Optimal Static Hedging of Price Risk in the Energy Market: Replicating Portfolio in a Robust Setting

Lilian Gonzalez Marulanda [1] and Javier Pantoja Robayo [2]

Abstract

This research explores risk managing in energy markets through the hedging of price risks considering different plain vanilla derivatives, using robust optimization. This proposal allows improving the hedging process through a robust methodology to find an optimal payoff without any assumption of underlying distributions, focusing on constructing a portfolio using only instruments available in the market. The numerical results show the proposed framework to measure the risks related to energy prices, establish an accurate, adequate and more realistic risk management for this market than previous proposals found in the literature.

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[1] School of Economics and Finance, Universidad EAFIT. Medellin, Colombia. E-mail: ligonzal@eafit.edu.co
[2] School of Economics and Finance, Universidad EAFIT. Medellin, Colombia. E-mail: jpantoja@eafit.edu.co

1 Introduction

Managing risk is a crucial feature in finance since it allows to improve the way enterprises and financial institutions face situations of vulnerability. For the energy market, the energy retailers (ER), who buy electricity at a variable wholesale price to supply an uncertain demand, face a high level of risk. The sources of those risks are due to the volatile nature of prices in electricity markets, where ER supply at a fixed regulated price on an inelastic demand or at a precommitment price for long-term contracts. This nature has led to the participants of this volatile and competitive market to look for certainty through hedging activities.

Energy Retailers or Electricity Suppliers conforms the last stage of electricity supply chain, they buy electricity from generators and sell it to consumers. Its existence is fundamental for the adequate provisioning of power and, ensuring the profitability of these activities allows to obtain a stable dynamics of the supply of electricity in this market. This paper explores risk managing for energy market through optimal hedging strategies approach using robust optimization. The hedging mechanism consists of transforming risk which is non-tradable, into tradable financial assets, transferring price risk exposition to capital markets. By taking into account the price information of the market, it is possible to quantify the risk inherent to spot energy price and allow managers to develop efficient hedging strategies to care for investments.

Hedging risk strategies for this kind of markets has been considering in recent literature. Regarding price and quantity risk, Näsäkkälä and Keppo (2005) extend the formulation of McKinnon (1967) to obtain the optimal hedge ratio, they develop a hedging strategy for quantity risk mitigation using forward contracts based on price. Oum and Oren (2009) extend the model of Näsäkkälä and Keppo (2005) to include vanilla derivatives in the hedging strategy. Other approaches use weather indexes, as Lee and Oren (2009) who extend Oum and Oren (2009) model, via the inclusion of forwards based on weather-linked indexes. Besides, Pantoja (2011) and Id Brik and Roncoroni (2015) consider the case of price and quantity hedging assuming independence between price and weather index. Later, Pantoja and Vera (2018) step aside from the assumption of independence, considering the existence of the correlation between price and weather, they achieve a discrete closed form solution for this general model.

This research is in line with hedging strategies established by Oum and Oren (2009) and Kleindorfer and Li (2005). They propose a strategy for hedging an ER who wants to maximize a Mean-Variance utility function of his net profit subject to a self-financing constraint. The hedging strategy consists of optimal derivative pay-offs written respectively on electricity price. The basis of this proposal is the Mean-variance utility function. Mean-variance Markowitz (1952) is the basis of the modern portfolio allocation strategies, leading the field of Decision Making in Financial Economics. In his seminal work Markowitz (1952) solves the problem of minimizing portfolio variance for a given level of expected return. Due to its simplicity, it is widely used as an approximation to more general expected utility functions (see, Morone et al. (2005)).

Considering a discrete setting, this research looks for hedging price risks strategies for ER by constructing a portfolio based on different plain vanilla derivatives. In this setting, the price and quantity take values on a discrete set. The method is based on Robust Optimization, which is an optimization model that takes into account the estimation parameter uncertainty. As the objective is to generate a model based on a set of assets traded in the market, the available data is discrete. Therefore, the model is developed in a discrete set which is more appropriate than the continuous one.

The contributions of this research are mainly two: First, it creates an optimal portfolio for the payoff of the hedge with financial instruments as forwards, put and call options. Second, this research bypasses the assumption about the knowledge of the marginal distribution of the aleatory variables by including directly the uncertainty related to these variables to the optimization process, this is done via robust optimization. In other words, this research obtains a hedge for energy retailers directly from the market data, in a first moment, assuming that quantity and prices follow a known probability measure called ψ and later, only making assumptions about the investment horizon of an Energy Retailer. Also, the model is simplified to a unique source of risk to be hedged, the spot price of electricity.

