed without estimating the profit function. In fact, it would be wrong to estimate the profit function because of the misspecification problem which we show explicitly. That is, the efficiency measure used in the NSPF literature is incorrect because it is estimated using an econometric model that is not consistent with the theoretical optimizing model.

We demonstrate that banks' profit efficiency is an overall measure of cost and revenue efficiencies only if both input and output price inefficiencies are present — price efficiency affects

revenues while input efficiency affects costs. Even if there is no revenue (cost) inefficiency, profit efficiency is not the same as cost (revenue) efficiency. This result follows from the optimizing model used in the original NSPF framework and not from any special feature of our modelling approach. Using the homogeneity properties of the nonstandard revenue function, we also show that the NSPF is not linear homogeneous in input prices, contrary to the common assumption made in the empirical literature.

We call our corrected measure the *composite* NSPF (CNSPF) efficiency measure to distinguish it from the traditional NSPF efficiency measure. The CNSPF decomposes profit efficiency into revenue and cost efficiencies. From this decomposition, we can examine the relative importance of improving revenue and cost efficiencies on profit efficiency.

In Section 3, we present the econometric model to estimate revenue and cost inefficiencies. We estimate revenue inefficiency using a *nonstandard revenue function* (NSRF, Berger et al., 1996) and cost inefficiency using a *standard cost function*. Using revenue and cost (in)efficiency estimates, we compute the CNSPF efficiency measure as the ratio between actual and optimal profits without estimating a profit function.<sup>3</sup>

In Section 4, we describe the data used in the estimation and in Section 5 we present our empirical results. We find that average revenue, cost, and profit efficiency estimates for U.S. commercial banks are around 95%, 90%, and 80%, respectively. Forgone rents due to inefficiencies amount to approximately 8% of banks' equity. If banks were to eliminate revenue inefficiencies, average profit efficiency would increase to 89% and forgone rents would decrease to 5%. On the other hand, if banks were to eliminate cost inefficiencies, average profit efficiency would increase to 90% and forgone rents would decrease to 3%. Thus, cost efficiency weighs more than revenue efficiency in estimated profit efficiencies. However, revenue inefficiency affects more overall profitability. For example, a 1% increase in revenue (cost) efficiency leads to a 2% (1%) increase in profit efficiency. This finding partially explains why revenue efficiency is higher than cost efficiency.

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<sup>3</sup>In addition, we can obtain estimates of the characteristics of the technology through the cost function (e.g., returns to scale, technical change, etc.) which is not possible from the NSPF because it does not satisfy the duality results.

## 2. Efficiency in the NSPF Framework

In this section, we model banks' output price and input inefficiencies and show that the econometric model used in the NSPF literature to estimate profit efficiency is misspecified. Using the NSPF framework, we propose an alternative method that is free from this econometric problem and allows researchers to investigate the relation among profit, revenue, and cost efficiencies.

### 2.1. Modelling Efficiency in the NSPF Framework

In the NSPF framework, banks maximize profits,  $\Pi = \sum_m p_m y_m - \sum_j w_j x_j$ , by setting output prices ( $p_m$ ;  $m = 1, \dots, M$ ) and choosing input quantities ( $x_j$ ;  $j = 1, \dots, J$ ). Output quantities ( $y_m$ ) and input prices ( $w_j$ ) are taken as given. Output and input quantities are related through the production possibility frontier that banks face which is defined in terms of a transformation function,  $Af(y, \theta \cdot x; \beta) = 1$  where  $A$  is a productivity parameter,  $f(\cdot)$  is the core transformation function,  $0 \leq \theta \leq 1$  captures input inefficiency (also known as input-oriented technical inefficiency), and  $\beta$  represents parameters of the transformation function. Output and input prices are related through the price possibility frontier (PPF),  $g(p, w, z; \alpha) = 1$ , where  $g(\cdot)$  is a functional form,  $z$  represents factors affecting banks' price setting strategies (e.g., output level, risk, etc.), and  $\alpha$  represents parameters of the PPF.

The PPF is the distinctive feature of the NSPF framework. [Humphrey and Pulley \(1997\)](#) posit the existence of a price opportunity set (POS) containing all feasible combinations of input and output prices and other factors. The PPF is thus the frontier of the POS and contains the highest feasible output prices given input prices and other factors that affect it. The PPF constrains the relation among output and input prices and other factors included in  $z$ . According to [Humphrey and Pulley \(1997, p.81\)](#), the PPF reflects the bank's assessment of their competitive position, customers' willingness to pay for the bank's products and services, and pricing rules that the bank may follow, among other factors influencing banks' price setting strategies. However, [Humphrey and Pulley](#) did not specify how failing to set optimal prices (prices lying on the PPF) contributes to banks' profit efficiency through lower revenue.

Unlike [Berger et al. \(1996\)](#) and [Humphrey and Pulley](#), we explicitly introduce price in-

efficiency into the PPF.<sup>4,5</sup> We measure output price efficiency by comparing observed output prices with the highest feasible output prices (optimal prices) that are unobserved. We define the optimal price vector as  $p^* = \eta p$ , with  $\eta \geq 1$ . A bank is price efficient if  $\eta = 1$ . Price inefficiency is the percentage shortfall of price from its optimal value (i.e.,  $\ln \eta = \ln p^* - \ln p$ ). Thus, price efficiency is given by  $0 \leq \eta^{-1} \leq 1$  and measures how close  $p$  is to its optimal value,  $p^*$ . The PPF becomes  $g(p^*, w, z, \alpha) = 1$  and the PPF frontier is  $g(p, w, z; \alpha) = 1$ . Note that since outputs are given, the presence of revenue inefficiency implies that observed output prices are lower than optimal prices. This causes actual revenue to be less than optimal revenue. The presence of revenue inefficiency (due to lower than optimal prices) can be tested econometrically.

As we argued before, we explicitly model the missing connection between price and profit efficiencies. We show below that output price efficiency ( $\eta^{-1}$ ) affects revenues, while input efficiency ( $\theta$ ) affects costs. Jointly, they affect profits. Without explicitly modeling output price and input efficiency, the NSPF efficiency measure, as routinely estimated in the literature, cannot be interpreted as an overall measure of cost and revenue efficiencies. More specifically, the NSPF cannot be used to measure inefficiency no matter what the source of inefficiencies are, price or input. This is shown in the next two sub-sections.

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<sup>4</sup>Banks use three main pricing strategies: *risk-based loan pricing*, *relationship-based pricing*, and *relationship-lending pricing* (see Berger and Udell, 2002; Edelberg, 2003; McCoy, 2007; Berger and Udell, 2006; Cowan and Cowan, 2006; Gan and Riddiough, 2008; Dick and Lehnert, 2010; and Berger and Black, 2011). However, the adoption of these strategies differ regarding banks' characteristics (e.g., risk tolerance, geographic diversification, bank size, loan portfolio, borrowers pool, among others.), competition, economic conditions, specific demand factors, and funding costs (see the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices). These pricing strategies rely on difficult to observe borrowers characteristics and other variables that are outside banks' control. The subjective nature of pricing strategies makes them complex and subject to errors. Therefore, it is natural to assume that banks may err in setting optimal prices, and become price inefficient.

<sup>5</sup>Berger et al. (1996) consider the possibility that banks may fail to set prices to maximize revenues but focus on economies of scope rather than on inefficiency. Koetter, Kolari, and Spierdijk (2012) consider a framework in which banks with market power fail to set prices optimally. In their model, profit inefficiency arises since the observed price function, or inverse demand function, is everywhere lower than the optimal price function. Nonetheless, they use the traditional NSPF and do not explicitly estimate price inefficiency as we do in this paper.

## 2.2. Solution of the Maximization Problem in the NSPF Framework

The Lagrangian associated with the profit maximization problem is:<sup>6</sup>

$$\max_{p,x} \mathcal{L} = \sum_m p_m y_m - \sum_j w_j x_j + \lambda [Af(y, \theta \cdot x; \beta) - 1] + \mu [g(\eta \cdot p, w; \alpha) - 1] \quad (1)$$

Defining  $p^* = p \cdot \eta$  and  $x^* = x \cdot \theta$ , the first order conditions (FOCs) for  $p_m$  and  $x_j$  are:

$$y_m + \mu \frac{\partial g(p^*, w; \alpha)}{\partial p_m^*} \frac{\partial p_m^*}{\partial p_m} = 0 \quad \forall m : 1, \dots, M. \quad (2)$$

$$-w_j + \lambda \cdot A \frac{\partial f(y, x^*; \beta)}{\partial x_j^*} \frac{\partial x_j^*}{\partial x_j} = 0 \quad \forall j : 1, \dots, J. \quad (3)$$

From (2) we get:

$$\frac{p_m y_m}{p_1 y_1} \equiv \frac{p_m^* y_m}{p_1^* y_1} = \frac{\frac{\partial \ln g(p^*, w; \alpha)}{\partial \ln p_m^*}}{\frac{\partial \ln g(p^*, w; \alpha)}{\partial \ln p_1^*}} \quad \forall m : 2, \dots, M. \quad (4)$$

Likewise, from (3) we get:

$$\frac{w_j x_j}{w_1 x_1} \equiv \frac{w_j x_j^*}{w_1 x_1^*} = \frac{\frac{\partial \ln f(y, x^*; \beta)}{\partial \ln x_j^*}}{\frac{\partial \ln f(y, x^*; \beta)}{\partial \ln x_1^*}} \quad \forall j : 2, \dots, J. \quad (5)$$

Since  $x_j^*$  does not appear in (4), we can solve for  $p_m^*$  from (4) together with the price opportunity set  $g(p^*, w; \alpha) = 1$  in terms of  $w$ ,  $\tilde{y}$ , and the parameters of the PPF, i.e.,  $\eta p_m = p_m^* = \phi_m(w, \tilde{y}; \alpha)$  where  $\tilde{y}_m = y_m/y_1$ . This expression relates optimal prices to output quantities and input prices.