The structure of the paper is the following: Section 2 describes some preliminaries of risk hedging in electricity markets and the hedging model proposed, Section 3 shows the methodology proposed for the hedging portfolio problem, Section 4 presents the results of numerical evaluation and conclusions are presented in Section 5.

2 Preliminaries

2.1 Risk hedging in the Electricity market

Diversification is a main principle of modern portfolio theory, it implies positive properties for risk management. According to Markowitz (1952), diversification is a rule of investor behavior and implies a feedback between his desire to maximize expected return portfolio and minimize both the variance of the portfolio and the covariance between assets. This approach is called Mean-Variance efficiency, it offers a framework to rationalize the value of diversification (Michaud, 1989). Nowadays, the development of financial markets leaves asset allocation problem as a central topic of investment decisions.

Nonetheless Mean-Variance methodology is recognized as an important tool for asset allocation, it has some fundamental limitations because of the way it is conceived. As Michaud (1989) affirms, Mean Variance optimizers use the estimation of risk and returns as the fundamental inputs for optimization process, in this sense Mean Variance approach should be thought as estimation error maximizer. Besides, according to Michaud (1989), many of the results of this optimization process are unintuitive because they ignore other factors which are important to investment management as the liquidity or the capitalization of assets in the porfolio.

This research will develop a more accurate measure of the inputs which determine the hedging of expected profits of energy retailers and improving the asset allocation process to risk managing through a robust methodology. Therefore, the aim is to explore hedging of price risk through asset allocation approach using robust optimization, which permits to establish more accurate and adequate risk management for energy market. The robust approach incorporate the uncertainty about the parameters which allows to overcome the main limitation of Mean Variance framework.

Some authors follow the methodology proposed by Markowitz in their researches about energy markets. Näsäkkälä and Keppo (2005) and Woo et al. (2004), determined optimal hedging strategies through a Mean-Variance model, they analyzed the interaction between electricity prices and stochastic consumption volumes. Lee and Oren (2009), develop an equilibrium pricing model for weather derivatives in a multicommodity setting. They constructed a model in the context of a stylized economy where agents include weather derivatives to optimize their hedging portfolios, under mean-variance approach. Most literature address their hedging strategies using vanilla derivatives.

2.2 Hedging model

According to the structure of electricity markets, the ER's profit could be written as $y(p, q) = (r - p)q$. Thus, to serve the customers' demand q, the ER buys electricity at the spot price p and sells it at fixed retail price r at a fixed time T. The retail price r is fixed and known, while the demand q and the spot price p are random.

2.2.1 The nature of the payoff

In the model of Oum and Oren (2009) to hedge risks, the ER hedges the profit $y(p, q)$ using a payoff function $x(p)$, which is a function of the electricity spot price p at time T. Notice that in this section the attention is over payoff functions, and disregard the details on how to construct replicating portfolios composed by vanilla derivatives for the payoff in a discrete setting. In this section (2.2), it is presented the main conceptual framework about price risk hedging in electricity markets. Meanwhile, in section 3 it will be discussed the issues concerning the construction of the optimal portfolio for the hedge.

The hedged profit is then,

$$Y(p, q, x(p)) = (r - p)q + x(p) \tag{1}$$

Assume ER's preference utility is the Mean-Variance utility function $U(Y) = E^{\psi}[Y] - a\, Var^{\psi}[Y]$ of the hedged profit $Y = Y(p, q, x(p))$ at time T, where $a \geq 0$ is the coefficient of absolute risk aversion. The ER seeks to maximize its expected utility, under the constrain that the cost of constructing the financial portfolios with payoff $x(p)$ is zero, that is those portfolios are *self-financing.* Thus the following optimization problem is obtained:

$$\max_{x(p)} \quad E^{\psi}[y(p, q) + x(p)] - a\, Var^{\psi}[y(p, q) + x(p)] \tag{2}$$

$$st \quad E^{\phi}[x(p)] = 0$$

Where the probability measure ψ supported on $\{(p, q) : p \geq 0,\ q \geq 0\}$ represents the ER beliefs on the realization of p and q at time T, and ϕ is a risk neutral probability measure. Besides, $Var^{\psi}[Y]$ is equal to $E^{\psi}[Y^2] - E^{\psi}[Y]^2$. Notice that we do not assume ϕ is unique, since the electric power market is incomplete. The expectations $E^{\psi}[.]$ and $E^{\phi}[.]$ denote expectations under the probability measure ψ and ϕ, respectively.

Based on this framework, Oum and Oren (2009) obtained a closed form solution for the optimal payoff, they show the case where the probability measures ψ and ϕ are equal, implying that there is not a risk premia in the electricity market. The standard assumption in the existing literature is that ψ and ϕ are known (Oum et al. (2006), Oum and Oren (2009), Lee and Oren (2009), Pantoja (2011) and Id Brik and Roncoroni (2015)). In this research we do not use this assumption. In section 3.1 only ψ is assumed to be known, while the only information on ϕ is the available data from the market, that is the price of traded derivatives. Moreover, in section 3.2 the assumption about ψ is weakened, instead of assume ψ to be know, we assume that only a set of possible values for p and q are given, and use a robust optimization approach.

2.2.2 The replicating portfolio

The most frequently used framework in literature is the replication of the optimal payoff applying the model proposed by Carr and Madan (2001), they consider that a continuous twice differentiable function x(p) could be represented as follows:

$$\begin{aligned} x(p) = x(F)\cdot 1 - x'(F)(p-F) + \int_0^F x''(K)(K-p)^+ dK \\ + \int_F^\infty x''(K)(p-K)^+ dK \end{aligned} \tag{3}$$

where F is the forward price at time 0 and the terms 1, $(p-F)$, $(K-p)^+$ and $(p-K)^+$ are respectively the unit payoffs of a bond, forward, European put options and call options with strike price K. Besides, the terms $x(F)$, $x'(F)$ and $x''(K)dK$ correspond to the quantities of the instruments in the replicating portfolio $x(p)$, which is the same payoff because the replication is exact. This precision is possible because the instruments are conformed to have continuous strike prices. In discrete settings, there are some proposal for discrete strike prices like Oum et al. (2006), Su (2008) and Kirkby and Deng (2019).

3 Hedging through an optimal portfolio

Here the attention is focused on the issue of portfolio optimization. Portfolio optimization refers to combining a set of assets in a way that it has the adequate mix of assets to suit some required circumstances or investment aims. For the problem raised in this investigation, the assets correspond to plain vanilla derivatives and the individual conditions coincide with the development of a hedging.

In contrast with existing literature (Oum et al. (2006), Oum and Oren (2009), Lee and Oren (2009), Pantoja (2011) and Id Brik and Roncoroni (2015)), for empirical applications of hedging strategies, a limited number of instruments are available, and thus optimal payoff cannot be completely replicated by a portfolio. In this case, to hedge risks the ER hedges the profit $y(p, q)$ using a portfolio constructed by instruments written on electricity price. The hedged profit is then,

$$\begin{aligned} Y\left(p,\, q,\, u\right) &= \left(r-p\right)q + \sum_{j=1}^{J} X_j\left(p\right)u_j \\ &= \left(r-p\right)q + X\left(p\right)^T u \end{aligned}$$

Here $X\left(p\right)^T u$ is the payoff resulting of the optimal portfolio that could be conformed by bonds, forwards, put and call options, all of them written on spot electricity price; where,

$$X\left(p\right) = \begin{bmatrix} X_1\left(p\right) \\ X_2\left(p\right) \\ \vdots \\ X_J\left(p\right) \end{bmatrix} \qquad \text{and} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_J \end{bmatrix}$$

Where u represents the amount of each instrument that the investor needs to acquire to form the optimal portfolio to hedge his risk, and $X_j\left(p\right)$ is the payoff of instrument j given a *realized* electricity price p in the future.

3.1 Mean-Variance approach

Assuming the distribution ψ is known and using Mean-Variance as the objective function, the following optimization problem is obtained:

$$\begin{aligned} &\max_{u} \;\; \mathbb{E}^{\psi}\left[Y\left(p,\, q,\, u\right)\right] - a\mathrm{Var}^{\psi}\left[Y\left(p,\, q,\, u\right)\right] \\ &\text{s.t.} \;\; \sum_{j=1}^{J} u_j \pi_j = 0 \end{aligned} \tag{4}$$

where π_j is the market price for the corresponding instrument.

The aim is to describe a situation from the real world where the observed prices and quantities are naturally discrete. The proposed discrete setting is in line with the proposal of Pantoja and Vera (2018), and it consists of a discrete setting with n possible prices and m possible values for quantity. This is a set $P = \{p_i : i = 1, ..., n\}$ and a set $\mathcal{Q} = \{q_i : i = 1, ..., m\}$. Here, ψ is a discrete probability distribution supported on $P \times \mathcal{Q}$. In other words, we can think about p as one realization out of the possible $p_i \in P$ that could occur. Where P correspond to an ordered vector in $\mathbb{R}^n$.