Likewise, since  $p^*$  does not appear in (5), we can solve for  $\theta x_j = x_j^*$  from (5) together with the transformation function  $Af(y, x^*; \beta) = 1$  in terms of  $\tilde{w}$ ,  $y$  and the parameters of the transformation function., i.e.,  $\theta x_j = x_j^* = \psi_j(\tilde{w}, y; \beta)$  where  $\tilde{w}_j = w_j/w_1$ . This expression represents

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<sup>6</sup>To ease the notation we drop  $z$  from the PPF. This is inessential for the results. Note that we do not require output quantities to be included in the PPF. The relation between input prices and output quantities, in the spirit of Berger et al. and Humphrey and Pulley, results naturally from the first order conditions of the profit maximization problem. Further, in the simple output case, if the PPF depends directly on output quantities, there is no need to solve the profit maximization problem to find out optimal prices. They could be derived directly from  $g(y, \eta \cdot p, w) = 1$ . In addition,  $y$  can be included in  $z$ , anyway.

the conditional factor demand functions.

These results follow from the fact that  $\mathcal{L}$  in (1) can be written as:

$$\max_{p,x} \mathcal{L} = \max_p \mathcal{L}_1 - \min_x \mathcal{L}_2 \quad (6)$$

$$\max_p \mathcal{L}_1 = \max_p \sum_m p_m y_m + \mu [g(\eta \cdot p, w; \alpha) - 1] \quad (7)$$

$$\min_x \mathcal{L}_2 = \min_x \sum_j w_j x_j - \lambda [Af(y, \theta \cdot x; \beta) - 1] \quad (8)$$

Note that (7) does not involve  $x$  and (8) does not involve  $p$ . Consequently, the optimization problem in (6) splits into two separate optimization problems, viz., a nonstandard revenue maximization problem in (7) and a standard cost minimization problem in (8). Since there are no common parameters this result is true even under the assumption of full efficiency. That is, the above result does not depend on any special feature of our modeling strategy.

The solutions of  $\theta x$  and  $\eta p_m$  from (6) - (8) can be used to define the following:

$$\max_p \mathcal{L}_1 \text{ gives } \sum p_m^* y_m = \sum \phi_m(w, \tilde{y}; \alpha) y_m = R(w, y; \alpha) \quad (9)$$

$$\min_x \mathcal{L}_2 \text{ gives } \sum w_j x_j^* = \sum w_j \psi_m(\tilde{w}, y; \beta) = C(w, y; \beta) \quad (10)$$

$$\max_{p,x} \mathcal{L} \text{ gives } \sum p_m^* y_m - \sum w_j x_j^* = R(w, y; \alpha) - C(w, y; \beta) \quad (11)$$

The relationship in (11) is written as  $\pi(w, y; \alpha, \beta)$  and is labeled as the NSPF which is given by the difference between the nonstandard maximum revenue function (NSRF)  $R(w, y; \alpha)$  in (9), and the standard cost function (SCF)  $C(w, y; \beta)$  in (10). Unlike the neoclassical revenue function, the NSRF depends on input prices, output quantities, and the parameters of PPF. The function  $C(w, y; \beta)$  in (10) is a SCF which depends on input prices, output quantities, and the parameters of the technology. If one writes  $\pi(w, y; \Theta) = R(w, y; \alpha) - C(w, y; \beta)$ , and views it as a profit function, its parameters cannot identify the  $\alpha$  and  $\beta$  parameters from  $\Theta$ . This is because both the NSRF and the SCF are functions of the same variables but their parameters are different. Consequently, the estimated parameters in the NSPF cannot be used to identify either the transformation function or the PPF. Thus the NSPF has no economic meaning: there are no duality results that can be drawn from it to know the features of either the transformation

function or the PPF. The neoclassical profit function, in contrast, does not separate additively between the revenue and the cost functions because both are functions of the parameters of the transformation function,  $\beta$ . In the neoclassical framework, going from the cost function to the profit function requires  $\max_y \{\sum_m p_m y_m - C(w, y; \beta)\}$  which makes  $y_m$  functions of the same parameters  $\beta$ . Thus, profits are not the difference between maximum revenues  $R(p, x; \beta)$  (obtained from maximizing revenue subject to the transformation function) and minimum costs  $C(w, y; \beta)$  (obtained from minimizing cost subject to the same transformation function). As we show below, complete separability (in terms of parameters) of the NSPF explains why the econometric model used in the literature to estimate NSPF efficiency is misspecified.<sup>7</sup>

### 2.3. Measuring NSPF Efficiency

Using (9) and (10), the relation among actual profit, maximum revenue, and minimum cost is given by:

$$\begin{aligned}
\pi^a &= R^a - C^a \\
&= \sum p_m y_m - \sum w_j x_j \\
&= (1/\eta) \sum \eta p_m y_m - (1/\theta) \sum w_j \theta x_j \\
&= (1/\eta) \sum \phi_m(w, \tilde{y}) y_m - (1/\theta) \sum w_j \psi_m(\tilde{w}, y) \\
&= (1/\eta) R(w, y) - (1/\theta) C(w, y)
\end{aligned} \tag{12}$$

where  $\pi^a$ ,  $R^a$ , and  $C^a$  stand for actual profit, actual revenue, and actual cost while  $R(w, y)$  and  $C(w, y)$  represent maximum revenues and minimum cost, respectively. Thus, since  $\eta \geq 1$  and  $0 \leq \theta \leq 1$ , actual revenues are a fraction of maximum revenues and minimum costs are a fraction of actual costs. Thus,  $0 \leq \eta^{-1} \leq 1$  is a measure of revenue efficiency and  $0 \leq \theta \leq 1$  is a measure of cost efficiency.<sup>8</sup> Then, actual profits are lower than optimal for two reasons: actual revenues are lower than maximum revenues due to output price inefficiency, and actual cost are higher than minimum costs due to input inefficiency.

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<sup>7</sup>From now on we drop  $\alpha$  and  $\beta$  from the nonstandard revenue and the cost functions to ease notational complexity.

<sup>8</sup>Consider  $\eta = 4/3$ , actual revenues will be 75% of maximum revenues,  $R^a = (3/4)R(w, y)$ . Thus, revenue efficiency will be 75%. Now, If  $\theta = 3/4$ , for example, minimum costs will be 75% of actual costs. Thus, a higher value of  $\theta$  indicates higher cost efficiency.

Using (11) and taking the ratio between actual ( $\pi^a$ ) and maximum profits ( $\pi^*$ ), we get our composite NSPF (CNSPF) efficiency measure:

$$\gamma(\eta, \theta) = \frac{\pi^a}{\pi^*} = \frac{(1/\eta)R(w, y) - (1/\theta)C(w, y)}{\pi^*} = \frac{1}{\eta}\omega_1 + \frac{1}{\theta}\omega_2 \quad (13)$$

where  $\omega_1 = R(w, y)/\pi^* \geq 0$  and  $\omega_2 = -C(w, y)/\pi^* \leq 0$  with  $\omega_1 + \omega_2 = 1$ . Thus, (13) makes it clear that our CNSPF efficiency measure can be decomposed into revenue efficiency ( $\eta^{-1}$ ) and (the inverse of) cost efficiency ( $\theta$ ) components, viz.,  $\frac{1}{\eta}\omega_1$  and  $\frac{1}{\theta}\omega_2$ . Profit efficiency,  $\gamma(\eta, \theta)$ , takes values in the unit interval if  $\pi^a \geq 0$  and is nondecreasing in revenue ( $\partial\gamma/\partial\eta^{-1} = \omega_1 \geq 0$ ) and cost efficiencies ( $\partial\gamma/\partial\theta = -\omega_2/\theta^2 \geq 0$ ).<sup>9</sup> The result in (13) shows that one needs to estimate the NSRF and cost function with inefficiencies to estimate profit efficiency. It is not necessary to estimate the profit function.

In earlier studies, researchers estimate profit efficiency using a NSPF which does not correspond to its theoretical counterpart given in (11). Rather, they attach a multiplicative efficiency term to the NSPF to get  $\pi^a = \pi_{\text{NSPF}}(w, y) \times \gamma$ , where  $\pi_{\text{NSPF}}(w, y)$  is an estimate of the NSPF and  $0 \leq \gamma \leq 1$  is a measure of profit efficiency ( $\pi^a / \pi_{\text{NSPF}}(w, y)$ ). However, according to (12) the NSPF with inefficiency  $(1/\eta)R(w, y) - (1/\theta)C(w, y) \neq \pi_{\text{NSPF}}(w, y) \times \gamma$ , unless  $\gamma = 1/\eta = 1/\theta$ . Since  $\eta \geq 1$  and  $0 \leq \theta \leq 1$ , the equality  $\gamma = 1/\eta = 1/\theta$  cannot hold. That means, it is impossible to express actual profit  $\pi^a$  as  $\pi^a = \pi_{\text{NSPF}}(w, y) \times \gamma$ . So, the economic model used in the NSPF literature is misspecified. If banks are fully price efficient  $\pi^a = R(w, y) - (1/\theta)C(w, y) = \pi_{\text{NSPF}}(w, y) + (1 - 1/\theta)C(w, y) \neq \pi_{\text{NSPF}}(w, y) \times \gamma$ , where  $0 \leq \gamma \leq 1$  is profit efficiency which does not depend on  $w$  and  $y$ . This shows that the NSPF is misspecified, even when banks are full price efficient.

Alternatively, if banks are fully price efficient  $R(w, y) = R^a$  and we can rewrite  $\pi^a$  as  $R^a - C^a = R^a - (1/\theta)C(w, y)$  which implies  $C^a = (1/\theta)C(w, y)$ . Thus one need to estimate only the cost function to estimate input inefficiency. Similarly, if banks are input efficient, i.e., ( $\theta = 1$ ),  $C^a = C(w, y)$  which implies that  $\pi^a = R^a - C^a = (1/\eta)R(w, y) - C(w, y)$ . This in turn implies

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<sup>9</sup>If  $\pi^a < 0$ ,  $\gamma$  will be ill defined. In that case, other measures of profit inefficiency can be computed using equation (13). For example, we can define return on equity inefficiency as  $ROE^* - ROE^a = (\pi^* - \pi^a)/\text{Total Equity}$ , which measures the return on equity foregone due to both revenue and cost inefficiencies. This measure is well defined as long as  $\pi^* \geq 0$ , which holds by the definition of maximum profits.

that  $R^a = (1/\eta)R(w, y)$  and one needs to estimate only the NSRF to estimate price inefficiency. In summary, there is no need to estimate a NSPF which can neither identify the sources of inefficiency nor estimate the overall profit efficiency. If banks are only price (input) inefficient then one needs to estimate only the NSRF (SCF) and estimate revenue (cost) efficiency.