Notice that the decision variable is a vector u indexed by j. This vector represents the amount of each instrument in the optimal hedging portfolio. Since we are uncertain about the price p to be realized, we have a discrete probability distribution associated with p, ψ_p. Then, the expected value of $X\left(p\right)$ is:

$$\mathbb{E}^{\psi}[X\left(p\right)] = \mathbb{E}^{\psi}\begin{bmatrix} X_1(p) \\ X_2(p) \\ \vdots \\ X_J(p) \end{bmatrix} = \begin{bmatrix} \mathbb{E}^{\psi}[X_1(p)] \\ \mathbb{E}^{\psi}[X_2(p)] \\ \vdots \\ \mathbb{E}^{\psi}[X_J(p)] \end{bmatrix} = V\left(p\right)$$

where the j^{th} element of $V\left(p\right)$, $\mathbb{E}^{\psi}[X_j(p)]$ is equal to $\sum_{i=1}^{n} X_j\left(p_i\right)\psi_{p_i}$. ψ_p is the marginal probability of ψ on P and $V\left(p\right)$ is the vector of financial instruments' expected payoffs. Thus, $V\left(p\right)$ represents the expected payoff function value at each $p \in P$ for each intrument j.

According to this, the expected value of $Y\left(p,\, q,\, u\right)$ would correspond to:

$$\begin{aligned} \mathbb{E}^{\psi}\left[Y\left(p,\, q,\, u\right)\right] &= \mathbb{E}^{\psi}\left[y\left(p,\, q\right) + X\left(p\right)^T u\right] \\ &= \mathbb{E}^{\psi}\left[y\left(p,\, q\right)\right] + \mathbb{E}^{\psi}\left[X\left(p\right)^T u\right] \\ &= \mu_y + \mathbb{E}^{\psi}\left[X\left(p\right)\right]^T u \\ &= \mu_y + V\left(p\right)^T u \end{aligned}$$

Now, the variance for this setting is:

$$\begin{aligned}\mathrm{Var}^{\psi}\left[Y\left(p,\,q,\,u\right)\right] &= \mathrm{Var}^{\psi}\left[y\left(p,\,q\right)+X\left(p\right)^{T}u\right]\\ &= \mathrm{Var}^{\psi}\left[y\left(p,\,q\right)\right]+\mathrm{Var}^{\psi}\left[X\left(p\right)^{T}u\right]\\ &\qquad + 2\mathrm{Cov}^{\psi}\left[y\left(p,\,q\right),X\left(p\right)^{T}u\right]\\ &= \sigma_y^2+u^T\left\{\mathbb{E}^{\psi}\left[X\left(p\right)\,X\left(p\right)^T\right]-\mathbb{E}^{\psi}\left[X\left(p\right)\right]\mathbb{E}^{\psi}\left[X\left(p\right)\right]^T\right\}u\\ &\qquad + 2\left\{\mathbb{E}^{\psi}\left[y\left(p,q\right)X\left(p\right)^T\right]-\mathbb{E}^{\psi}\left[y\left(p,q\right)\right]\mathbb{E}^{\psi}\left[X\left(p\right)\right]^T\right\}u\\ &= \sigma_y^2+u^TM(p)u+2M(y,p)^Tu\end{aligned}$$

Similarly to the expectations case, the j^{th} and h^{th} element of $M\left(p\right)$ is:

$$M\left(p\right)_{j,h}=\sum_{i=1}^{n}X_j\left(p_i\right)X_h\left(p_i\right)\psi_i^p-\sum_{i=1}^{n}X_j\left(p_i\right)\psi_i^p\sum_{i=1}^{n}X_h\left(p_i\right)\psi_i^p$$

In this case, the j^{th} element of $M\left(y,\,p\right)$ is:

$$M\left(y,\,p\right)_j=\sum_{i=1}^{n}\sum_{k=1}^{m}\left(r-p_i\right)q_kX_j\left(p_i\right)\psi_{i,\,k}^{p,\,q}-\sum_{i=1}^{n}\sum_{k=1}^{m}\left(r-p_i\right)q_k\psi_{i,\,k}^{p,\,q}\sum_{i=1}^{n}X_j\left(p_i\right)\psi_i^p$$

Then, the problem in equation 4 becomes:

$$\begin{aligned}\max_{u}\;\; & \mu_y-a\sigma_y^2+\left(V(p)-2aM\left(y,p\right)\right)^Tu-a\;u^TM\left(p\right)u\\ \text{s.t.}\;\; & \sum u_i\pi_i=0\end{aligned}$$

where u represents the amount of each derivative instrument in the optimal portfolio, $V(p)$ is the vector of financial instruments' expected payoffs, $M\left(y,p\right)$ is the vector of the covariance between the unhedged income and the payoff of each instrument, $M\left(p\right)$ is the variance and covariance matrix among all instruments in the portfolio. Finally, π is the vector of prices of each derivative.