It is clear from the above discussion that the correct approach to model inefficiency in a non-standard profit maximizing model (as outlined in [Humphrey and Pulley 1997](#)) is to use both the NSRF  $R(w, y; \alpha)$  and the SCF  $C(w, y; \beta)$  with inefficiencies but not the profit function as advocated in the literature. These two functions have no parameters in common and therefore, separate estimation yields parameter estimates that are consistent with the theoretical optimization framework underlying the NSPF. Inefficiency estimates from these two functions can be used to estimate profit efficiency without estimating a profit function. The NSPF  $\pi_{\text{NSPF}}(w, y; \Theta)$  is not consistent with the underlying optimization theory because it mixes parameters from two different functions (the transformation function and the PPF). This problem does not arise in the neoclassical framework since by duality theory the parameter estimates of the profit, revenue, and cost functions are tied to a unique set of parameters of the transformation function.

One can use a misspecified model to estimate profit efficiency but as shown above, the estimated  $\gamma$  from the misspecified model ( $\pi^a = \pi_{\text{NSPF}}(w, y) \times \gamma$ ) is not an estimate of profit efficiency. Constraining  $\gamma$  to be in the interval  $(0, 1)$  and interpreting it as profit efficiency might give wrong relationship among profit, revenue, and cost efficiencies. For example, although  $1/\eta \neq 1/\theta$ , erroneously constraining  $\gamma = 1/\eta = 1/\theta$  would imply that profit efficiencies are negatively correlated with cost efficiencies but positively correlated with revenue efficiencies. This is a common finding in the literature (e.g., [Berger and Mester, 1997](#) and [Rogers, 1998](#)) and it has been attributed to a negative correlation between revenue and cost efficiencies. Our empirical results using the misspecified model confirm this intuition. However, using the correct specification we show that despite a negative correlation between revenue and cost efficiencies, profit efficiencies are positively correlated with both cost and revenue efficiencies.

In the following, we refer to the correct specification of the NSPF in (11) as the Composite NSPF (CNSPF) because it separates into a NSRF and a cost function. We refer to its misspecified version,  $\pi_{\text{NSPF}}(w, y)$ , as the NSPF. Further, we refer to profit efficiency estimates derived

from the misspecified model as NSPF efficiencies. Likewise, we refer to  $\eta^{-1}$  as the NSRF efficiency measure and  $\theta$  as the cost efficiency measure.

Our CNSPF efficiency measure is linear in revenue efficiency and nonlinear in cost efficiency. It allows us to disentangle the individual effects that revenue and cost efficiencies have on profit efficiency. In addition, to compute CNSPF efficiencies, there is no need to estimate a profit function. Only the estimation of the NSRF and the cost frontiers is required. Finally, one can obtain estimates of the characteristics of the technology through the cost function (e.g., returns to scale, technical change, etc.).

From (13), we identify five fundamental sources of profit efficiency, *ceteris paribus*: i) revenue efficiency, ii) cost efficiency, iii) shifts of the cost frontier, iv) shifts of the revenue frontier, and v) shifts in the profit frontier — a combination of iii) and iv). Increases in NSRF efficiency ( $\uparrow \eta^{-1}$ ) or in cost efficiency ( $\uparrow \theta$ ) increase CNSPF efficiency ( $\uparrow \gamma$ ). Downward shifts of the NSRF frontier or upward shifts of the cost frontier increase CNSPF efficiency ( $\uparrow \gamma$ ). Upward shifts of NSRF and cost frontiers lead to upward shifts of the profit frontier if revenues increase by more than the increase in costs. All these effects may lead to different relations between CNSPF, NSRF, and cost efficiency estimates. In general, if the shifts of the frontiers are small, one would expect a positive correlation between CNSPF and both NSRF and cost efficiencies, regardless of the correlation between these two.

Note that (13) constrains the range of possible values that  $\theta$  and  $\eta$  can take in relation with  $\gamma$ . In Section 5, we show that NSPF efficiency estimates using the wrong specification of the NSPF frequently violate this constraint. In contrast, our CNSPF efficiency measure, by construction, is free from this problem.

#### 2.4. Relationship among Profit, Revenue, and Cost Efficiencies

Our CNSPF efficiency measure in (13) allows us to analyze profit efficiency changes due to changes in revenue, cost efficiency, or both. The ratio between proportional changes in CNSPF efficiency due to revenue efficiency and proportional changes in CNSPF due to cost efficiency is:

$$\Lambda = \frac{\partial \ln \gamma / \partial \ln \eta^{-1}}{\partial \ln \gamma / \partial \ln \theta} = \frac{R^a}{C^a} \quad (14)$$

$\Lambda \geq 1$  if  $R^a \geq C^a$ , i.e.,  $\pi^a \geq 0$ . This result indicates that revenue and cost efficiencies affect profit efficiency asymmetrically. A given percentage increase in revenue efficiency has a greater impact on profit efficiency compared to the same percentage change in cost efficiency, provided that profits are positive. If profits are negative, the opposite is true.

Using (13) we can compute the percentage change in profit efficiency due to a one percentage change in revenue efficiency using  $\frac{\partial \ln \gamma(\eta, \theta)}{\partial \eta^{-1}} = \frac{\eta^{-1}}{\gamma(\eta, \theta)} \times \omega_1 \geq 0$ . Similarly, the percentage change in profit efficiency due to a one percentage change in cost efficiency can be computed using  $\frac{\partial \ln \gamma(\eta, \theta)}{\partial \theta} = -\frac{1}{\gamma(\eta, \theta)} \times \omega_2 \geq 0$ . Thus, the percentage change in profit efficiency due to a one percentage change in both revenue and cost efficiencies is  $[\frac{1}{\gamma(\eta, \theta)}][\eta^{-1} \times \omega_1 - \theta^{-1} \times \omega_2]$ . Since  $\omega_1$  and  $\omega_2$  are bank-specific and also vary over time, these effects are bank- and year-specific.

Another way of looking at this is to think of an equiproportional change in revenue and cost efficiencies by  $x\%$ . This is equivalent to a change in revenue and cost efficiencies by  $k = 1 + x\%$ . Denoting the new profit efficiency level by  $\gamma(k \cdot \eta^{-1}, k \cdot \theta)$  and the proportional change in profit efficiency by  $\Delta\% \Gamma(\eta^{-1}, \theta) = \gamma(k \cdot \eta^{-1}, k \cdot \theta) / \gamma(\eta^{-1}, \theta) - 1$ , we have:

$$1 + \Delta\% \Gamma(\eta^{-1}, \theta) = \frac{\frac{k}{\eta} R(w, y) - \frac{1}{k\theta} C(w, y)}{\frac{1}{\eta} R(w, y) - \frac{1}{\theta} C(w, y)} \quad (15)$$

If only revenue efficiency increases:

$$1 + \Delta\% \Gamma(\eta^{-1}) = \frac{\frac{k}{\eta} R(w, y) - \frac{1}{\theta} C(w, y)}{\frac{1}{\eta} R(w, y) - \frac{1}{\theta} C(w, y)} \quad (16)$$

If only cost efficiency increases:

$$1 + \Delta\% \Gamma(\theta) = \frac{\frac{1}{\eta} R(w, y) - \frac{1}{k\theta} C(w, y)}{\frac{1}{\eta} R(w, y) - \frac{1}{\theta} C(w, y)} \quad (17)$$

Substituting (17) in (16):

$$1 + \Delta\% \Gamma(\eta^{-1}) = k [1 + \Delta\% \Gamma(\theta)] \quad (18)$$

Equation (18) is the discrete counterpart of (14). Provided that  $\pi^a \geq 0$ , a  $x\%$  change in revenue (cost) efficiency has a bigger (smaller) effect on profit efficiency. To see this note that  $k = 1 + x\%$ . If  $x\% \geq 0$ , then  $k \geq 1$ , implying  $\Delta\% \Gamma(\eta^{-1}) \geq \Delta\% \Gamma(\theta)$ . This result holds for any given level of revenue and cost efficiencies. Thus, revenue efficiency gains are more important than cost efficiency gains for improving overall profit efficiency.

Using (15), (16), and (17), is easy to show that  $\Delta\% \Gamma(\eta^{-1}, \theta) = \Delta\% \Gamma(\theta) + \Delta\% \Gamma(\eta^{-1})$ , indicating that proportional changes in profit efficiency are the sum of proportional changes in profit efficiency due to revenue and cost efficiency changes.

We can also compute the marginal effects of determinants of revenue and cost inefficiencies on profit inefficiencies. Suppose that revenue and cost inefficiencies are functions of a set of variables  $z_j$ ,  $j = 1, \dots, J$ . We denote NSRF, cost, and CNSPF inefficiencies as  $\ln \eta(z)$ ,  $-\ln \theta(z)$ , and  $-\ln \gamma(z)$ , respectively. Then, the marginal effect of a variable  $z_j$  on CNSPF inefficiency,  $\partial(-\ln \gamma)/\partial z_j$ , is given by:

$$\frac{\partial(-\ln \gamma(z))}{\partial z_j} = \frac{1}{\pi^a} \left[ \frac{\partial(1/\eta(z))}{\partial z_j} R(w, y) + \frac{\partial(1/\theta(z))}{\partial z_j} C(w, y) \right] = \frac{1}{\pi^*} \left[ \frac{\partial \ln \eta}{\partial z_j} R^a + \frac{\partial(-\ln \theta)}{\partial z_j} C^a \right] \quad (19)$$

where  $\partial \ln \eta / \partial z_j$  and  $\partial(-\ln \theta) / \partial z_j$  are the estimated marginal effects of  $z_j$  on revenue and cost inefficiencies.

## 2.5. Homogeneity Properties of the NSPF

In the neoclassical framework, the properties of the profit function follow solely from the assumption of profit maximization. The reason is that the arguments of the neoclassical profit function (input and output prices) do not enter into the transformation function. In contrast, the arguments of the NSPF, input prices and outputs, enter into the transformation function or the PPF. Thus, the properties of the NSPF depend crucially on the assumptions about the

technology and the pricing opportunity set.

From the definition of  $R$  and the assumption that  $y$  does not enter into the PPF in (1), we can establish that the NSRF is homogeneous of degree one in outputs. To see this, note that  $R/y_1 = p_1 + \sum_{m=2} \tilde{p}_m \tilde{y}_m \equiv \phi(w, \tilde{y})$  since  $p_m = \phi_m(w, \tilde{y})$ . Further,  $\phi(w, \tilde{y})$  is homogeneous of degree zero in  $y$  which means that  $R = y_1 \phi(w, \tilde{y})$  is homogeneous of degree one in  $y$ . If  $y$  enters into the PPF, but the PPF is homogeneous of degree zero in  $y$ ,  $R$  will still be homogeneous of degree one in  $y$ . Otherwise, the NSRF would not be homogeneous in outputs.<sup>10</sup>

The next issue is whether the NSRF is linear homogeneous in input prices or not. We argue that it is not. The NSRF would be linear homogeneous in input prices only if  $\phi_m(w, \tilde{y})$  in (9) is linear homogeneous in  $w$ . That is, optimal output prices have to move in unison with input prices. However, this assumption seems unrealistic given that most empirical studies find evidence of incomplete pass-through of market rates to banks' lending rates, specially in bank-based financial systems (see [Cottarelli and Kourelis, 1994](#), [Berger and Udell, 1995](#), [Berlin and Mester, 1999](#), [De Bondt, 2005](#), and [Hofmann and Mizen, 2004](#)).<sup>11</sup> In addition, in the [Panzar and Rosse \(1987\)](#) framework of competitive structure, the assumption that revenues are homogeneous of degree one in input prices would correspond to the case of perfect competition; which conflicts with the key assumption of imperfect competition of the NSPF framework.

The cost function is linear homogeneous in input prices, which implies that a proportional increase in input prices yields an equiproportional increase in total costs. Since the NSRF is linear homogeneous in outputs quantities but not in input prices, from (11) the NSPF is neither homogeneous in input prices nor in output quantities. Therefore, we do not see any reasons for researchers to impose linear homogeneity conditions on the NSPF because it goes against the theoretical framework on which the NSPF is based. In the next subsection, and in our empirical application, we show that imposition of the linear homogeneity condition on the NSPF might distort the dynamics between profit and cost efficiency estimates.

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<sup>10</sup>Note that in the end,  $p_m$  and therefore  $R$  and/or  $\Pi$  are functions of  $y$  and  $w$ , no matters whether  $y$  appears in the PPF or not.

<sup>11</sup>Loan rates adjust sluggishly to changes in funding costs, specially in bank-based financial systems. The lending rates pass-through literature shows that banks shelter customers from funding rates shocks by smoothing loan rates. For instance, [Berger and Udell \(1992\)](#) point out that loan interest rates may be sticky due to long-term lending relationships or recontracting with financially distress borrowers. These contractual mechanisms induce banks to hold interest rates relatively constant over time for commitment loans which have less asymmetric information problems for banks.

### 3. Econometric Estimation

To keep our results comparable to those in the literature, we estimate the NSRF, cost, and NSPF frontiers using translog functional forms and follow the standard stochastic frontier estimation procedure. The econometric specification of a typical stochastic frontier model is:

$$\ln Q_{it} = f(\ln \mathbf{y}_{it}, \ln \mathbf{w}_{it}, t) + u_{it} + v_{it} \quad (20)$$

$$v_{it} \sim N(0, \sigma_v^2) \quad (21)$$

$$u_{it} \sim N^+(0, \sigma_{it}^2) \quad (22)$$

$$\sigma_{it}^2 = \exp(\mathbf{z}_{it} \boldsymbol{\delta}) \quad (23)$$

Depending on the frontier to be estimated, the left-hand-side variable of (20) will change (e.g.,  $Q_{it}$  is either total profits or total revenue or total costs for bank  $i$  at time  $t$ ). Usually the variables in the right-hand-side of (20) are outputs ( $\mathbf{y}$ ), input prices ( $\mathbf{w}$ ), and time ( $t$ ).<sup>12</sup> However, these variables differ across the three specifications given that the NSPF, the NSRF, and the cost function have different homogeneity properties. The cost function is homogeneous of degree one in input prices, the NSRF is homogeneous of degree one in output quantities; and, as we showed in Section 2, the NSPF is neither homogeneous in input prices nor in output quantities. Therefore, in addition to the symmetry restrictions, we impose linear homogeneity of the cost function by dividing total costs and all input prices by one of the input prices. For the NSRF, we impose linear homogeneity by dividing total revenues and all outputs by one of the outputs. In both cases, the choice of the normalizing variable is innocuous since they are mathematically equivalent.

In the stochastic frontier model specified in (20) and (22),  $u_{it}$  is a one-sided error term typically assumed to be half-normally distributed with mean zero and variance  $\sigma_{it}^2$ . In our case,  $u_{it} = -\ln \eta_{it} \leq 0$ ,  $u_{it} = -\ln \theta_{it} \geq 0$ , and  $u_{it} = \ln \gamma_{it} \leq 0$  for NSRF, cost, and NSPF inefficiencies (see (12) and (13)). Finally,  $v_{it}$  is a two-sided error term normally-distributed with mean zero and variance  $\sigma_v^2$ .<sup>13</sup>

<sup>12</sup>Following the literature, we include banks' equity as an additional control variable (see Berger and Mester, 2003 and Wheelock and Wilson, 2012)

<sup>13</sup>See Kumbhakar and Lovell, 2003 for details on estimating stochastic frontiers. Distributional assumptions

Following [Caudill, Ford, and Gropper \(1995\)](#), [Hadri \(1999\)](#), and [Wang \(2002\)](#), we assume that the variance of the inefficiencies has the functional form given in (23), where  $\delta$  is a vector of parameter to be estimated and  $\mathbf{z}_{it}$  is a vector of bank characteristics that are of interest to explain bank-specific inefficiency. Specifically, we include time and time squared to capture the inefficiency trend over time; non performing loans (NPL) as an *ex-post* measure of credit risk; the ratio of fees and loan income over total operating income as a measure of core income from loans; a proxy for off-balance sheet activities (the ratio of non interest income over total income) to capture the effect of nontraditional activities; the fraction of real estate loans over total assets and the ratio of total loans over total assets as a measure of output mix, and the log of total assets to control for bank size.

The parametrization of  $\sigma_{it}^2$  allows us to account for time varying determinants of inefficiency via the  $\mathbf{z}_{it}$  variables (see [Wang, 2002](#)). In this setting,  $E[u_i] = \sigma_{it} [\phi(0)/\Phi(0)] = \sqrt{2/\pi} \exp(\mathbf{z}_{it}'\delta)$ , where  $\phi$  and  $\Phi$  are the probability and cumulative density functions of a standard normal distribution, respectively. Therefore, the marginal effects of the  $\mathbf{z}$  variables on the unconditional mean of inefficiency are given by:

$$\frac{\partial E[u_i]}{\partial z_k} = \frac{\partial (z_i' \delta)}{\partial z_k} \sqrt{(0.5/\pi) \exp(z_i' \delta)} \quad (24)$$

To accommodate possible heteroscedasticity of the error term  $v_{it}$ , we follow [Hadri \(1999\)](#) and [Wang \(2003\)](#) and make  $\sigma_v^2 = \exp(\mathbf{h}_{it}'\varphi)$ ; where  $\mathbf{h}_{it}$  is a vector of time and bank characteristics that may induce heteroscedasticity in  $v_{it}$ , and  $\varphi$  is a vector of parameters to be estimated. We include time, NPL, and leverage in  $\mathbf{h}$ . For all stochastic frontiers we estimate, we reject the hypothesis of homoscedasticity at the 1% level of statistical significance.

#### 4. Data

We use data from the Report of Conditions and Income (Call Reports) from the Federal Reserve Bank of Chicago. We include all FDIC insured commercial banks with available data between 2001Q1 and 2010Q4. We exclude banks reporting negative values for assets,

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of these one-sided errors have little impact on estimated efficiency ranks (e.g., [Kumbhakar and Lovell, 2003](#))

equity, outputs and prices, standalone internet banks, commercial banks conducting primarily credit card activities, and banks chartered outside continental U.S. territory. Our data set is an unbalanced panel with 63,120 bank-year observations for 8,483 banks. We deflate all nominal quantities using the 2005 Consumer Price Index for all urban consumption published by the Bureau of Labor Statistics.

Table 1: Summary Statistics

Variable	Mean	sd	Percentiles				
			5 <sup>th</sup>	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	95 <sup>th</sup>
$y_1$	73,832	1,700,000	614	2,051	4,489	10,079	47,747
$y_2$	361,000	5,310,000	5,697	20,471	49,531	121,000	554,000
$y_3$	225,000	4,710,000	2,930	8,332	17,099	37,325	170,000
$y_4$	300,000	7,980,000	4,558	13,205	26,858	57,704	256,000
$y_5$	39,388	1,020,000	302	779	1,585	3,483	18,051
$x_1$	227	3,437	9	19	36	76	302
$x_2$	11,750	147,000	153	743	2,017	4,908	19,837
$x_3$	300,000	6,710,000	2,512	8,935	21,451	52,966	279,000
$x_4$	26,228	243,000	1,666	4,921	9,985	20,063	53,747
$x_5$	552,000	10,900,000	10,849	25,726	49,863	116,000	600,000
$w_1$	52.84	13.49	36.42	43.79	50.11	58.83	79.16
$w_2$	0.348	0.342	0.103	0.167	0.242	0.388	0.968
$w_3$	0.034	0.011	0.017	0.026	0.034	0.043	0.053
$w_4$	0.010	0.007	0.002	0.005	0.008	0.014	0.025
$w_5$	0.026	0.010	0.011	0.018	0.025	0.033	0.046
$\Pi$	33,795	598,000	643	1,776	3,510	7,723	37,021
$R$	73,325	1,290,000	1,771	4,299	8,245	17,790	81,558
$C$	39,530	751,000	979	2,401	4,673	10,027	44,623
Assets	1,070,000	20,900,000	25,303	59,017	115,000	247,000	1,110,000
NPL	0.009	0.015	0.000	0.001	0.005	0.011	0.033
Leverage	0.895	0.035	0.831	0.883	0.903	0.916	0.929
Offbal	0.059	0.066	0.008	0.022	0.040	0.072	0.167
Loan/Assets	0.663	0.144	0.395	0.575	0.683	0.768	0.863
Int.Ratio	0.703	0.131	0.459	0.630	0.721	0.796	0.882
REL/Loans	0.088	0.082	0.008	0.032	0.066	0.117	0.240
Equity	102,000	1,900,000	2,700	6,245	11,774	24,334	106,000