Let $\mu_y=\mathbb{E}^{\psi}[y(p,q)]$ the expected value of $y(p,q)$ under ψ, and $\sigma_y^2=\mathrm{Var}^{\psi}[y(p,q)]$.

Model (3.1) is equivalent then to

$$\begin{aligned}\mu_y-a\sigma_y^2+\max_{u}\;\; & \left(V(p)-2aM\left(y,p\right)\right)^Tu-a\;u^TM\left(p\right)u\\ \text{s.t.}\;\; & u^Tp=0\end{aligned}\tag{5}$$

The matrix M is positive semidefinite and for $a > 0$ model (5) is a Quadratic Optimization Problem (QOP), which is efficiently solved through an optimization process. According to Mansini et al. (2014), a Mean-Variance model can be readily resolved through a quadratic programming method.

3.2 Robust optimization approach

The existing literature about static hedging in electricity market has assumed the knowledge of the marginal density functions of the expected value of the ER's utility and the expected value of the payoff of the priced hedging instruments. This research avoids this assumption by including the uncertainty related to electricity spot price to the optimization process, through a robust optimization approach.

The robust portfolio asset allocation incorporates data uncertainty to the optimization process, to obtain an optimal portfolio which is good for all possible realizations of the uncertain parameters (Cornuejols and Tütüncü, 2006). Tütüncü and Koenig (2004) and Scutellà and Recchia (2013) propose a robust approach to mean-variance portfolio optimization commonly used in research about robust asset allocation.

The start point is an uncertainty set $\mathcal{S}$ which contains different possible values of the uncertain parameters. For instance, consider an uncertainty set corresponding to spot price p:

$$\mathcal{S} = \left\{p : p^L \leq p \leq p^U\right\},$$

where extreme values for intervals of p can be extracted from historical data, analyst's opinions, confidence intervals, among others sources.

It is possible to formulate an optimization problem which looks for an optimal hedged through an optimal portfolio, for the values of the uncertainty sets, it is the maximum for the feasible hedge. The corresponding model would be:

$$\max_{u} \left\{ \min_{p \in \mathcal{S}} \mathcal{Y}(p, q, u) \right\} \tag{6}$$

given $\mathcal{S}$ some uncertainty set.

Specifically, the model to be optimized is:

$$\begin{array}{cc} \max\limits_{u} & \min_{p\in\mathcal{S}} \mathcal{Y}(p,q,u) \\ st & \pi_i^T u = 0 \end{array} \tag{7}$$

where $\mathcal{S}$ is a given uncertainty set.
We consider an interval as uncertainty set on the price p,

$$\mathcal{S} = \left\{p : p^L \leq p \leq p^U\right\},$$

$\mathcal{Y}(p,q,u)$ for u and q fixed, is pice-wise linear (in p) with breaking points $p^L < K_1 < K_2 < \ldots < K_i < p^U$, where K_i are the strikes. The minimum of a pice-wise linear function is attained at a breaking point. Thus, the inner problem reduces to:

$$\begin{array}{cc} \max\limits_{u} & \min_{p\in\mathcal{S}} \left\{\mathcal{Y}(p^L,q,u), \mathcal{Y}(K_1,q,u), \mathcal{Y}(K_2,q,u), \cdots, \mathcal{Y}(K_i,q,u), \mathcal{Y}(p^U,q,u)\right\} \\ st & \pi_i^T u = 0 \end{array} \tag{8}$$

Thus, the robust optimization model becomes,

$$\begin{array}{cl} \max\limits_{t} & t \\ st & \mathcal{Y}(p^L,q,u) \geq t \\ & \mathcal{Y}(K_i,q,u) \geq t \quad i = 1, ..., J \\ & \mathcal{Y}(p^U,q,u) \geq t \\ & \pi_i^T u = 0 \end{array} \tag{9}$$

4 Numerical Evaluation

4.1 Using synthetic data to compare the 3 models.

The objective is to compare the three models described in previous sections. The used distributions and data parameters describe the situation of a hypothetical energy retailer following Oum et al. (2006). The comparison will be made between Model 2, Model 5 and Model 9. Notice that to apply Model 2, it is necessary to know the distribution ϕ and ψ, while to apply 5 is necessary the distribution ψ and the vector of prices π. Finally, for Model 9 only the vector of prices π is necessary.