Notes: This table shows the average (Mean), standard deviation (sd), the median, and the 5, 25, 75, and 95<sup>th</sup> percentiles. The data used for estimation include 63,120 year-bank observations for 8,483 banks with information for at least four years between 2001 and 2010. Nominal values are in thousands of 2005 dollars. The output variables are: household and individual loans ( $y_1$ ), real estate loans ( $y_2$ ), loans to business and other institutions ( $y_3$ ), federal funds sold and securities purchased under agreements to resell ( $y_4$ ), and other assets ( $y_5$ ). The input variables are: labor quantity ( $x_1$ ), premises and fixed assets ( $x_2$ ), purchased funds ( $x_3$ ), interest-bearing transaction accounts ( $x_4$ ), and non-transaction accounts ( $x_5$ ). For each input  $x_j$  its price,  $w_j$ , is computed by dividing total expenses by the corresponding input quantity. Profits,  $\Pi$ , equals revenues minus costs,  $R - C$ . Assets: Total Assets, NPL: Non Performing Loans, Leverage: Total liabilities/Assets, Offbal: non interest income/ total income, Int. Ratio: Interest income/Total income, REL: Real estate loans, Equity: Total equity.

To estimate profit, revenue, and cost efficiencies, we must specify a model to map banks' activities to output and input quantities and their corresponding prices. We follow the literature and model banks' activities using the balance-sheet approach of [Sealey and Lindley \(1977\)](#). In this framework, a bank's balance-sheet captures the essential structure of banks' core business. First, liabilities, together with physical capital and labor, are inputs into the bank production

process. Second, assets, other than physical assets, are outputs. Liabilities are composed of core deposits and purchased funds. Assets include loans and trading securities. Therefore, banks use labor, physical capital, and debt to produce loans, invest in financial assets, and facilitate other financial services.

Table 1 presents summary statistics for all the variables we use. Nominal variables are in thousands. Output and input variables for each year are computed as the quarterly average of balance-sheet nominal (stock) values. We define five output variables: household and individual loans ( $y_1$ ), real estate loans ( $y_2$ ), business loans ( $y_3$ ), securities (e.g. federal funds sold and securities purchased under agreements to resell) ( $y_4$ ), and other assets ( $y_5$ ). These outputs are essentially the same as those used in [Berger and Mester \(2003\)](#).

We define five input variables: labor (number of full-time equivalent employees at the end of each quarter) ( $x_1$ ), physical capital (e.g., premises and fixed assets including capitalized leases) ( $x_2$ ), purchased funds (federal funds purchased and securities sold under agreements to repurchase, total trading liabilities, other borrowed money, and subordinated notes and debentures) ( $x_3$ ), interest-bearing transaction accounts ( $x_4$ ), and non-transaction accounts ( $x_5$ ). For each input  $x_j$  its price,  $w_j$ , is computed by dividing total expenses by the corresponding input quantity. Total costs,  $C$ , equal the sum of expenses for five inputs; total revenues,  $R$ , equal the sum of revenues for each output category; and, profits,  $\Pi$ , equal total revenues minus total costs.

On the revenue side, real estate loans account for about 42% of total banks' revenues, loan to business and other institutions 16% , securities 15% , and other assets 16%. Loans to individuals and households only account for 7% of total revenues. On the cost side, expenditures on non-transaction accounts represent 29%, labor 41%, premises and fixed assets 10%, purchased funds 17%, and transaction accounts 3% of total cost.

We include the following variables as possible determinants of inefficiencies: time, time squared, log of total assets (Assets), non performing loans (NPL), leverage (Leverage:Total liabilities/Assets), a proxy for off-balance sheet activities (Offbal: non interest income/ total income), the ratio between interest income and total income (Int. Ratio), the ratio of real estate loans (REL) over total assets, and total equity.

## 5. Empirical Results

### 5.1. Efficiency Estimates

We summarize our main results in Table 2.<sup>14</sup> Our proposed CNSPF efficiency measure indicates that, on average, banks' profits are 79.5% of their potential maximum. Average CNSPF efficiency decreases with bank size. Big banks (assets > \$1 billion) are more profit efficient than medium (\$100 million < assets < \$1 billion), and small (assets < \$100 million) banks. The empirical distribution of CNSPF efficiencies is well behaved (see the top left plot of Figure 1) and its shape is similar to those that are usually found in the literature.

The second set of efficiency measures in Table 2 corresponds to NSPF efficiencies estimated using the NSPF stochastic frontier. The results mirror those using the CNSPF approach. Average NSPF efficiency is 80.8%, slightly higher than average CNSPF efficiencies. NSPF efficiencies also indicate that big banks are more profit efficient than medium and small banks. The density plot of NSPF efficiencies in Figure 1 shows that its distribution is similar to the distribution of CNSPF efficiencies. We further investigate the similarities and differences between these two measures below.

The empirical distribution of NSRF efficiencies is clustered above 80% (see Figure 1), indicating that NSRF efficiencies are high for most banks. Mean NSRF efficiency is 95.3%. NSRF efficiencies are even higher for big banks. These results are robust to different specifications of the NSRF frontier. For all of them, we reject the hypothesis of no revenue inefficiency.<sup>15</sup> Table 3 shows that NSRF efficiencies are stable and slightly decreased over time (-0.74% from 2001 to 2010).

Cost efficiencies are also high with a mean value of about 90%. In contrast to profit and revenue efficiencies, cost efficiencies are slightly higher for medium and small banks compared to big banks. However, the differences are small. Table 3 shows that cost efficiencies decreased from 2001 to 2004 and also in 2009, and increased in the other years. Overall, cost

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<sup>14</sup>Table 2 in the supplementary material accompanying this paper presents the estimated parameters of the NSPF, NSRF, and Cost stochastic frontiers.

<sup>15</sup>Formally, we test if the variance of the inefficiency term in equation (20) is zero,  $\sigma_u = 0$ . We use likelihood ratio tests following Coelli and Battese (1996) and Wang (2003). Under the null hypothesis, these tests follow a mixture Chi-squared distribution whose critical values are given in Kodde and Palm (1986, Table 1).

Table 2: Summary Statistics for Estimated Efficiencies

Efficiencies		mean	sd	Percentiles				
				5 <sup>th</sup>	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	95 <sup>th</sup>
CNSPF	All banks	0.795	0.104	0.609	0.753	0.817	0.863	0.912
	Big Banks	0.853	0.083	0.694	0.826	0.873	0.906	0.942
	Medium Banks	0.807	0.098	0.633	0.770	0.829	0.870	0.915
	Small Banks	0.773	0.108	0.581	0.728	0.795	0.845	0.897
NSPF	All banks	0.808	0.113	0.597	0.768	0.830	0.878	0.939
	Big Banks	0.870	0.092	0.714	0.827	0.885	0.936	0.978
	Medium Banks	0.809	0.110	0.600	0.770	0.833	0.877	0.933
	Small Banks	0.799	0.117	0.583	0.760	0.823	0.871	0.931
NSRF	All banks	0.953	0.033	0.894	0.936	0.958	0.977	0.996
	Big Banks	0.981	0.025	0.942	0.975	0.988	0.996	1.000
	Medium Banks	0.960	0.030	0.907	0.946	0.965	0.981	0.996
	Small Banks	0.942	0.034	0.883	0.924	0.946	0.965	0.989
Cost	All banks	0.896	0.076	0.749	0.879	0.919	0.942	0.960
	Big Banks	0.883	0.099	0.700	0.865	0.914	0.940	0.961
	Medium Banks	0.901	0.075	0.766	0.886	0.922	0.944	0.962
	Small Banks	0.893	0.075	0.740	0.874	0.916	0.939	0.958
CNSPF (No revenue ineff.)	All banks	0.878	0.090	0.712	0.851	0.902	0.934	0.963
	Big Banks	0.887	0.080	0.724	0.862	0.907	0.937	0.967
	Medium Banks	0.882	0.084	0.727	0.856	0.903	0.934	0.963
	Small Banks	0.873	0.097	0.694	0.842	0.900	0.934	0.963
CNSPF (No cost ineff.)	All banks	0.892	0.090	0.740	0.858	0.910	0.951	0.991
	Big Banks	0.959	0.055	0.867	0.945	0.976	0.992	1.000
	Medium Banks	0.905	0.084	0.758	0.876	0.924	0.958	0.992
	Small Banks	0.869	0.093	0.717	0.833	0.886	0.927	0.976
NSPFW (homogeneous in input prices)		0.807	0.128	0.565	0.757	0.832	0.890	0.961

Notes: This table shows the average (mean), standard deviation (sd), and the 5, 25, 50, 75, and 95 percentiles of the estimated efficiency measures for all, big (assets > \$1 billion), medium (\$100 million < assets < \$1 billion), and small (assets < \$100 million) banks. CNSPF efficiencies ( $\Pi/\Pi^*$ ) are computed using equation (13). NSPF ( $\gamma$ ), NSRF ( $1/\eta$ ), and cost ( $\theta$ ) efficiencies are estimated using equation (20). No revenue inefficiency indicates that only cost inefficiency is taken into account. Homogeneous in input prices indicates that the wrong assumption of linear homogeneity is imposed. The data used for estimation include 63,120 bank-year observations for 8,483 banks. Profit efficiencies are computed only from 62,577 observations for which profits are positive.

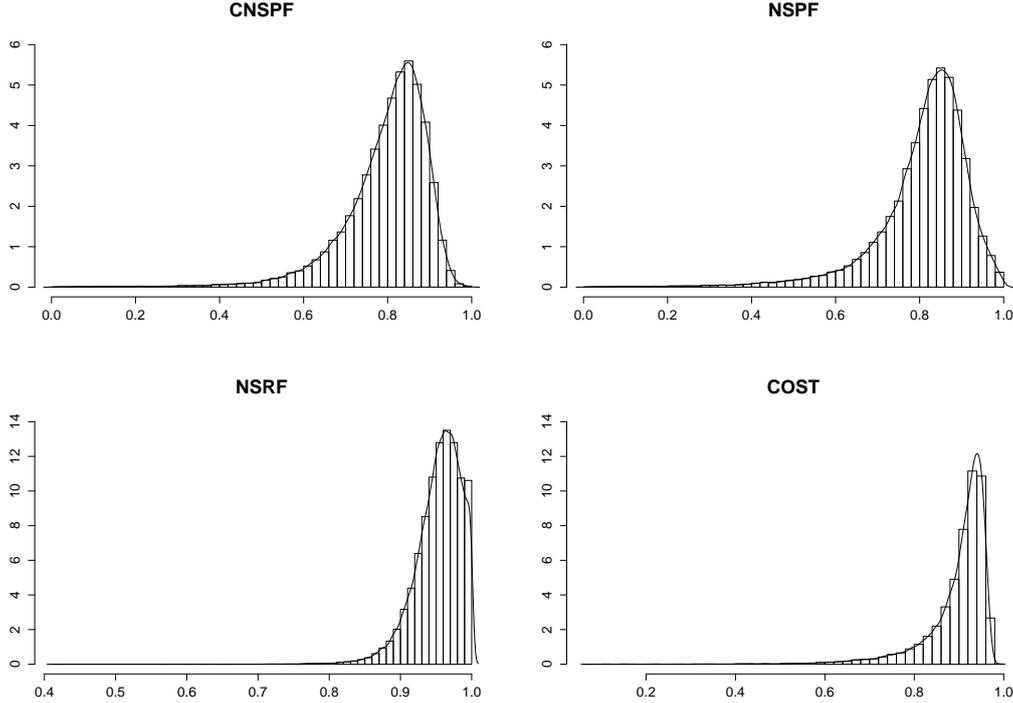


Figure 1: Histogram and density plots for efficiency measures (see definitions and computation in Table 2).

efficiency increased 1.55% from 2001 to 2010. We also find that CNSPF, NSPF, NSRF and cost efficiencies are persistent over time.

To examine how cost and revenue efficiencies contribute to our CNSPF efficiency measure, we report summary statistics for two additional measures of CNSPF efficiency in Table 2. We compute CNSPF using equation (13) but with the assumption that there is no revenue inefficiency ( $\eta = 1$ ). Likewise, we compute CNSPF assuming that there is no cost inefficiency ( $\theta = 1$ ). We find that if banks were fully revenue efficient, profit efficiency would increase, on average, by 8.3 percentage points from 79.5% to 87.8%. The gains for big, medium, and small banks would be 3.4, 7.5, and 10 percentage points, respectively. On the other hand, if banks were fully cost efficient, profit efficiency would increase, on average, by 9.7 percentage points from 79.5% to 89.2%. The gains for big, medium, and small banks would be 16.4, 9.8, and 9.6 percentage points, respectively. These findings suggest that cost inefficiencies weigh more than revenue inefficiencies in estimated profit efficiencies.

For completeness, we present summary statistics of the traditional measure of profit efficiency used in the literature. We also estimate the NSPF in (20) imposing the wrong assumption

that the NSPF is linear homogeneous in input prices. We label it as NSPFW. The results are similar to those without imposing the homogeneity restriction. However, they follow a different pattern over time (see Table 3) and correlate differently with cost efficiencies (see Table 4).

The top right plot of Figure 1 shows that the empirical distribution of NSPF efficiencies is similar to that of CNSPF efficiencies. The similarity between these two measures is surprising given that they were obtained using different approaches. Common statistical tests reject that both distributions are equal. Therefore, we further investigate if the small differences between our CNSPF and NSPF efficiency estimates can lead to economically significant differences that are relevant for empirical studies.

Quantile-quantile and empirical cumulative distribution plots (not reported) between NSPF and CNSPF efficiency estimates reveals that both distributions have similar quantiles except for those observations lying on the left tail of the distributions. Further, transition probability matrices (not reported) show that CNSPF and NSPF efficiencies categorized by size deciles are comparable. In summary, both efficiency estimates yield similar conclusions regarding the efficiency level of big, medium, and small banks. However, both measures rank banks differently.

Table 3: Mean Efficiency Measures

Year	C-NSPF	% $\Delta$	NSPF	% $\Delta$	NSRF	% $\Delta$	COST	% $\Delta$	NSPFW	% $\Delta$
2001	0.776		0.857		0.951		0.903		0.892	
2002	0.796	2.58	0.840	-1.98	0.953	0.21	0.887	-1.77	0.870	-2.47
2003	0.801	0.63	0.819	-2.50	0.955	0.21	0.876	-1.24	0.848	-2.53
2004	0.806	0.62	0.806	-1.59	0.956	0.10	0.873	-0.34	0.831	-2.00
2005	0.807	0.12	0.807	0.12	0.957	0.10	0.889	1.83	0.803	-3.37
2006	0.794	-1.61	0.809	0.25	0.957	0.00	0.904	1.69	0.772	-3.86
2007	0.776	-2.27	0.797	-1.48	0.955	-0.21	0.909	0.55	0.748	-3.11
2008	0.779	0.39	0.775	-2.76	0.952	-0.31	0.910	0.11	0.736	-1.60
2009	0.798	2.44	0.773	-0.26	0.949	-0.32	0.909	-0.11	0.760	3.26
2010	0.817	2.38	0.780	0.91	0.944	-0.53	0.917	0.88	0.786	3.42
<b>Total</b>	<b>0.795</b>	<b>5.280</b>	<b>0.808</b>	<b>-8.980</b>	<b>0.953</b>	<b>-0.740</b>	<b>0.896</b>	<b>1.550</b>	<b>0.807</b>	<b>-11.88</b>

Notes: This table shows annual means of the estimated efficiency measures and their annual percentage changes. NSPF stands for non standard profit function (traditional method), CNSPF correspond to Composite NSPF (our method) computed using equation (13). NSRF refers to efficiency estimates from the non standard revenue function and Cost to efficiency estimates using a cost function. NSRF efficiencies are computed as  $1/\eta$  and Cost efficiency as  $\theta$  (See equation (20)). CNSPF efficiencies are computed using equation (13) as  $\Pi/\Pi^*$ . No revenue inefficiency indicates that only cost inefficiency is taken into account. The data used for estimation include 63,120 bank-year observations for 8,483 banks with information for at least four years between 2001 and 2010. Estimated profit efficiencies are computed only from 62,577 observations for which profits are positive.

The rank correlation coefficient between CNSPF and NSPF efficiency estimates is only 0.4437 with a standard error of 0.003 (see Table 4). In addition, the correlation between these two measures and both NSRF and cost efficiency estimates differ markedly. For example, the rank correlations of CNSPF efficiency with NSRF and cost efficiency estimates are 0.505 and 0.352, respectively. In contrast, the rank correlation between NSPF efficiency and cost efficiency estimates is significantly negative (-0.293). Table 3 provides further evidence that CNSPF and NSPF efficiencies are different. Overall, CNSPF efficiency increased by 5.28% from 2001 to 2010. NSPF efficiency, in contrast, decreased by 8.98%. We think that these differences are economically relevant for empirical studies.

Given that both CNSPF and NSPF efficiencies may lead to different conclusions, we advocate using the CNSPF efficiency measure over the NSPF for two main reasons. First, the economic model used in the estimation of CNSPF efficiencies is correctly specified, as opposed to the one used to estimating NSPF efficiencies. Second, the CNSPF efficiency measure in (13) clearly identifies the underlying sources of profit efficiency, which is not possible from the NSPF. That is, one cannot identify the source of profit inefficiency from the estimated NSPF.

Equation (13) imposes constraints on the relation between cost, revenue, and cost efficiencies. In the NSPF approach, maximum profits ( $\pi^*$ ) equal maximum revenues,  $R(w,y)$ , minus minimum costs,  $C(w,y)$ . For given levels of cost and profit efficiency, revenue efficiency can be computed as  $R^a/R(w,y) = (\pi^a + C^a)/(\pi^* + C(w,y))$ . This relation helps in verifying the consistency of different measures of cost, revenue, and profit efficiency. By construction, our CNSPF efficiency measure always satisfies this relation. However, this is not the case for the traditional NSPF efficiency measure. To show this, we compute the implicit revenue efficiency values that are consistent with the estimated NSPF and cost efficiencies. That is, we use the profit efficiency measures derived from the NSPF, along with the cost efficiency measures derived from the cost function, and compute the implicit revenue efficiency measure. We find that implicit revenue efficiency measures are higher than 100% for 22% of the observations. This result implies that for some banks actual revenues are higher than the maximum possible revenues. This exercise allows us to conclude that the traditional NSPF approach gives implausible levels of revenue efficiency. In contrast, CNSPF efficiency measures are always well

Table 4: Spearman Rank Correlation Coefficients

	CNSPF	NSPF	NSRF	COST	NSPFW	RTS
<b>NSPF</b>	0.4437 (0.0036)					
<b>NSRF</b>	0.5050 (0.0035)	0.5196 (0.003)				
<b>COST</b>	0.3521 (0.0037)	-0.2930 (0.0038)	-0.3174 (0.0038)			
<b>NSPFW</b>	0.2694 (0.0038)	0.8629 (0.0016)	0.4210 (0.0036)	-0.4428 (0.0039)		
<b>RTS</b>	0.0374 (0.0040)	-0.1466 (0.0040)	-0.1252 (0.0040)	0.0438 (0.0040)	-0.1897 (0.0039)	
<b>TC</b>	-0.0761 (0.0040)	0.0903 (0.0040)	-0.1366 (0.0040)	-0.1261 (0.0039)	0.1947 (0.0039)	0.3309 (0.0038)

Notes: This table shows the sample Spearman rank correlations between different estimated efficiency measures, returns to scale (RTS) and technical change (TC). Standard errors are into parenthesis. All correlations are statistically different from zero at the 99% confidence level. NSPF stands for non standard profit function (traditional method), CNSPF correspond to Composite NSPF (our method) computed using equation (13). NSRF refers to efficiency estimates from the non standard revenue function and Cost to efficiency estimates using a cost function. NSRF efficiencies are computed as  $1/\eta$  and Cost efficiency as  $\theta$  (See equation (20)). NSPFW denote efficiency estimates imposing homogeneity restrictions on the NSPF. The data used for estimation include 63,120 bank-year observations for 8,483 banks with information for at least four years between 2001 and 2010. Estimated profit efficiencies are computed only from 62,577 observations for which profits are positive.

behaved by construction.