The first step is to find an optimal hedged profit through an optimal portfolio which represents the result of Model 5 and compare it with an optimal hedged profit through a replicated portfolio, corresponding to the framework proposed in traditional literature (Model 2). The main difference between both frameworks is that Model 5 optimizes directly over the portfolio and Model 2 does it over a payoff to be replicated. Therefore, models 5 and 2 are estimated using discretized distributions and data parameters. As in Pantoja and Vera (2018), we use a discretized version of the distribution ψ and ϕ from Oum et al. (2006), with the following assumptions:

- Price follows a lognormal distribution with a mean of 4 and a standard deviation of 0.7 in real and risk-neutral probabilities. That would be represented as $log\ p \sim N^{\psi}(4, 0.7^2)$ and $log\ p \sim N^{\phi}(4, 0.7^2)$.
- Load follows a lognormal distribution with a mean of 7.99 and a standard deviation of 0.2: $log\ p \sim N^{\psi}(7.99, 0.2^2)$.
- Energy Retailer faces a regulated demand who pay a fix price of $r = \$120$ per MWh for their consumption of electricity.

The financial instruments in the portfolio will be: a bond, a forward and put options with i strikes. The payoff vector is then,

$$X\left(p\right) = \begin{bmatrix} 1+r \\ p - F_1 \\ (K_1 - p)^+ \\ \vdots \\ (K_i - p)^+ \end{bmatrix}$$

In this setting, we take a sample of 1 strike that is equal to the minimum price plus 0.1 times the difference between the range of the maximum less the minimum price. On the other hand, we compute the prices of each instrument (π_i) by calculating the expected value of the payoff under risk-neutral probabilities in order to keep the three approaches comparable. To compute the replication portfolio u^*, it is necesssary to solve the following optimization problem for Model 2:

$$\begin{aligned} \min_{u^*} \quad & X(p)^T u^* - x(p) \qquad (10) \\ st \quad & E^{\phi}[x(p)] = 0 \end{aligned}$$

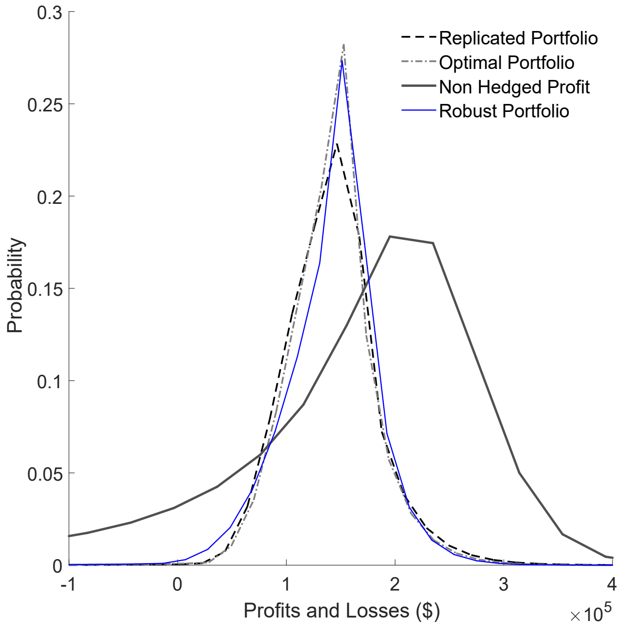
Now, to include Model 9 it is necessary to set up the uncertainty set for the electricity prices. Following the methodology of Tütüncü and Koenig (2004), who use box uncertainty sets in robust asset allocation problems. Their algorithm considers a box uncetarinty set which compute different quantiles of the extreme values of the box p^L and p^U. The bounds of their uncertainty sets are the 5 and 95 percentiles ot the given data for the uncertain variable. For this application, we construct p^L and p^U from a confidence interval for p based on a sample from ψ, taking the 5 and 95 percentiles of the simulated prices.

Again, notice the difference in assumptions: while Model 2 assumes perfect knowledge of ψ and ϕ, Model 5 assumes only a discrete approximation to ψ and Model 9 only assumes an interval for the possible values of p.