## 5.2. Relationship among Profit, Revenue, and Cost Efficiencies

We compute forgone rents as a fraction of banks' total equity due to inefficiencies. Table 5 presents the results. Forgone rents due to profit inefficiency amount to 8.2% of banks equity. The second and third panels in Table 5 shows that 3.5 percentage points of forgone rents are due to revenue inefficiency and 4.7 percentage points due to cost inefficiency. Based on average annual equity for all banks during the sample period, we estimate that forgone rents due to cost and revenue inefficiencies are about \$31.5 and \$23.5 billion, respectively. Therefore, contrary to previous studies, we find that the actual levels of banks' profitability are more affected by cost inefficiency than by revenue inefficiency.

Forgone rents for big, medium, and small banks are 7.1%, 7.7%, and 8.8%, respectively. If banks were fully cost efficient, forgone rents would decrease to 1.5%, 3.1%, and 4.2% for big, medium, and small banks. The corresponding values would be 5.6%, 4.6%, and 4.7% if banks were fully revenue efficient.

Table 5: Summary Statistics for Forgone Rents due to Inefficiency

Forgone rents due to:	mean	sd	Percentiles					
			5 <sup>th</sup>	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	95 <sup>th</sup>	
Revenue and cost inefficiencies								
All Banks	0.082	0.061	0.031	0.051	0.070	0.097	0.162	
Big Banks	0.071	0.073	0.020	0.036	0.052	0.081	0.178	
Medium Banks	0.077	0.060	0.030	0.049	0.066	0.090	0.152	
Small Banks	0.088	0.058	0.034	0.057	0.077	0.106	0.170	
Revenue inefficiency								
All Banks	0.035	0.025	0.003	0.017	0.031	0.047	0.080	
Big Banks	0.015	0.021	0.000	0.003	0.009	0.019	0.047	
Medium Banks	0.031	0.023	0.003	0.015	0.027	0.042	0.071	
Small Banks	0.042	0.026	0.008	0.025	0.039	0.055	0.089	
Cost inefficiency								
All Banks	0.047	0.058	0.011	0.021	0.033	0.054	0.120	
Big Banks	0.056	0.073	0.010	0.022	0.036	0.063	0.162	
Medium Banks	0.046	0.060	0.011	0.022	0.034	0.053	0.117	
Small Banks	0.047	0.054	0.011	0.021	0.033	0.054	0.120	

Notes: This table shows the average (mean), standard deviation (sd), and the 5, 25, 50, 75, and 95 percentiles of the estimated forgone rents as a fraction of banks' total equity due to inefficiencies for all, big (assets > \$1 billion), medium (\$0.1 < assets < \$1 billion), and small (assets < \$0.1 billion) banks. The data used for estimation include 63,120 bank-year observations for 8,483 banks. Forgone rents are computed only from 62,577 observations for which profits are positive.

Now, we estimate the effect of a 1% increase in revenue, cost, or both efficiencies on CNSPF efficiency. For each bank, we multiply its NSRF and cost efficiency scores by 1.01. Then, we compute the percentage change in CNSPF efficiency due to the change in NSRF efficiency using (16), the change in cost efficiency using (17), and changes in both NSRF and cost efficiencies using (15). Table 6 reports the results.

For all banks, a 1% increase in NSRF efficiency leads to an average increase of 2.64% in CNSPF efficiency. The same increase in cost efficiency leads to an average increase of 1.62% in CNSPF efficiency. Thus, for any given level of efficiency, increasing revenue efficiency has a greater impact on profit efficiency than increasing cost efficiency. This result is consistent with our general finding that, on average, banks are highly revenue efficient. Over the years, banks may have realized that improving revenue efficiency or avoiding revenue inefficiency has a greater impact in overall profitability than improving cost efficiency or eliminating cost inefficiency.

A simultaneous increase in both NSRF and cost efficiencies by 1% leads to an increase in CNSPF efficiency by 4.26%, on average. Small banks experience bigger gains than medium

banks, and medium banks have bigger gains than big banks. Considering a 1% decrease, instead, we find similar results.

Table 6:  $\Delta\%$  in CNSPF efficiency due to a 1% Change in NSRF and Cost Efficiencies

Due to 1% $\Delta$ in:	mean	sd	Percentiles				
			5 <sup>th</sup>	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	95 <sup>th</sup>
<b>All Banks</b>							
NSRF Efficiency	2.63679	5.84964	1.69016	1.98710	2.26739	2.65642	3.80156
Cost Efficiency	1.62059	5.79173	0.68332	0.97733	1.25485	1.64002	2.77382
Both	4.25738	11.64137	2.37348	2.96443	3.52224	4.29644	6.57537
<b>Big Banks</b>							
NSRF Efficiency	2.24624	0.89011	1.60804	1.87737	2.12209	2.45668	3.16247
Cost Efficiency	1.23390	0.88130	0.60202	0.86869	1.11098	1.44225	2.14106
Both	3.48015	1.77141	2.21006	2.74606	3.23307	3.89893	5.30353
<b>Medium Banks</b>							
NSRF Efficiency	2.60303	4.34329	1.71204	2.02260	2.30241	2.68706	3.80529
Cost Efficiency	1.58716	4.30028	0.70499	1.01247	1.28951	1.67035	2.77752
Both	4.19018	8.64357	2.41704	3.03507	3.59192	4.35741	6.58281
<b>Small Banks</b>							
NSRF Efficiency	2.72576	7.51504	1.68162	1.96570	2.24319	2.64215	3.89412
Cost Efficiency	1.70867	7.44063	0.67487	0.95614	1.23088	1.62589	2.86546
Both	4.43443	14.95567	2.35649	2.92185	3.47408	4.26805	6.75958

Notes: This table shows the average (mean), standard deviation (sd), and the 5, 25, 50, 75, and 95 percentiles of the percentage change in profit efficiency measures if revenue and cost efficiency estimates, or both, were shifted up by 1% for all, big (assets > \$1 billion), medium (\$0.1 < assets < \$1 billion), and small (assets < \$0.1 billion) banks. The data used for estimation include 63,120 bank-year observations for 8,483 banks. Forgone rents are computed only from 62,577 observations for which profits are positive.

We repeat the previous exercise but instead of computing the change in profit efficiencies, we compute the change in returns on assets (ROE). The results are reported in Table 7. We find that a 1% increase in revenue efficiencies would yield a 0.79 percentage points increase in ROE. That is, ROE will increase from an average of 10.91% to 11.70%. From a 1% increase in cost efficiencies, ROE will increase by 0.38 percentage points to 11.29%, on average. From a 1% increase in both revenue and cost efficiencies, ROE will increase by 1.17 percentage points to 12.08%, on average. This effect is the sum of the two separate effects of revenue and cost efficiencies on ROE.

Similar to the effects of cost and revenue efficiencies on profit efficiency, the effect of revenue efficiencies on ROE is stronger than the effect of cost efficiency on ROE. The gains in ROE from a 1% simultaneous increase in revenue and cost efficiencies are similar for big and medium banks, but smaller for small banks.

Table 7: Percentage Points Change in ROE due to a 1% Change in NSRF and Cost Efficiencies

	mean	sd	Percentiles				
			5 <sup>th</sup>	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	95 <sup>th</sup>
<b>ROE (%)</b>							
All banks	10.91051	25.92201	-6.08869	7.38383	12.46615	17.51077	26.17512
Big Banks	14.76771	20.26284	-7.27529	10.43905	17.26188	22.67688	32.28473
Medium Banks	11.75311	29.51609	-5.71353	8.64264	13.74273	18.57407	27.14227
Small Banks	9.43269	21.57637	-6.32208	6.17151	10.66172	15.20543	23.20761
<b>Due to 1%Δ in:</b>							
<b>All Banks</b>							
NSRF Efficiency	0.79324	0.29200	0.39350	0.60486	0.76898	0.94638	1.26778
Cost Efficiency	0.38288	0.14706	0.17122	0.27830	0.36831	0.46863	0.64719
Both & cost shift	1.17612	0.41706	0.57597	0.89319	1.14249	1.41161	1.88499
<b>Big Banks</b>							
NSRF Efficiency	0.84281	0.30576	0.38806	0.64713	0.81729	1.00167	1.36699
Cost Efficiency	0.38515	0.14902	0.16294	0.28386	0.37212	0.46873	0.66060
Both & cost shift	1.22795	0.42665	0.56690	0.95017	1.19502	1.46378	1.98574
<b>Medium Banks</b>							
NSRF Efficiency	0.81056	0.29860	0.42125	0.62983	0.78506	0.95408	1.27069
Cost Efficiency	0.39965	0.14704	0.18506	0.29700	0.38552	0.48370	0.66533
Both & cost shift	1.21021	0.41772	0.61839	0.93658	1.17830	1.43466	1.90973
<b>Small Banks</b>							
NSRF Efficiency	0.76661	0.27978	0.36962	0.57271	0.73848	0.92803	1.25085
Cost Efficiency	0.36285	0.14432	0.15989	0.25876	0.34676	0.44778	0.62249
Both & cost shift	1.12945	0.41037	0.54107	0.83897	1.09275	1.37137	1.84414

Notes: This table shows the average (mean), standard deviation (sd), and the 5, 25, 50, 75, and 95 percentiles of returns on equity (ROE) (first panel) for all, big (assets > \$1 billion), medium (\$0.1 < assets < \$1 billion), and small (assets < \$0.1 billion) banks. It also presents summary statistics of the increase in ROE when, revenue, cost, or both type of efficiency estimates are shifted up by 1%. The data used for estimation include 63,120 bank-year observations for 8,483 banks. Forgone rents are computed only from 62,577 observations for which profits are positive.

Now, we explore the effect of some bank characteristics (determinants of inefficiency) on revenue, cost, and profit efficiencies. We parameterize the unconditional variance of the inefficiency term for the NSPF, NSRF, and cost frontiers following Wang (2002) (see (23)). The estimated parameters are presented in Table 8. The sign of each parameter gives the direction of the marginal effects of the corresponding variable on the unconditional mean of inefficiencies. We compute the marginal effects using (24) for each inefficiency measure. Table 9 presents the results in elasticity form.<sup>16</sup>

Table 8: Parameter Estimates of the Variance of Inefficiency for NSPF, NSRF, and Cost Stochastic Frontiers

Variable	NSPF		NSRF		Cost Function	
	Parameter	Std. Err.	Parameter	Std. Err.	Parameter	Std. Err.
$t$	0.160	0.027	-0.238	0.053	0.281	0.071
$t^2$	-0.022	0.005	0.048	0.011	-0.062	0.018
NPL	12.26	0.589	11.80	1.283	15.95	1.437
Int. Ratio	5.624	0.231	5.462	0.573	-5.308	0.328
Offbal	-3.623	0.281	-31.75	1.631	6.620	0.353
RSL/Assets	-12.34	0.637	-5.824	0.601	3.955	0.414
Loans/Assets	-5.188	0.218	-8.174	0.5767	0.427	0.281
$\ln$ Assets	-0.206	0.009	-0.176	0.017	-0.171	0.015
Log Likelihood	-15,303.679		27,730.598		28,669.082	
Obs.	62,579		63,120		63,120	

Notes: This table shows the estimated parameters of the variance of inefficiency for the three translog stochastic frontiers: NSPF, NSRF, and Cost (see (20)). NPL: Non Performing Loans. Leverage equals total liabilities over total assets. Int. Ratio: Interest ratio equals total interest income from loans over total income. Offbal: off balance-sheet activities equals total non interest income over total income. REL: Real estate loans. Assets: Total assets.

The marginal effect of time on CNSPF inefficiencies shows an average decline of 3.2% per year from 2001 to 2010. NSRF and cost inefficiencies also decrease 1% and 0.2% per year, respectively. In contrast, NSPF inefficiencies increase on average by 0.5% per year.

As expected, NPL has a positive effect on inefficiencies. A 10% increase in NPL contributes to a 0.9% increase in CNSPF inefficiency, 0.57% increase in NSRF inefficiency, and 0.9% increase in cost inefficiency.

The marginal effect of interest ratio weighs heavily on NSRF and cost inefficiencies. A 10% change in interest ratio a yields 21% decrease in cost inefficiency and a 20% increase in NSRF inefficiency. These two effects offset each other to yield a slightly negative effect

<sup>16</sup>Although these marginal effects are observation-specific, we are reporting the mean values and their standard deviation to conserve space. Details are available from the authors upon request.

Table 9: Marginal Effects of Bank Characteristics on Inefficiency Estimates

Variable	Inefficiency Measures							
	CNSPF		NSPF		NSRF		COST	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
<b>Time</b>	-0.022	0.153	0.005	0.008	-0.010	0.008	-0.002	0.022
<b>NPL</b>	0.064	0.654	0.056	0.352	0.057	0.101	0.090	0.644
<b>Interest ratio</b>	-0.037	1.234	2.488	1.205	1.994	0.504	-2.101	1.003
<b>Off balance-sheet activities</b>	-0.080	1.983	-0.128	0.149	-0.954	1.054	0.268	1.651
<b>Real estate loans</b>	0.001	0.567	-0.630	0.551	-0.265	0.250	0.209	0.439
<b>Loans/Assets</b>	-1.117	0.916	-2.158	1.041	-2.807	0.728	0.161	0.116
<b>Assets</b>	-0.082	0.081	-0.129	0.052	-0.092	0.016	-0.098	0.078

Notes: This table shows the mean and standard deviation of the estimated marginal effects (in elasticity form except for time) on inefficiency of time and banks' characteristics: non performing loans (NPL), interest ratio (Interest and fee income on loans / Total income), off balance-sheet activities, real estate loans over total assets, total loans over assets, and banks' size (assets) on estimated inefficiencies. CNSPF: Composite NSPF, NSPF: non standard profit function, NSRF: non standard revenue function, COST: cost function. The variance of the inefficiency term is parametrized following [Caudill et al. \(1995\)](#), [Wang \(2002\)](#), and [Schmidt \(2011\)](#) as  $\exp(z_i'\delta)$  with  $z$  being a vector of time and banks' characteristics. The marginal effects on inefficiency of the associated variables are computed using (24). The marginal effects on CNSPF are computed using (19).

on CNSPF efficiency. Banks with relative high off-balance sheet activities tend to be more cost inefficient and less revenue inefficient. Overall, the marginal effects of off-balance sheet activities on revenue and cost inefficiencies offset each other and generate a small negative effect on CNSPF efficiency. Banks with a higher concentration on real estate loans are also less revenue inefficient and more cost inefficient. The overall effect on CNSPF efficiencies is small.

Banks with a large loan portfolio relative to total assets tend to be less revenue inefficient but more cost inefficient. A 10% change in loans to total assets ratio leads to a 28% (11.17%) reduction in revenue (CNSPF) inefficiencies. Finally, a 10% change in total assets would lead to a 0.10% decrease in CNSPF profit inefficiency.

Overall, the marginal effects on CNSPF inefficiency capture the joint dynamics of the marginal effects on revenue and cost inefficiencies. In contrast, the effects on NSPF inefficiency seem to be entirely due to the marginal effects on revenue inefficiencies. Since NSPF efficiencies are estimated using a misspecified model, NSPF efficiencies may confound the revenue and cost efficiency effects. More importantly, note that the estimated NSPF is likely to be biased because it is misspecified. Consequently, estimated NSPF inefficiencies are likely to be incorrect, and a comparison of NSPF and CNSPF inefficiencies might be wrong.

### 5.3. Homogeneity Restrictions on the NSPF

For estimation, researchers often impose the restriction that the NSPF is homogeneous of degree one in input prices (e.g. [Berger and Mester, 1997](#), [Altunbas, Evans, and Molyneux, 2001](#), [Clark and Siems, 2002](#), [Berger, Hasan, and Zhou, 2009](#), [Koetter et al., 2012](#)). In most cases, the authors state that their results are unaffected by such restrictions. [Humphrey and Pulley \(1997, p.81, 84\)](#) explicitly state that the NSPF is not linear homogeneous in input prices and [Kumbhakar \(2006\)](#) formally proves this point. In Section 2, we showed that the same results holds in our framework.

We find that imposing linear homogeneity in input prices is not innocuous. As shown in Table 4, the rank correlation between NSPF and cost efficiencies—labeled NSPF and COST, respectively, is  $-0.293$ . The rank correlation between NSPFW and cost efficiency is significantly lower ( $-0.4428$ ). The difference between them is statistically significant and might be economically significant in empirical studies. For example, studies usually take profit and cost efficiencies and draw conclusions depending on how they correlate with other variables. Imposing wrong homogeneity restrictions may lead to misleading results.

## 6. Conclusions

Research on bank's profit efficiency using the nonstandard profit function sprang after the publication of [Humphrey and Pulley \(1997\)](#). Since then, more than fifty published studies have contributed significantly to the current stock of knowledge regarding some aspects of the evolution of the banking industry over the last three decades. In this paper we show that the econometric model used in the literature to estimate [Humphrey and Pulley's](#) model is misspecified. Using the same theoretical framework, we propose an alternative method that solves this problem and allows researchers to study the contribution that both revenue and cost efficiencies have on overall banks' profit efficiency.

Our empirical results indicate that the measured levels of bank's profit efficiency are influenced more by cost inefficiencies than by revenue inefficiencies. We estimate the combined annual losses for all U.S. commercial banks due to cost and revenue inefficiencies at about \$31.5 and \$23.5 billion, respectively. We demonstrate theoretically and show empirically that

changes in revenue efficiency have a greater impact on profit efficiency than equivalent changes in cost efficiency; a result consistent with the measured high levels of revenue efficiency we present.

Our alternative method allows us to investigate the relation between profit, revenue, and cost efficiencies across banks and over time. We find that revenue and cost efficiencies tend to be negatively correlated, but both correlate positively with profit efficiency. As suggested by [Berger and Mester \(1997\)](#), the negative correlation between revenue and cost efficiencies may indicate that less cost efficient banks offset their related losses with higher revenue efficiency. Perhaps because less efficient banks incur higher costs in producing higher quality outputs that bring in higher revenues or because high revenue efficient banks exert less effort in controlling costs. In contrast to the results obtained using the misspecified econometric model, we find that profit and cost efficiencies tend to be positively correlated. Thus, the negative relation between cost and profit efficiencies reported in the literature (e.g., [Rogers 1998](#) and [Berger and Mester 1997](#)) stems from misspecification of the econometric model which, as we show, implicitly constraints profit and cost efficiencies to be negatively correlated.

Using U.S. commercial bank data from 2001 to 2010, we find that revenue and cost efficiency estimates are around 95% and 90%, respectively; leading to an overall profit efficiency level of about 80%. Big banks are more revenue efficient than medium, and small banks. However, average cost efficiency levels are similar across the bank size distribution, except for the very small banks. Profit inefficiency represents about 8.15% of bank equity of which 3.49% is due to revenue inefficiencies and 4.66% due to cost inefficiencies. For big, medium, and small banks average forgone rents are 7.1%, 7.7%, and 8.8% of banks' equity, respectively.

If banks were fully revenue efficient, profit efficiency would increase from 79.5% to 87.8%. If banks were fully cost efficient, profit efficiency would increase from 79.5% to 89.2%. A 1% increase in revenue efficiency would increase profit efficiencies by 2.64%. In comparison, the same increase in cost efficiencies would increase profit efficiencies by only 1.6%. Thus, for any given level of cost and revenue efficiencies, revenue efficiencies weigh more than cost efficiencies in overall profit efficiency.

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