4.2 Synthetic example

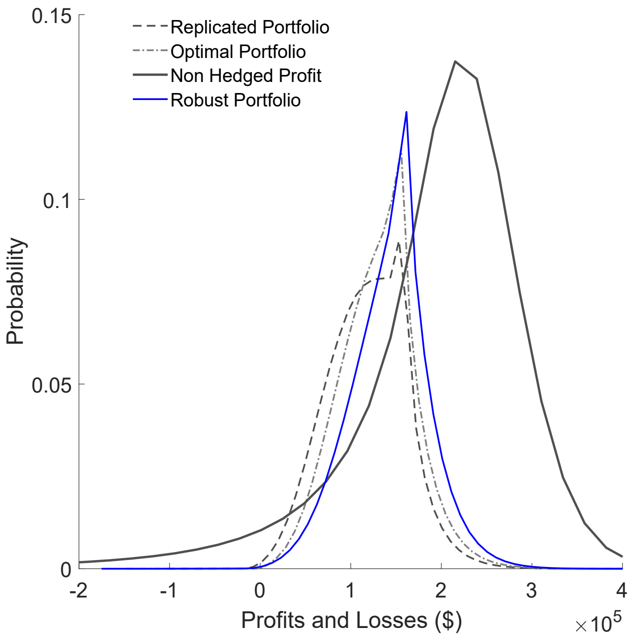
Figure 1 shows the main results regarding the comparison of the unhedged and hedged profit distributions for the replicated payoff (Model 2) and the optimal portfolios (Model 5 and Model 9). The results exhibit that three strategies to obtain optimal hedged profits can mitigate the risk of losses. However, the approach through the optimal payoff shows a better performance concerning a higher level of profits given the price of electricity.

Figure 1: Unhedged and Hedged Profit Distributions for Training Data.



Nonetheless, in the previous approach it is assumed that both distributions are known. To do a fairer comparison we will compare the performance of the found portfolios over a new set of data. For this new numerical results, we will use a price which follows a lognormal distribution with a mean of 3.8 and a standard deviation of 0.6 in real probabilities, risk-neutral probabilities considers a mean of 4.1 and the same standard deviation of 0.6. That would be represented as $log\ p \sim N^{\psi}(3.8, 0.6^2)$ and $log\ p \sim N^{\phi}(4.1, 0.6^2)$. Figure 2 shows the results for the comparison of the unhedged and hedged profit distributions for different portfolios for the evaluation data.

Figure 2: Unhedged and Hedged Profit Distributions for Evaluation Data.



In this case, the three different approaches to hedge the profit of an energy retailer reaches to mitigate the losses, making the expected return much more certain. However, the robust portfolio is better for hedging than the optimal discrete portfolio and the replicated one. The main reason for this result is the availability of information for each optimization process. The replicated and optimal portfolios are established over the complete information of the variables behavior, reaching the best results for that data. Meanwhile, the optimal portfolio approach has fewer assumptions making it a more general solution when we do not know the exact behavior of the uncertain variables.

Now, to make an additional evaluation about the performance of portfolios resulting from Model 2, Model 5 and Model 9, it is necessary to make a comparison of the unhedged and hedged profit for an energy retailer using the previously proposed evaluation sample of prices. For this comparison, we use a set of different electricity prices from \$ 80 to \$ 150 per MWh. The fix retail price r is equal to \$ 120 per MWh.

Figure 3: Performance of different portfolios.

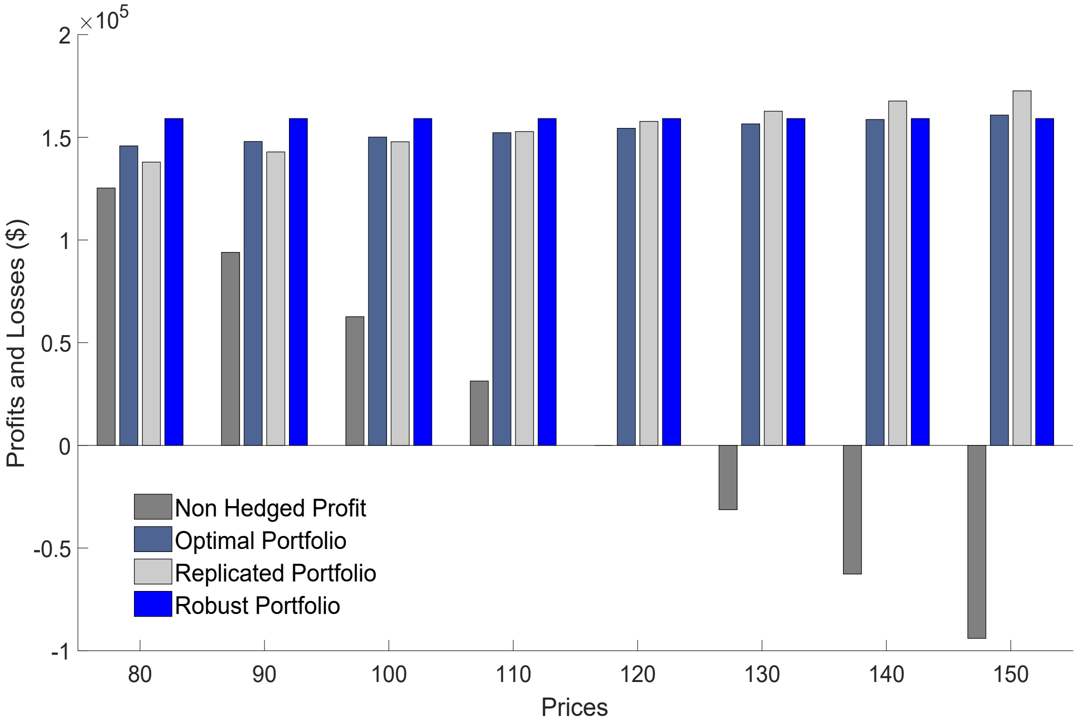


Figure 3 shows the results of the performance of optimal portfolio for this new sample of electricity prices. The three methodologies present great improvement respect to the unhedged profit result, nonetheless robust portfolio shows a better and more stable hedged profit performance than the other methodologies.

Previous proposals found in the literature and traditional portfolio optimization techniques are settled over different assumptions about the knowledge of the variables behavior that are not available in the real world. Particularly, the assumptions about the distributions of the uncertain variables and the existence of an optimal hedge for a Mean-Variance approach. Nonetheless, Mean-Variance optimization techniques have some fundamental limitations because of the way it is conceived.

As Michaud (1989) affirms, Mean-Variance optimizers use the estimation of risk and returns as the fundamental inputs for the optimization process, in this sense Mean-Variance approach should be thought as estimation error maximizer. Whereas, the robust optimization technique proposed in this article accomplish a more adequate risk managing, just considering an uncertainty set that contains all possible values of the uncertain prices of electricity. These results show that the proposed framework to measure the risks related to energy prices, establish a more accurate and realistic risk management for this market than previous proposals found in the literature.

5 Conclusions

Energy markets represent a challenge in terms of risk management. Agents who trade in those markets face an important vulnerability in terms of prices, demand, and availability of electricity. Specifically, the development of optimal profit hedging is a main financial activity for Energy Retailers, who are exposed directly to prices volatility and changes in demand, since the selling price is regulated. Hedging mechanism consists of transforming non-tradable risk into financial assets, which are tradable, transferring risk exposition to capital markets.

This research looks for hedging price risks strategies for ER by constructing a portfolio based on different plain vanilla derivatives. The method is based on Robust Optimization, which is an optimization model that takes into account the estimation parameter uncertainty. As the objective is to generate a model based on a set of assets traded in the market, the available data is discrete in nature. Then the model is developed in a discrete set which is more appropriate than a continuous one.

The results showed the proposed framework to measure risks related to energy prices establish a more accurate and adequate risk management for this market. Previous proposals found in the literature and traditional portfolio optimization techniques are settled over different assumptions about the knowledge of the variables behavior that are not available in the real world. Particularly, the assumptions about the distributions of the uncertain variables and the existence of an optimal hedge for a Mean-Variance approach. Nonetheless, Mean-Variance optimization techniques have some fundamental limitations because of the way it is conceived.

The robust optimization technique proposed in this article accomplishes a more accurate and realistic optimal portfolio, just considering an uncertainty set that contains all possible values of the uncertain prices of electricity. These findings are based on two issues: First of all, it introduces a framework to find an optimal self-financing portfolio to hedge the profits of Energy Retailers with instruments directly from the market as forwards, put options and bonds. Second, this research does not make any assumption about the marginal distribution functions for the aleatory variables by including directly the uncertainty related to these variables to the optimization process, this kind of approach is called robust optimization.

In other words, this research obtained a hedge for energy retailers directly from the market data, in a first moment, assuming that quantity and prices follow a known probability measure called ψ and later, only making assumptions about the uncertainty set of the electricity prices. Future research should include additional uncertainty sets to extend the analysis of robust optimization in asset allocation to hedge this market.

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